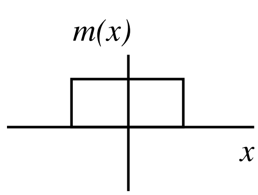
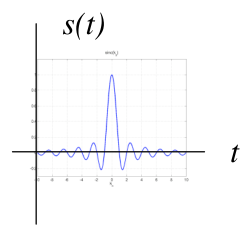
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
MRI Lecture 3

MR signal is Fourier Transform

$$s(t) = \int_x \int_y m(x,y) \exp\left(-j2\pi\left(k_x(t)x + k_y(t)y\right)\right) dxdy$$
$$= M\left(k_x(t), k_y(t)\right)$$
$$= F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}$$





K-space

At each point in time, the received signal is the Fourier transform of the object

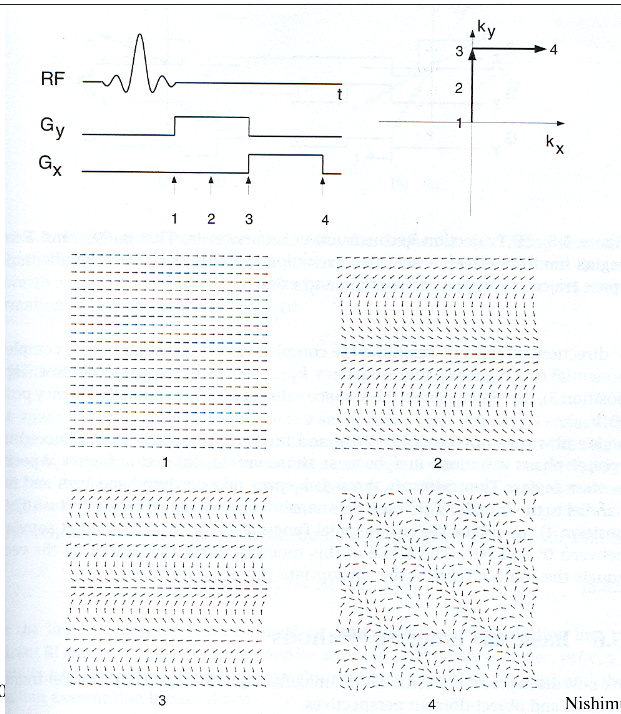
$$s(t) = M\left(k_x(t), k_y(t)\right) = F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}$$

evaluated at the spatial frequencies:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

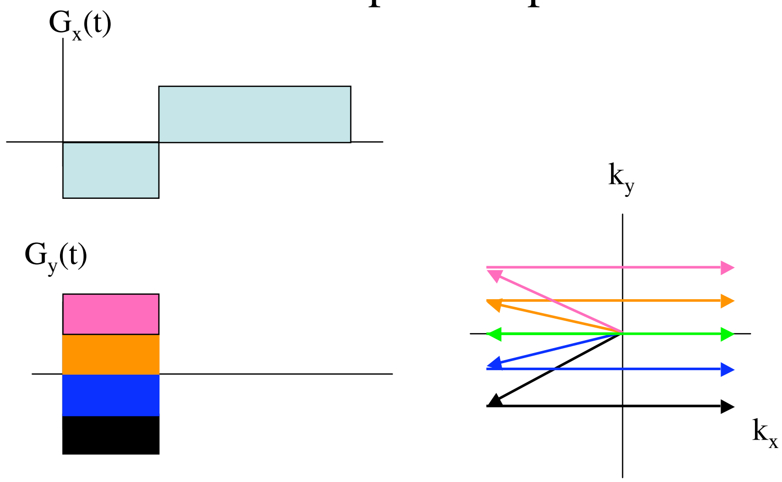
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Thus, the gradients control our position in k-space. The design of an MRI pulse sequence requires us to efficiently cover enough of k-space to form our image.



