Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2004
MRI Lecture 1

Today's Topics

- The concept of spin
- Precession of magnetic spin
- Relaxation
- Bloch Equation

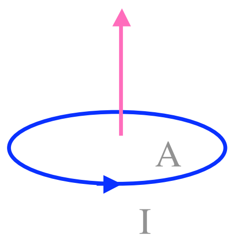
Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

The History of Spin

- 1921 Stern and Gerlach observed quantization of magnetic moments of silver atoms
- 1925 Uhlenbeck and Goudsmit introduce the concept of spin for electrons.
- 1933 Stern and Gerlach measure the effect of nuclear spin.
- 1937 Rabi predicts and observes nuclear magnetic resonance.

Classical Magnetic Moment



$$\vec{\mu} = IA\hat{n}$$

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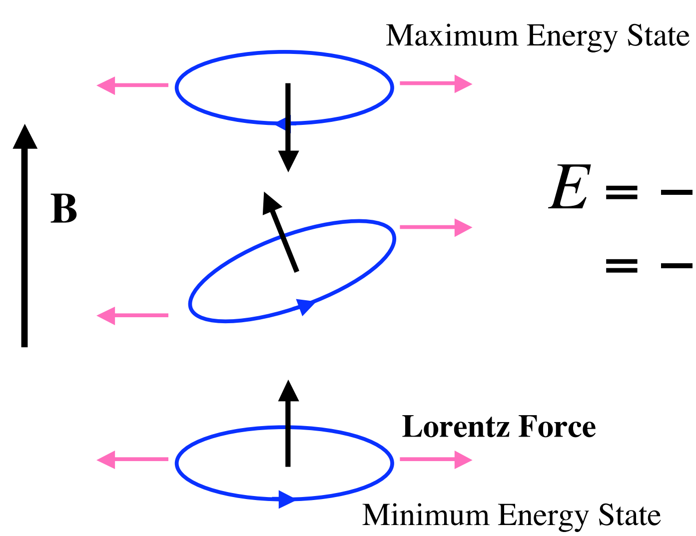
Energy in a Magnetic Field



$$E = -\vec{\mu} \bullet \mathbf{B}$$
$$= -\mu_z B$$

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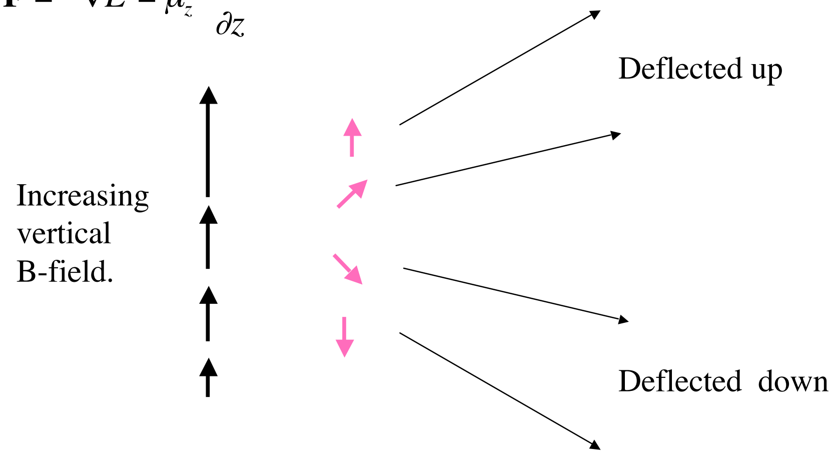
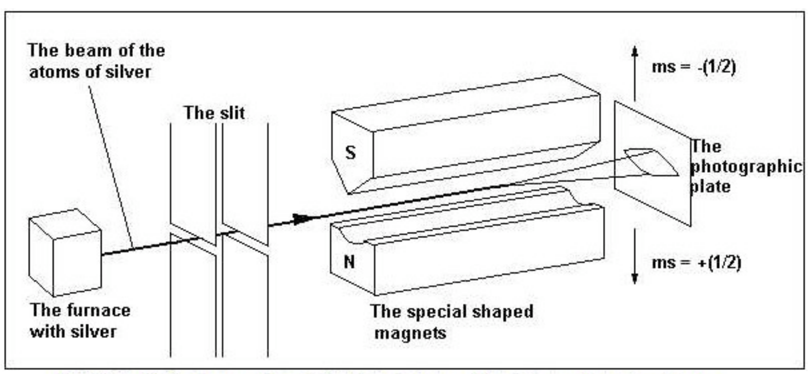
Force in a Field Gradient

$$\mathbf{F} = -\nabla E = \mu_z \frac{\partial B_z}{\partial z}$$



Stern-Gerlach Experiment



The Stern-Gerlach experiment. On the photographic plate are two clear tracks.

