Superposition Integral

What is the response to an arbitrary function $g(x_1, y_1)$?

Write $g(x_1, y_1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi,\eta)\delta(x_1 - \xi, y_1 - \eta)d\xi d\eta.$

The response is given by

$$I(x_2, y_2) = L\left[g_1(x_1, y_1)\right]$$

$$= L\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi,\eta)\delta(x_1 - \xi, y_1 - \eta)d\xi d\eta\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi,\eta)L\left[\delta(x_1 - \xi, y_1 - \eta)\right]d\xi d\eta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi,\eta)h(x_2, y_2; \xi,\eta)d\xi d\eta$$