Nishimura 1996

Spin-Warp



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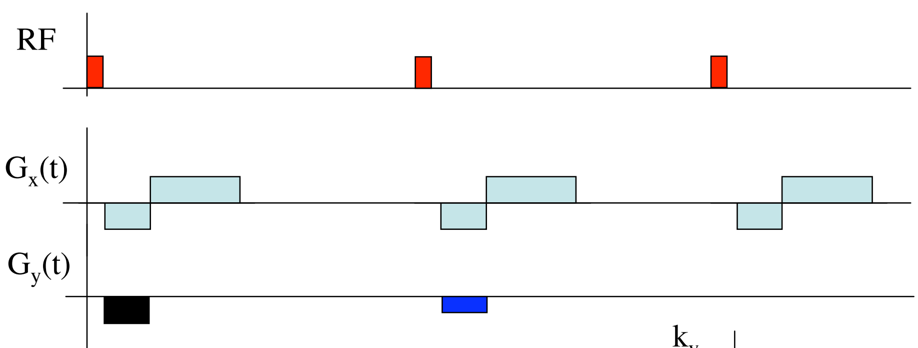
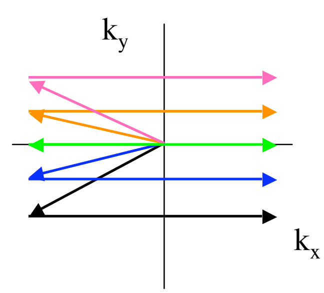
Spin-Warp Pulse Sequence





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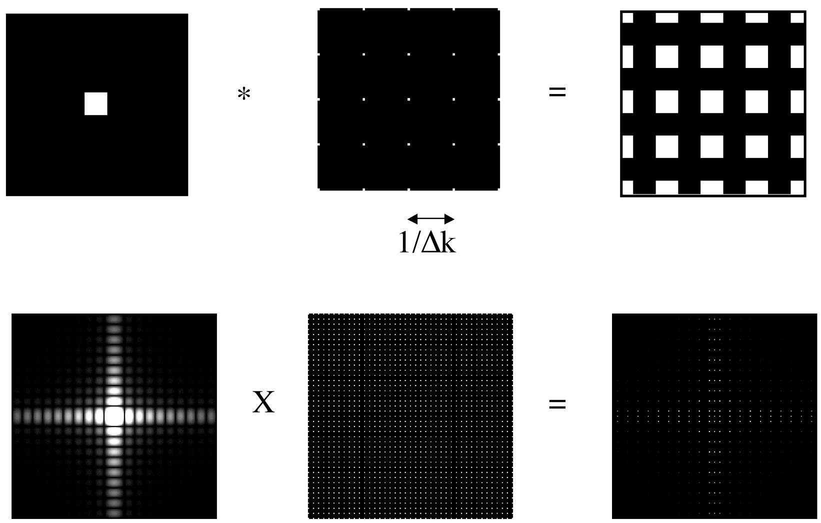

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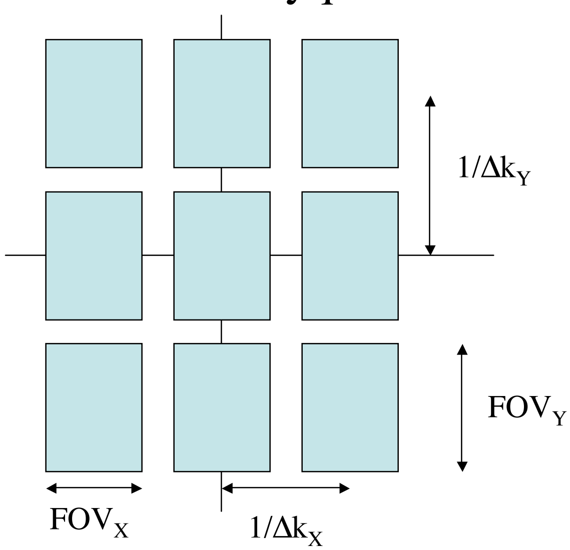
Nyquist Conditions



$1/\Delta k_Y > FOV_Y$

$1/\Delta k_X > FOV_X$

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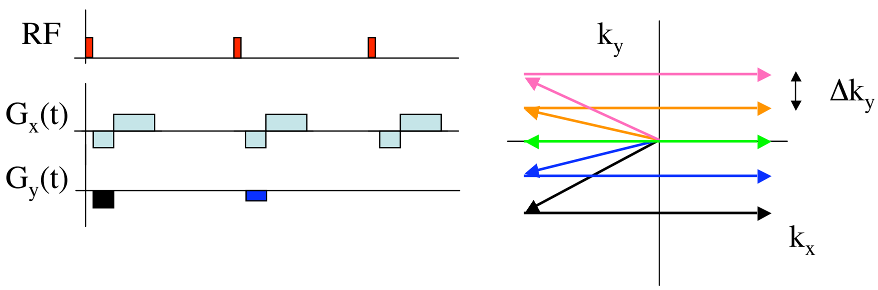
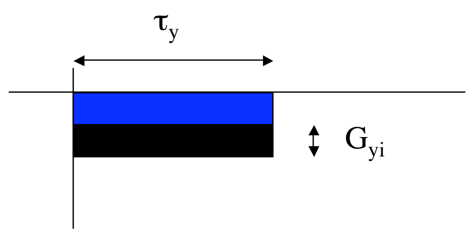
Sampling in k_y