Image from http://library.thinkquest.org/19662/high/eng/exp-stern-gerlach.html?tqskip=1

Stern-Gerlach Experiment

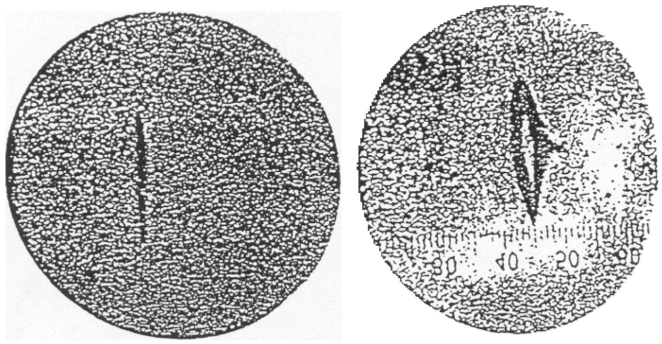


Image from http://library.thinkquest.org/19662/high/eng/exp-stern-gerlach.html?tqskip=1

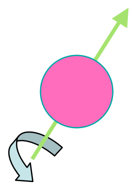
Quantization of Magnetic Moment

The key finding of the Stern-Gerlach experiment is that the magnetic moment is quantized. That is, it can only take on discrete values.

In the experiment, the finding was that

$$\mu_z = +\mu_0 \text{ OR } -\mu_0$$

Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where γ is the gyromagnetic ratio and $\mathbf{S}$ is the spin angular momentum.

Quantization of Angular Momentum

Because the magnetic moment is quantized, so is the angular momentum.

In particular, the z-component of the angular momentum Is quantized as follows:

$$S_z = m_s \hbar$$

$$m_s \in \{-s, -(s-1), \ldots s\}$$

s is an integer or half intege

Nuclear Spin Rules

Number of Protons	Number of Neutrons	Spin	Examples
Even	Even	0	^{12}C, ^{16}O
Even	Odd	j/2	^{17}O
Odd	Even	j/2	^{1}H, ^{23}Na, ^{31}P
Odd	Odd	j	^{2}H

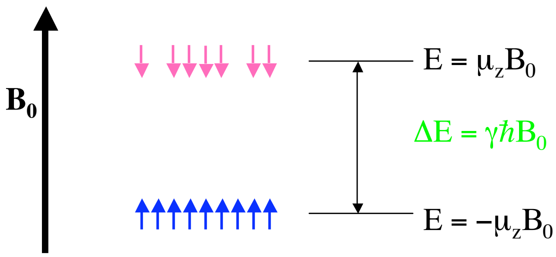
Hydrogen Proton

Spin 1/2

$$S_z = \begin{cases} +\hbar/2 \\ -\hbar/2 \end{cases}$$

$$\mu_z = \begin{cases} +\gamma\hbar/2 \\ -\gamma\hbar/2 \end{cases}$$

Boltzmann Distribution



$$\frac{\text{Number Spins Up}}{\text{Number Spins Down}} = \exp(-\Delta E/kT)$$

Ratio = 0.999990 at 1.5T !!!

Corresponds to an excess of about 10 up spins per million

Equilibrium Magnetization

$$\begin{aligned}
\mathbf{M}_0 &= N\langle \mu_z \rangle = N\left(\frac{n_{up}(-\mu_z) + n_{down}(\mu_z)}{N}\right) \\
&= N\mu \frac{e^{\mu_z B/kT} - e^{-\mu_z B/kT}}{e^{\mu_z B/kT} + e^{-\mu_z B/kT}} \\
&\approx N\mu_z^2 B/(kT) \\
&= N\gamma^2 \hbar^2 B/(4kT)
\end{aligned}$$

N = number of nuclear spins per unit volume

Magnetization is proportional to applied field.

