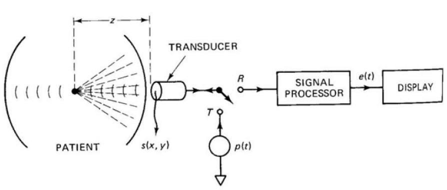
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
Ultrasound Lecture 1



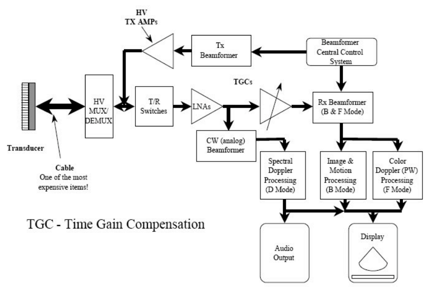

Basic System



Echo occurs at $t=2z/c$ where c is approximately 1500 m/s or 1.5 mm/µs

Macovski 1983

Basic System

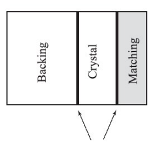


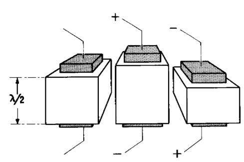
TGC - Time Gain Compensation

Brunner 2002

Macovski 1983

Transducer

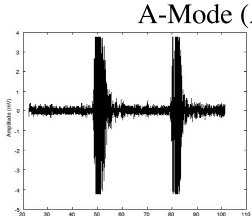


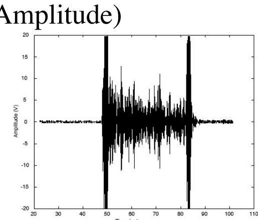


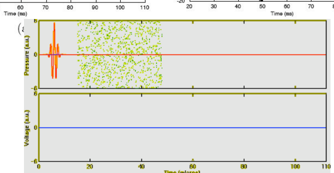
Seutens 2002

www.engineering.uiowa.edu/~bme_285/ Lecture/Notes%20on%20Ultrasound.pdf

A-Mode (Amplitude)







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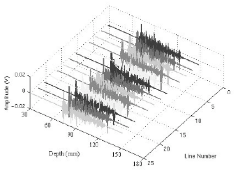
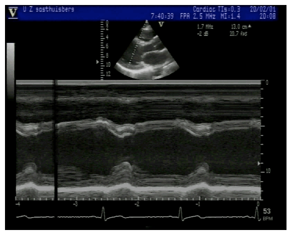
M-Mode (Motion)





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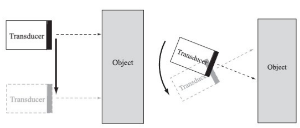
B-Mode (Brightness)



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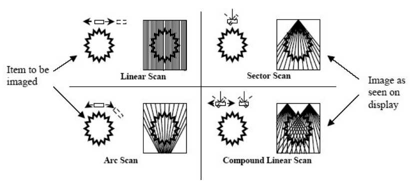

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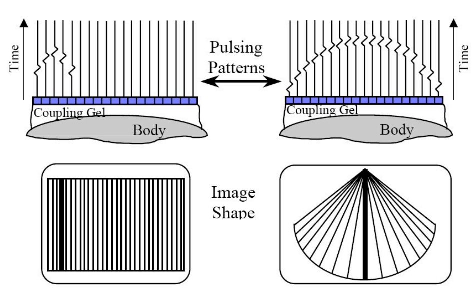

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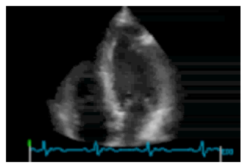
B-Mode



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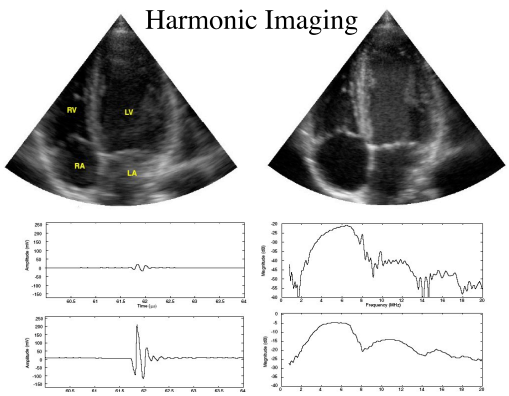
Harmonic Imaging