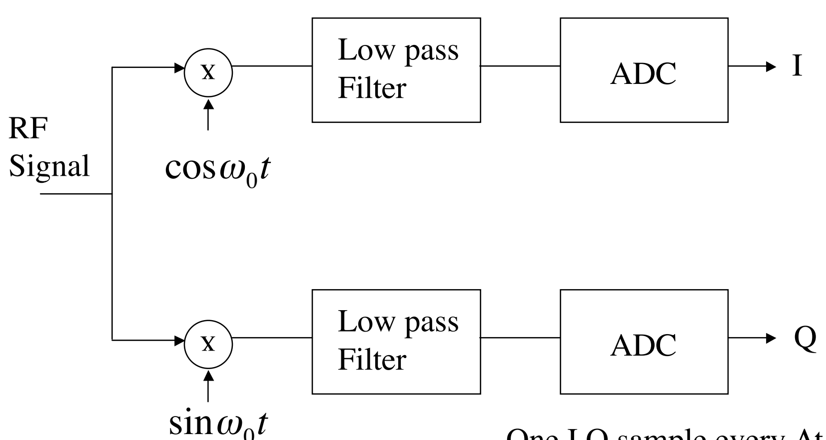




$$\Delta k_y = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

$$FOV_y = \frac{1}{\Delta k_y}$$

Sampling in k_x

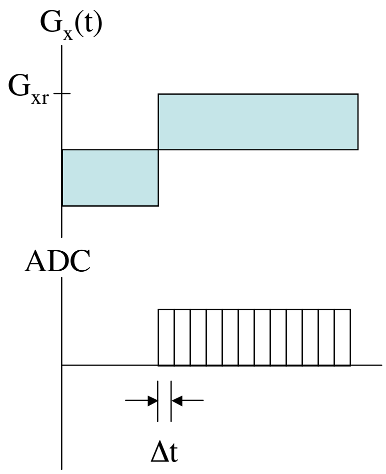


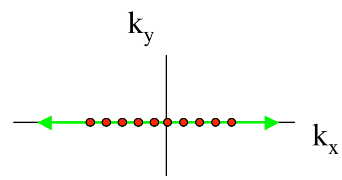
One I,Q sample every Δt

M= I+jQ

Note: In practice, there are number of ways of implementing this processing.