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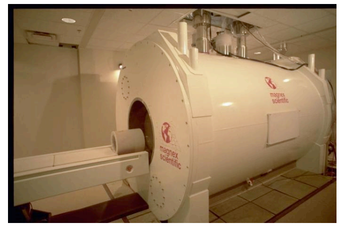
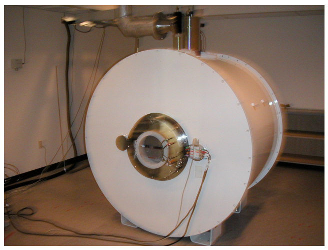
Bigger is better


3T Human imager at UCSD.


7T Human imager at U. Minn.


7T Rodent Imager at UCSD

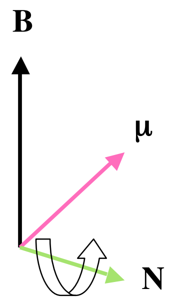

9.4T Human imager at UIC

Gyromagnetic Ratios

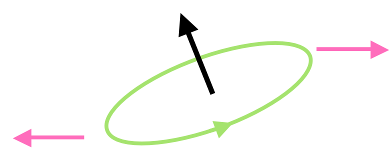
Nucleus	Spin	Magnetic Moment	$\gamma/(2\pi)$ (MHz/Tesla)	Abundance
^{1}H	1/2	2.793	42.58	88 M
^{23}Na	3/2	2.216	11.27	80 mM
^{31}P	1/2	1.131	17.25	75 mM

Source: Haacke et al., p. 27

Torque



For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)



$\mathbf{N} = \mu \times \mathbf{B}$

Torque

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Precession

Torque

$$\mathbf{N} = \mu \times \mathbf{B}$$

$$\frac{d\mathbf{S}}{dt} = \mathbf{N}$$

Change in Angular momentum

$$\frac{d\mathbf{S}}{dt} = \mu \times \mathbf{B}$$

$$\mu = \gamma \mathbf{S}$$

Relation between magnetic moment and angular momentum

$$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

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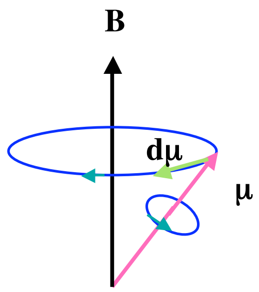
Precession

$$\frac{d\mu}{dt} = \mu \times \gamma \mathbf{B}$$



Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma B$$

This is known as the **Larmor** frequency.

Larmor Frequency

$\omega = \gamma B$ Angular frequency in rad/sec

$f = \gamma B / (2\pi)$ Frequency in cycles/sec or Hertz, Abbreviated Hz

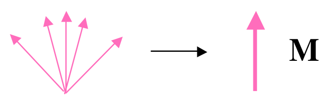
For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Magnetization Vector

$$\mathbf{M} = \frac{1}{V} \sum_{\substack{\text{protons} \\ \text{in V}}} \mu_i$$

Vector sum of the magnetic moments over a volume.



For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$

Equation of motion is the same form as for individual moments.

RF Excitation

At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

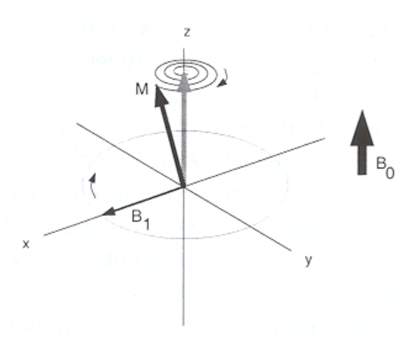
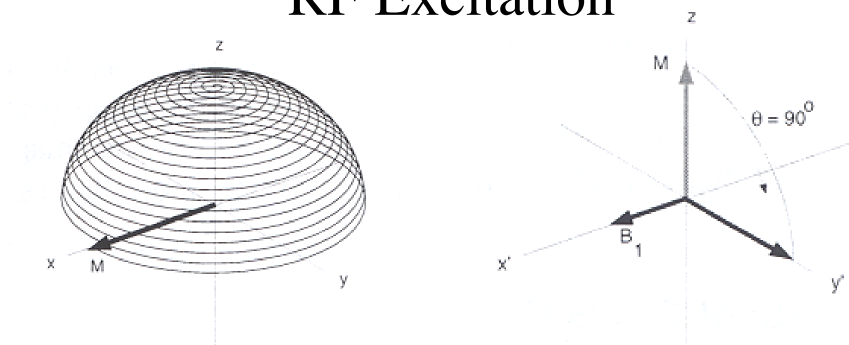