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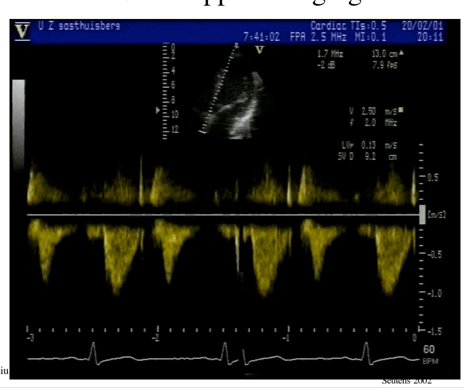
CW Doppler Imaging



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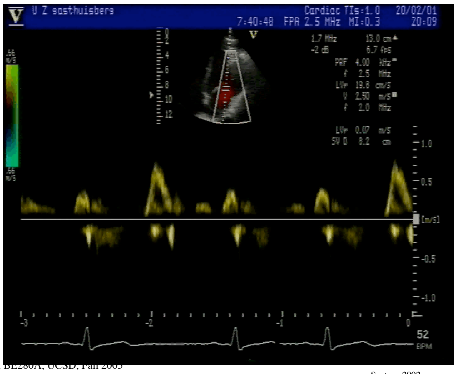
PW Doppler Imaging



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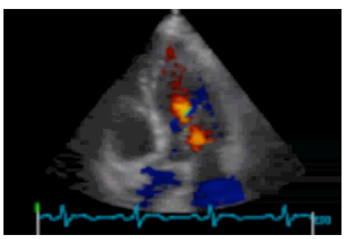
Color Doppler Imaging



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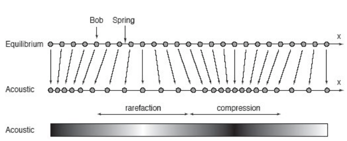
Acoustic Waves



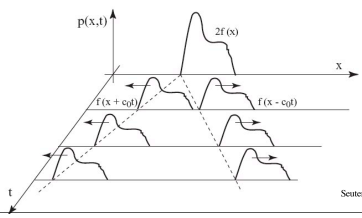
Suetens 2002

Acoustic Wave Equation

$$\nabla^2 p = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

Solutions are of the form

$$p(x,t) = A_1 f_1(x - ct) + A_2 f_2(x + ct)$$



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Acoustic Wave Equation

Solutions of the wave equation

Plane wave

$p(z,t) = \exp(j2\pi f(t - z/c))$

Superposition of plane waves

$p(z,t) = \int_{-\infty}^{\infty} P(f)\exp(j2\pi f(t - z/c))df$

At $z = 0$: $p(0,t) \equiv p(t) = \int_{-\infty}^{\infty} P(f)\exp(j2\pi ft)df = F^{-1}\left(P(f)\right)$

$p(z,t) = p(0, t - z/c) = p(t - z/c)$

Spherical Wave

$p(r,t) = \frac{1}{r}\exp(j2\pi f(t - r/c))$

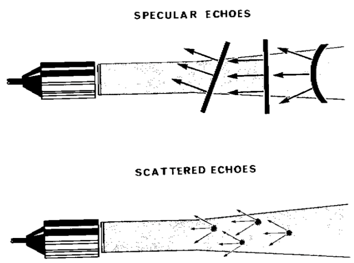
Impedance

Impedance Z $= \dfrac{\text{Pressure}}{\text{Velocity}} = \dfrac{P}{v} = \rho c$

density kg/m^3

speed of sound
Brain 1541 m/s
Liver 1549
Skull bone 4080 m/s
Water 1480 m/s

Echos



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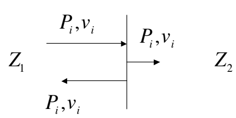
Specular Reflection



Material	Reflectivity
Brain-skull	0.66
Fat-muscle	0.10
Muscle-blood	0.03
Soft-tissue-air	.9995

$v_i - v_r = v_t$ (velocity boundary condition)

$$\frac{P_i}{Z_1} - \frac{P_r}{Z_1} = \frac{P_t}{Z_2}$$

$P_i + P_r = P_t$ (pressure boundary condition)

$$R = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \approx \frac{\Delta Z}{Z_0}$$

