BENG280A, Principles of Biomedical Imaging 11/3/04
Fall Quarter 2004

HOMEWORK #5 (Corrected 11/9/04)

Due Wednesday 11/10/04

Readings:
1. Read Suetens chapters 4 and 5. .

Problem 1

Define the object $m = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 2 & 6 \\ 7 & 2 & 9 \end{bmatrix}$.

(a) Consider the following 12 projections: 3 horizontal projections, 3 vertical projections, 3 left-to-right diagonal projections (involving at least two numbers), and 3 right-to-left diagonal projections (involving at least two numbers). Write down the matrix form of these projections. Use MATLAB to solve for the object from its projections. Hint: If $\mathbf{y} = \mathbf{Ab}$ and $\mathbf{A}$ is rectangular with more rows than columns, the least squares estimate of $\mathbf{b}$ is $\hat{\mathbf{b}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{y}$.

(b) Since there are only 9 numbers in the object, we only need 9 projections. Find 9 independent projections. Write down the matrix form and use MATLAB to solve for the object from its 9 projections.

(c) Using the 12 projections from part (a), derive the backprojection (unfiltered) estimate of the object. Compare this to the original object. You may find it useful to use MATLAB here.

(d) Using the 12 projections from part (a), compute the estimate of the object after one iteration of the algebraic reconstruction technique (ART). Hint: Follow the example shown in class. First go through all the horizontal projections, followed by the vertical projections, etc. Compare this to the original object. You may find it useful to use MATLAB here.

Problem 2

(a) Define the object $m(x,y) = rect(x)rect(y/2)$. Sketch the sinogram for this object. Give explicit expressions for the projections at 0, 90 degrees, and 26.5651 degrees (e.g. the projection that goes through the diagonals of the rectangle). For other angles, either a sketch or an explicit expression is okay.

(b) Now define $m(x,y) = rect(x/2)rect(y/2)$ Show that the projection theorem holds for theta = 45 degrees. In other words, derive the 2D Fourier transform and the projection at theta = 45 degrees and show how these are related.

Problem 3

Dr. M and Dr. C are having a discussion about whether MRI or CT is better. Dr. M claims that MRI is better because you can reconstruct the object with both better resolution and fewer data samples than CT. Dr. C argues that CT is better. Which one is correct?

(a) Which modality requires fewer data samples? Consider a 2D object with an FOV of 30 cm by 30 cm and a resolution of 1 mm by 1 mm.

(b) For the number of data samples derived in (a), which modality provides better resolution? Hint: look at the coverage of k-space.

Problem 4

Derive the convolution kernel $q(r)$ for the both the Hamming and Hanning windowed ramp filters shown in Figure 5.12. The windows are defined in Eqn. 5.40 of the text. Also, derive the effective widths of these kernels and indicate which one provides better resolution.