Image & caption: Nishimura, Fig. 3.2

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

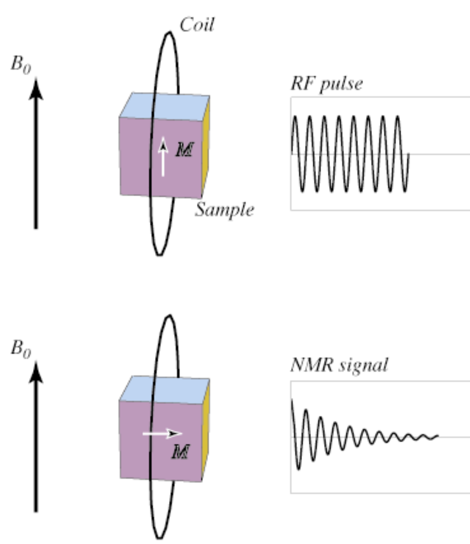
RF Excitation



a) Laboratory frame behavior of **M** b) Rotating frame behavior of **M**

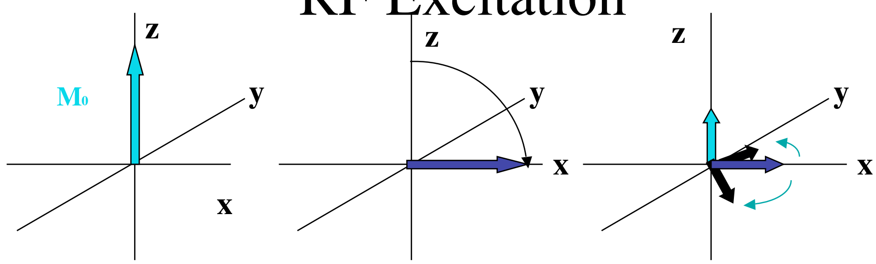
B_1 induces rotation of magnetization towards the transverse plane. Strength and duration of B_1 can be set for a 90 degree rotation, leaving **M** entirely in the *xy* plane. Images & caption: Nishimura, Fig. 3.3

RF Excitation

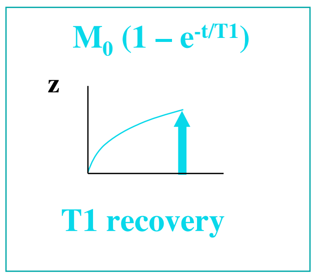


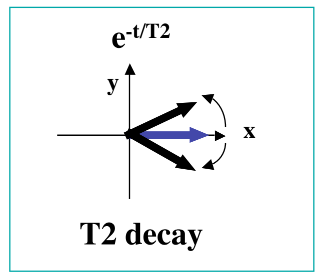
From Buxton 2002

RF Excitation



Doing nothing **Excitation** **Relaxation**





Credit: Larry Frank

Relaxation

An excitation pulse rotates the magnetization vector away from its equilibrium state (purely longitudinal). The resulting vector has both longitudinal $\mathbf{M_z}$ and tranverse $\mathbf{M_{xy}}$ components.

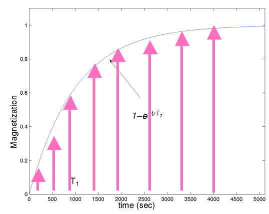
Due to thermal interactions, the magnetization will return to its equilibrium state with characteristic time constants.