Sampling in k_x

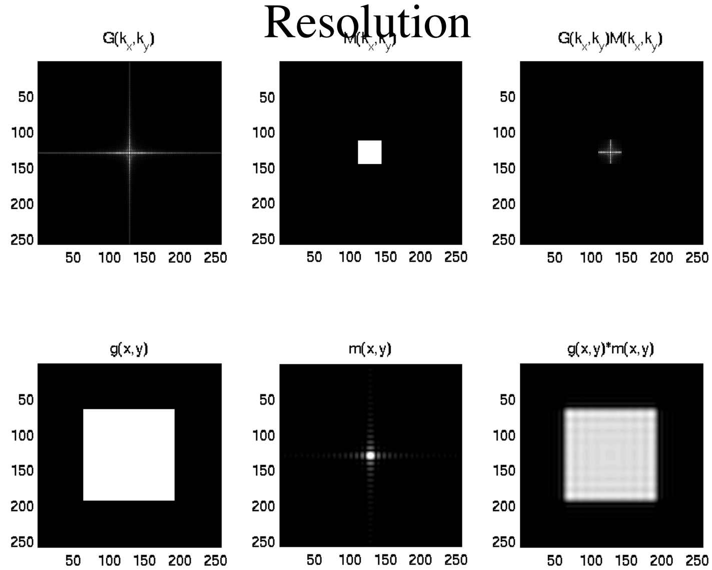




$$\Delta k_x = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

$$FOV_x = \frac{1}{\Delta k_x}$$

Resolution



Effective Width

$$w_E = \frac{1}{w(0)} \int_{-\infty}^{\infty} w(x)dx$$

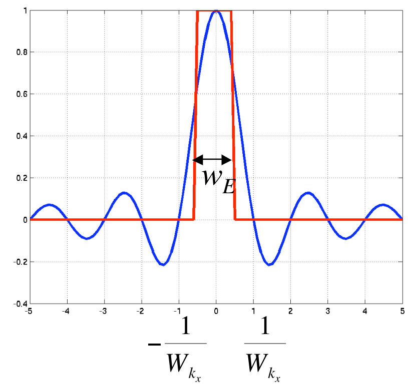
Example

$$w_E = \frac{1}{1} \int_{-\infty}^{\infty} \text{sinc}(W_{k_x} x)dx$$

$$= F\left[\text{sinc}(W_{k_x} x)\right]\Big|_{k_x=0}$$

$$= \frac{1}{W_{k_x}} rect\left(\frac{k_x}{W_{k_x}}\right)\Bigg|_{k_x=0}$$

$$= \frac{1}{W_{k_x}}$$

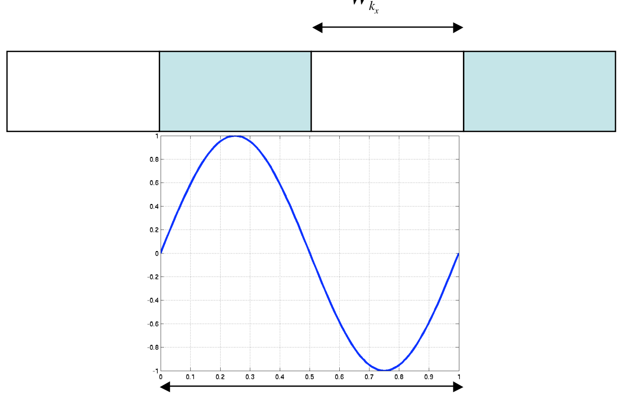


Resolution and spatial frequency

With a window of width W_{k_x} the highest spatial frequency is $W_{k_x}/2$.

This corresponds to a spatial period of $2/W_{k_x}$.

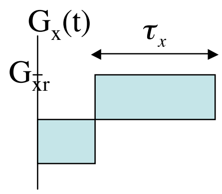
$$\frac{1}{W_{k_x}} = \text{Effective Width} = \delta_x = \text{Resolution}$$

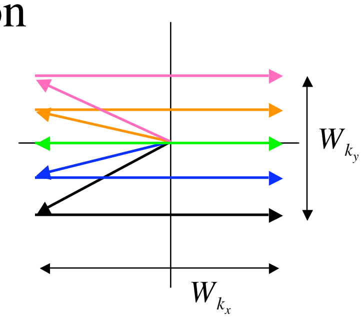


$$\frac{2}{W_{k_x}}$$

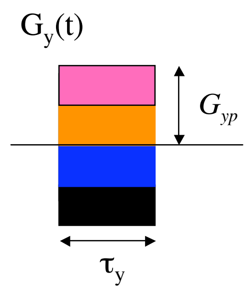
Resolution

$$\delta_x = \frac{1}{W_{k_x}} = \frac{1}{2k_{x,\max}} = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$





$$\delta_y = \frac{1}{W_{k_y}} = \frac{1}{2k_{y,\max}} = \frac{1}{\frac{\gamma}{2\pi} 2G_{yp} \tau_y}$$



Example

Goal :

$FOV_x = FOV_y = 25.6$ cm

$\delta_x = \delta_y = 0.1$ cm

Readout Gradient :

$$FOV_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t}$$

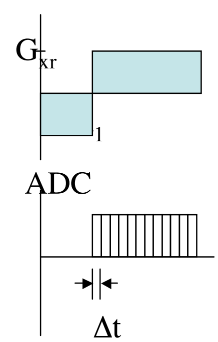
Pick $\Delta t = 32\ \mu\text{sec}$

$$G_{xr} = \frac{1}{FOV_x \frac{\gamma}{2\pi} \Delta t} = \frac{1}{(25.6cm)(42.57 \times 10^6 T^{-1} s^{-1})(32 \times 10^{-6} s)}$$

$$= 2.8675 \times 10^{-5}\ \text{T/cm}$$

$$= .28675\ \text{G/cm}$$

1 Gauss $= 1 \times 10^{-4}$ Tesla



Example

Readout Gradient :

$$\delta_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$

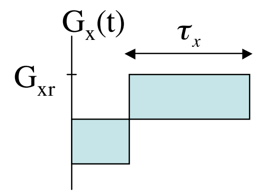
$$\tau_x = \frac{1}{\delta_x \frac{\gamma}{2\pi} G_{xr}} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(0.28675\ \text{G/cm})}$$

$$= 8.192\ \text{ms}$$

$$= N_{read}\Delta t$$

where

$$N_{read} = \frac{FOV_x}{\delta_x} = 256$$



Example

Phase - Encode Gradient :