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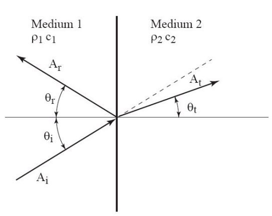
Reflection and Refraction



$$\frac{\sin\theta_i}{c_1} = \frac{\sin\theta_r}{c_1} = \frac{\sin\theta_t}{c_2}$$

$$R = \frac{P_r}{P_i} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta}$$

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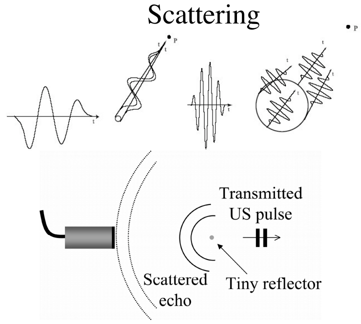
Scattering



Point scatterers retransmit the incident wave equally in all direction (e.g. isotropic scattering).

Attenuation

Loss of acoustic energy during propagation.
Conversion of acoustic energy into heat.

$H(f,z) = \exp(-\alpha(f)z) \approx \exp(-\alpha_0 f^n z)$

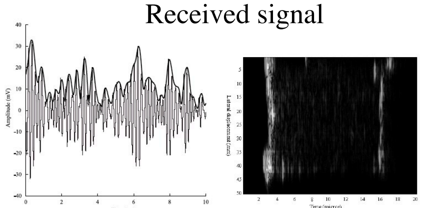
For frequencies used in medical ultrasound, $n \approx 1$.

In liver, typical value is $\alpha_0 = 0.5$ db/cm/MHz
For tissues in general, $\alpha_0 \approx 1.0$ db/cm/MHz

Example: Liver at 2 MHz, attenuation 1 dB/cm.
After the 6 cm, 6 dB of attenuation.
10^(-3/20) = 0.5

Recall dB $= 20\log_{10}\left(A_z / A_0\right)$

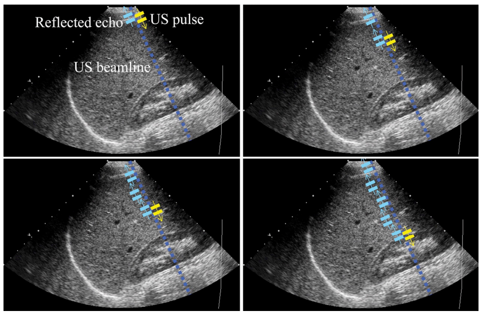
Received signal



$$e(t) = K\int\int\int \frac{e^{-2\alpha z}}{z} R(x,y,z)\,s(x,y)\,p(t-2z/c)\,dx\,dy\,dz$$

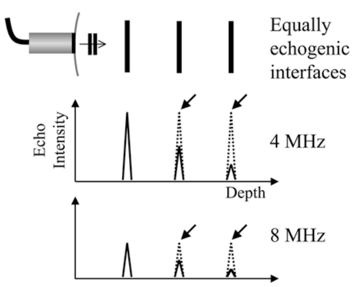
Attenuation — Reflection/Scattering — Beam width — Pulse

Received signal



http://radiographics.rsnajnls.org/content/vol23/issue4/images/large/g03jl25c1x.jpeg

Attenuation Correction



Attenuation Correction and Signal Equation

$$
\begin{aligned}
e(t) &= K\iiint \frac{e^{-2\alpha z}}{z} R(x,y,z)s(x,y)p(t-2z/c)\,dxdydz \\
&\approx K\frac{e^{-\alpha ct}}{ct/2}\iiint R(x,y,z)s(x,y)p(t-2z/c)\,dxdydz
\end{aligned}
$$

$$
\begin{aligned}
e_c(t) &= cte^{\alpha ct}e(t) \\
&\approx K\iiint R(x,y,z)s(x,y)p(t-2z/c)\,dxdydz \\
&= \frac{c}{2}\iiint R(x,y,c\tau/2)s(x,y)p(t-\tau)\,dxdyd\tau \\
&= K\frac{c}{2}\left[R(x,y,ct/2) *** s(-x,-y)p(t)\right]\Big|_{x=0,y=0}
\end{aligned}
$$