T_1 spin-lattice time constant, return to equilibrium of $\mathbf{M_z}$

T_2 spin-spin time constant, return to equilibrium of $\mathbf{M_{xy}}$

Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$

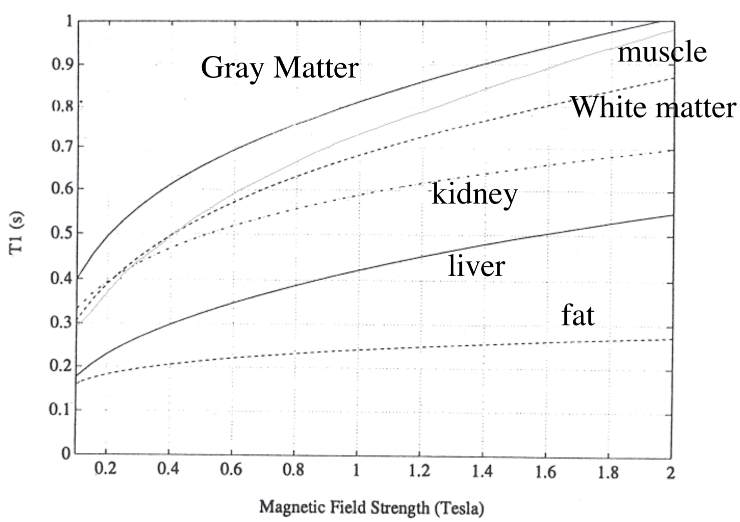


After a 90 degree pulse $M_z(t) = M_0(1 - e^{-t/T_1})$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that T_1 increases with **B**.

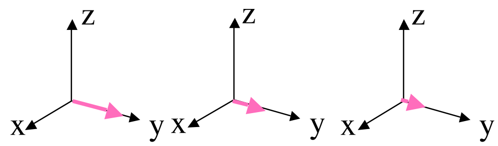
T1 Values



Image, caption: Nishimura, Fig. 4.2

Transverse Relaxation

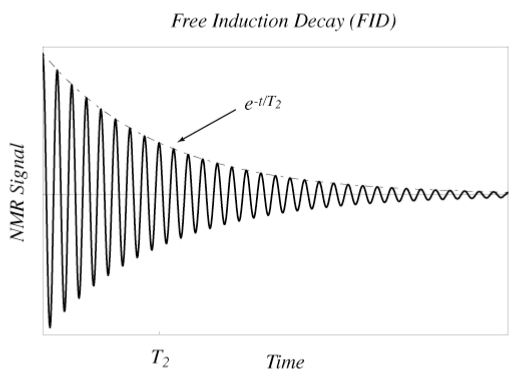
$$\frac{d\mathbf{M}_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$



Each spin's local field is affected by the z-component of the field due to other spins. Thus, the Larmor frequency of each spin will be slightly different. This leads to a dephasing of the transverse magnetization, which is characterized by an exponential decay.

T_2 is largely independent of field. T_2 is short for low frequency fluctuations, such as those associated with slowly tumbling macromolecules.

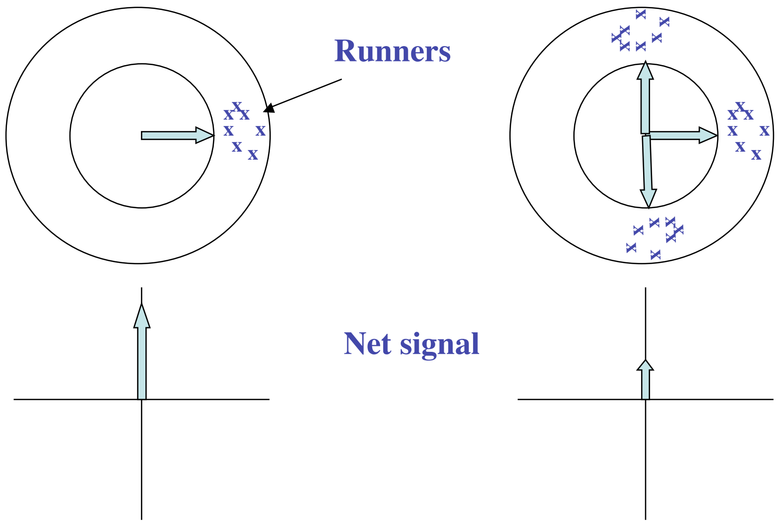
T2 Relaxation



After a 90 degree excitation

$$M_{xy}(t) = M_0 e^{-t/T_2}$$

T2 Relaxation



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Credit: Larry Frank

T2 Values

Tissue	T_2 (ms)
gray matter	100
white matter	92
muscle	47
fat	85
kidney	58
liver	43
CSF	4000

Table: adapted from Nishimura, Table 4.2

Solids exhibit very short T_2 relaxation times because there are many low frequency interactions between the immobile spins.

