BENG280A, Principles of Biomedical Imaging 10/18/05
Fall Quarter 2005

HOMEWORK #4
Due at the start of Class on Thursday 10/27/05

Readings:
1. Review last week's reading as necessary. Read sections 3.1 through 3.3 and sections 13.3.1 through 13.4.2.

Problems:

1. Find the 1D Fourier transform of $m(x) = \exp(-x^2)\delta(x)\cos^2(2\pi\sqrt{x})$.

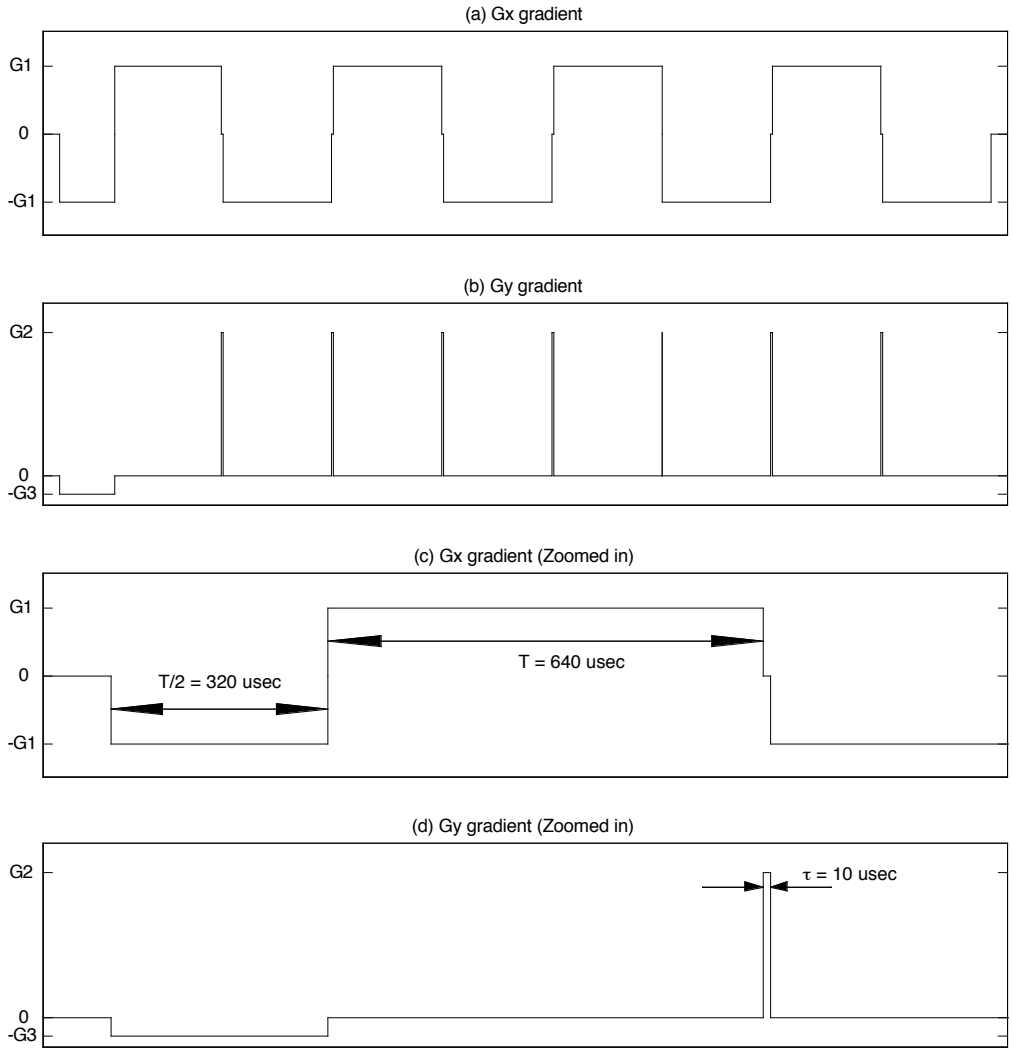
2. Consider the function $g(x) = \cos^2(2\pi k_0 x)$. Sketch this function. You sample this signal in the spatial domain with a sampling rate $K_S = 1/\Delta x$ (e.g. samples spaced at intervals of Δx). What is the minimum sampling rate that you can use without aliasing? Give an intuitive explanation for your answer.

3. Consider the 2D object $m(x,y) = \delta(x - x_0)\left(\delta(y - L/2) + \delta(y + L/2)\right)$ consisting of two impulses.
   (a) Find and sketch the Fourier transform of this object.
   (b) The Fourier transform of the object is sampled in the k_y direction at the Nyquist rate, i.e. $\Delta k_y = 1/FOV_y$. At what FOV will the reconstructed image be equal to zero? HINT: The sampling is uniform but the center of k-space is not necessarily sampled.
   (c) At what FOV will the image be a single impulse?

4. A 2D object has an FOV of 19.2 cm in the *x* direction and 25.6 cm in the *y* direction. We sample the 2D Fourier transform of the object. If we want to achieve a resolution of *1* mm in the *x* direction and *2* mm in the *y* direction, how should we sample *k*-space? (i.e. give the sampling intervals and the extent of the sampling region).

5. Consider the gradient waveforms shown in the figure on the next page. The full waveforms are shown in panels (a) and (b), and zoomed-in views are shown in (c) and (d). The analog-to-digital converter (ADC) is turned on during the flat parts of the readout (Gx) gradients with a sampling rate of Δt.

   (a) Draw the k-space trajectory.
   (b) Determine the sequence parameters (G1, G2 and G3, and Δt) to achieve the following image specifications: $FOV_x = FOV_y = 256$mm, $\delta_x = 4$ mm and $\delta_y = 32$ mm.


(a) Gx gradient
G1
0
-G1
(b) Gy gradient
G2
0
-G3
(c) Gx gradient (Zoomed in)
G1
0
-G1
T/2 = 320 usec
T = 640 usec
(d) Gy gradient (Zoomed in)
G2
0
-G3
τ = 10 usec

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