

HOMEWORK #6
Due at the start of Class on Thursday 11/17/05

Readings: Read Chapters 4 and 6 and sections 5.1 and 5.2.

Problems:

1. In class, we derived the zeroth and first order moments for a bipolar gradient pulse.
 - (a) Derive the 2nd order moment for this pulse.
 - (b) Design a bipolar pulse that results in 90 degrees of phase shift for an object with a velocity of 20 cm/sec and zero acceleration. Use a gradient amplitude of 4 Gauss/cm.
 - (c) What is the total phase shift using the pulse designed in (b) if the object is also accelerating at 300 cm/sec²?
 - (d) Modify the design of your pulse so that the total phase shift due to both velocity and acceleration is 90 degrees. If necessary, you may adjust the gradient amplitude.

2. Define the object $m(x,y) = \text{rect}(2x)\text{rect}(2y)$.
 - (a) Sketch the sinogram for this object. Give explicit expressions for the projections at 0, 90 degrees, and 45 degrees (e.g. the projection that goes through the diagonals of the square). For other angles, either a sketch or an explicit expression is okay.
 - (b) Show that the projection theorem holds for $\theta = 45$ degrees. In other words, derive the 2D Fourier transform and the projection at $\theta = 45$ degrees and show how these are related.

MATLAB Exercise

Define a 257x257 object where the center 61x61 square is 1 and the object is zero everywhere else.

- (a) Use the **radon** function to compute the projections for angles from 0 to 180. Try angular increments of 1 degree, 0.5 degree, and 0.25 degrees (e.g. $\theta = 0:0.5:179.5$). Examine the sinograms. Does the projection at 45 degrees agree with what you found in problem 2?
- (b) Use the **iradon** function to compute the filtered backprojection reconstruction of your image. What is the effect of the varying angular increments?
- (c) Experiment with the different filtering and interpolation options in **iradon**. What is the effect of using a Hamming or Hann filter?