On the other hand, liquids show relatively long T_2 values, because the spins are highly mobile and net fields average out.

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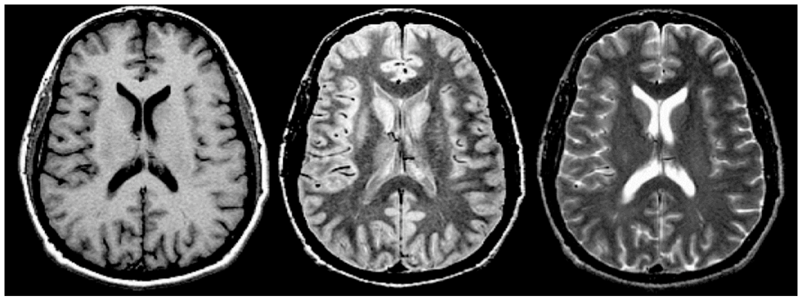
Example



T_1-weighted Density-weighted T_2-weighted

Bloch Equation

$$\frac{d\mathbf{M}}{dt} = \underbrace{\mathbf{M} \times \gamma \mathbf{B}}_{\text{Precession}} - \underbrace{\frac{M_x \mathbf{i} + M_y \mathbf{j}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\mathbf{k}}{T_1}}_{\text{Longitudinal Relaxation}}$$

i, j, k are unit vectors in the x,y,z directions.

Free precession about static field

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma\mathbf{B}$$
$$= \gamma \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{vmatrix}$$
$$= \gamma \begin{pmatrix} \hat{i}\left(B_z M_y - B_y M_z\right) \\ -\hat{j}\left(B_z M_x - B_x M_z\right) \\ \hat{k}\left(B_y M_x - B_x M_y\right) \end{pmatrix}$$

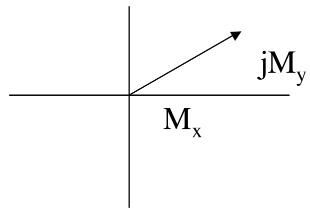
Free precession about static field

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} B_z M_y - B_y M_z \\ B_x M_z - B_z M_x \\ B_y M_x - B_x M_y \end{bmatrix}$$
$$= \gamma \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Precession

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} 0 & B_0 & 0 \\ -B_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Useful to define $M \equiv M_x + jM_y$



$$\begin{aligned} dM/dt &= d/dt\left(M_x + iM_y\right) \\ &= -j\gamma B_0 M \end{aligned}$$

Solution is a time-varying phasor

$$M(t) = M(0)e^{-j\gamma B_0 t} = M(0)e^{-j\omega_0 t}$$

Precession

$$\begin{aligned} M(t) &= M(0)e^{-j\omega_0 t} \\ &= \left(M_x(0)\cos\omega_0 t + M_y(0)\sin\omega_0 t\right) + j\left(M_y(0)\cos\omega_0 t - M_x(0)\sin\omega_0 t\right) \end{aligned}$$

In matrix form this is $\begin{bmatrix} M_x(t) \\ M_y(t) \end{bmatrix} = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t \\ -\sin\omega_0 t & \cos\omega_0 t \end{bmatrix} \begin{bmatrix} M_x(0) \\ M_y(0) \end{bmatrix}$

The full solution is then a rotation about the z-axis.

$$\begin{aligned} \begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix} &= \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x(0) \\ M_y(0) \\ M_z(0) \end{bmatrix} \\ &= R_z\left(\omega_0 t\right) \begin{bmatrix} M_x(0) \\ M_y(0) \\ M_z(0) \end{bmatrix} \end{aligned}$$