BENG280A, Principles of Biomedical Imaging 10/5/03
Fall Quarter 2004

HOMEWORK #2 (Corrected version)
Due by 4 p.m.,Wednesday 10/13/04
(Place in Anna's mailbox in Graduate Student Lounge of BENG bldg)

Homework Policy: Homeworks turned in late will be marked down 20% for every day that they are late. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA or instructor know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material.

Readings:
1. Review chapter 2 in Suetens.

Problems:

1. Sketch the function $m(x) = \left(\delta(x) + \frac{2}{3}\left(\delta(x-6) + \delta(x+6)\right) + \frac{1}{3}\left(\delta(x-3) + \delta(x+3)\right)\right) * \text{sinc}(x)$.
   Also find and sketch the 1D Fourier transform (you may use MATLAB to help with the plotting).

2. Find the 1D Fourier transform of $m(x) = \exp(-x^2)\delta(x)\cos(2\pi\sqrt{x})$.

3. Find and graph the 2D Fourier Transform of $g(x,y) = \exp(-j2\pi(3x + 4y))\cos(2\pi 5x)$. Hint: the function is separable.

4. Let $h(x)$ and $H(k_x)$ be a Fourier transform pair.
   (a) Show that the area of $H(k_x)$ is given by $\int_{-\infty}^{\infty} H(k_x)dk_x = h(0)$.
   (b) Let $H(k_x) = \frac{1}{2}\left(1 + \cos(2\pi k_x / W)\right)rect(k_x / W)$. Find $h(x)$ and also the area of $H(k_x)$.

5. Consider the 2D object $m(x,y) = \sin c(x)\sin c(y)\left[\cos(8\pi x) + \cos(16\pi y)\right]$
   (a) Derive and sketch the 2D Fourier transform of *m(x,y)*.
   (b) Define $g(x,y) = m(x,y) ** \sin c(4x)\delta(y)$. Sketch and give a simple expression for *g(x,y)*.
   (c) Define $h(x,y) = \left[m(x,y)\cos(8\pi x)\right] ** \left[\sin c(2x)\sin c(2y)\right]$. Sketch and give a simple expression for *h(x,y)*.

MATLAB exercise begins on next page.

MATLAB Exercise: This week's exercise uses the same image set as was used in Homework 1. The goal is to develop familiarity with relationship between 2D image space and 2D Fourier space, and to see for yourself how some of the images shown in lecture were derived.

Steps:

1. First download the file BE280Ahw1im.mat from the course website.
2. Load the image into MATLAB with the command: *load BENG280Ahw1im.*

3. Compute the 2D Fourier transform of the image with the command *Mf = fft2(Mimage);* where the 2D transform will now be stored in the variable *Mf*. Remember to add the semi-colon at the end of the command, otherwise MATLAB will display all the numbers in the matrix! The command *fft2* puts the zero-frequency value of the transform at the first indices of the matrix. For display it's convenient to put the zero-frequency value in the center of the matrix. To do this, type *Mf = fftshift(Mf);*

4. ***Resolution***. What happens when we zero out the outer regions of the Fourier transform?

(a) Resolution reduction in the x-direction.

```
>> res_span = 129+(-16:16);
>> Mf2 = zeros(256,256);
>> Mf2(:,res_span) = Mf(:,res_span);
>> Mf2 = fftshift(Mf2);
>> M_resx = ifft2(Mf2);
>> imagesc(abs(M_resx)); % This will show reduction of resolution in the x-direction.
```

(b) Demonstrate resolution reduction in the y-direction**. Hand in code and image.**

(c) Demonstrate resolution reduction in the x and y directions. **Hand in code and image**

5. ***Missing data in k-space***. We can also zero out the inner regions of the Fourier Transform.

```
>>Mfzero = Mf;
>>Mfzero(ky,kx) = 0;
>>iMFzero= ifft2(fftshift(Mfzero)); % look at resulting image.
```

Zero out the following:

(a) kx = 129; ky = 129

(b) kx= 1:256; ky = 129+(-16:16);

(c) kx = 129+(-16:16); ky = 1:256;

(d) kx = 129+(-16:16); ky = 129 + (-16:16);

(e) kx = 1:2:256;ky = 1:256;

For each set of parameters, plot out the Fourier transform and the resulting image. Give a qualitative explanation of why the image looks the way it does.

6. ***Spikes in the data***. You can put a spike at location (kx,ky) in Fourier space with the following commands

```
>>Mfspike = Mf;
>>spike = 100e6;
>>Mfspike(ky,kx) = Mf(ky,kx) + spike; % add spike
>>iMFspike = ifft2(fftshift(Mfspike)); % look at resulting image.
```

Put spikes at:

(a) kx = 129; ky = 129 – note this point corresponds to the center of k-space (i.e. kx = 0, ky = 0 in terms of the coordinates we used in class).

(b) kx= 161; ky = 129

(c) kx = 161; ky = 161

For each spike location, plot out the Fourier transform and the resulting image. Give a qualitative explanation for why the image looks the way it does. For example, what accounts for the direction of the artifacts?