Signal Equation Example

$$e_c(t) \approx K \int\int\int R(x,y,z)s(x,y)p(t-2z/c)dxdydz$$
$$= K\frac{c}{2}\left[R(x,y,ct/2) *** s(-x,-y)p(t)\right]\Big|_{x=0,y=0}$$

Let $R(x,y,z) = \delta(x)\delta(y)\delta(z-z_0) + \delta(x)\delta(y)\delta(z-z_1)$
and $s(x,y) = rect(x/L)rect(y/L)$

$$e_c(t) = K \int\int\int \left[\delta(x)\delta(y)\delta(z-z_0) + \delta(x)\delta(y)\delta(z-z_1)\right]rect(x/L)rect(y/L)p(t-2z/c)dxdydz$$
$$= K\left[p(t-2z_0/c) + p(t-2z_1/c)\right]$$

Signal Equation Example

$$e_c(t) \approx K \int\int\int R(x,y,z)s(x,y)p(t-2z/c)dxdydz$$
$$= K\left[R(x,y,ct/2) *** s(-x,-y)p(t)\right]\Big|_{x=0,y=0}$$

Let $R(x,y,z) = \delta(x)\delta(y)\delta(z-z_0) + \delta(x)\delta(y)\delta(z-z_1)$
What happens to the signal as we move the transducer
to some arbitrary position x_0, y_0?

$$e_c(t,x_0,y_0) = K \int\int\int \left[\delta(x)\delta(y)\delta(z-z_0) + \delta(x)\delta(y)\delta(z-z_1)\right]s(x-x_0,y-y_0)p(t-2z/c)dxdydz$$
$$= K\left[p(t-2z_0/c)s(-x_0,-y_0) + p(t-2z_1/c)s(-x_0,-y_0)\right]$$

Signal Equation Summary

In general, we can write

$$e_c(t,x_0,y_0) = K \int\int\int R(x,y,z)s(x-x_0,y-y_0)p(t-2z/c)dxdydz$$
$$= K\frac{c}{2}\left[R(x,y,ct/2) *** s(-x,-y)p(t)\right]\Big|_{x=x_0,y=y_0}$$

$$e_c(z',x_0,y_0) = K \int\int\int R(x,y,z)s(x-x_0,y-y_0)p(2(z'-z)/c)dxdydz$$
$$= \left[R(x,y,z') *** s(-x,-y)p(2z'/c)\right]\Big|_{x=x_0,y=y_0}$$

Response to a point target $\delta(x-x_1)\delta(y-y_1)\delta(z-z_1)$ is
$s(x_1-x_0, y_1-y_0)p(2(z'-z_1)/c)$

Signal Equation Summary

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$$e_c(t,x_0,y_0) = K\int\int\int R(x,y,z)s(x-x_0,y-y_0)p(t-2z/c)dxdydz$$
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Signal Equation Summary

Response to a point target $\delta(x-x_1)\delta(y-y_1)\delta(z-z_1)$ is
$s(x_1-x_0,y_1-y_0)p(2(z'-z_1)/c)$

Thus, $s(x,y)$ determine the lateral response and $p(t)$ determines the depth response.

Depth Resolution

$p(t) = p(2z/c)$ determines the depth resolution

Pulses are of the form $a(t)\cos(2\pi f_0 t+\theta)$ where $a(t)$ is the envelope function and f_0 is the resonant frequency of the transducer.

The duration of T of $a(t)$ is typically chosen to be about 2 or 3 periods (e.g. $T = 3/f_0$). If the duration is too short, the bandwidth of the pulse will be very large and much of its power will be attenuated.

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c/f_0 = 1.5\lambda$.

Depth Resolution

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c/f_0 = 1.5\lambda$.

Example:

For $f_0 = 5$ MHz, $\Delta z = (1.5)(1500 m/s)/(5\times10^6 Hz) = 0.45$ mm

Trade-off

Higher $f_0 \Rightarrow$ Smaller $\Delta z \Rightarrow$ but more attenuation

Example: Assume 1dB/cm/MHz

For 10 cm depth, 20 cm roundtrip path length.

At 1 MHz 20 dB of attenuation $\Rightarrow$ Attenuation = 0.1

At 10 MHz 200 dB of attenuation $\Rightarrow$ Attenuation = 1×10^{-10}

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