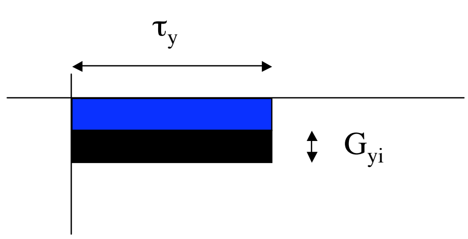
$$FOV_y = \frac{1}{\frac{\gamma}{2\pi} G_{yi} \tau_y}$$

Pick $\tau_y = 4.096\ m\text{sec}$

$$G_{yi} = \frac{1}{FOV_y \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(25.6cm)(42.57 \times 10^6 T^{-1} s^{-1})(4.096 \times 10^{-3} s)}$$

$$= 2.2402 \times 10^{-7}\ \text{T/cm}$$

$$= .00224\ \text{G/cm}$$



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Example

Phase - Encode Gradient :

$$\delta_y = \frac{1}{\frac{\gamma}{2\pi} 2 G_{yp} \tau_y}$$

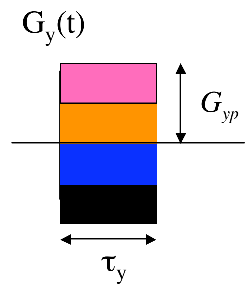
$$G_{yp} = \frac{1}{\delta_y 2 \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(4.096 \times 10^{-3} s)}$$

$$= 0.2868 \text{ G/cm}$$

$$= \frac{N_p}{2} G_{yi}$$

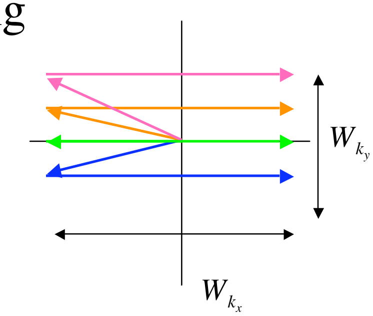
where

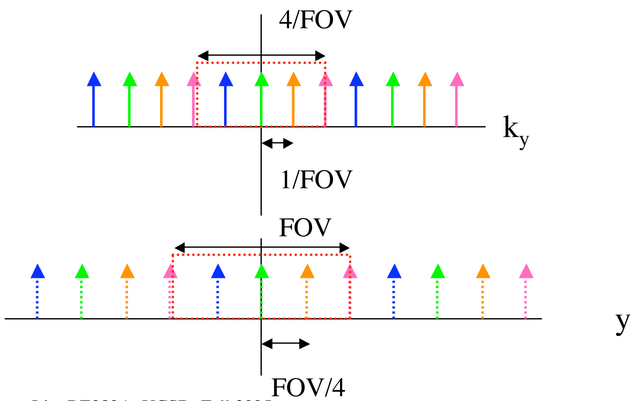
$$N_p = \frac{FOV_y}{\delta_y} = 256$$



Sampling

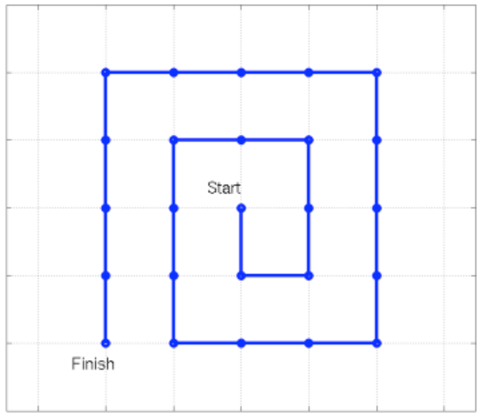
In practice, an even number (typically power of 2) sample is usually taken in each direction to take advantage of the Fast Fourier Transform (FFT) for reconstruction.





Example

Consider the k-space trajectory shown below. ADC samples are acquired at the points shown with $\Delta t = 10\ \mu\text{sec}$. The desired FOV (both x and y) is 10 cm and the desired resolution (both x and y) is 2.5 cm. Draw the gradient waveforms required to achieve the k-space trajectory. Label the waveform with the gradient amplitudes required to achieve the desired FOV and resolution. Also, make sure to label the time axis correctly.



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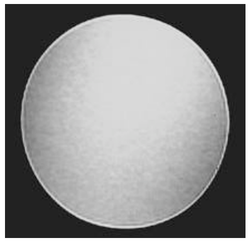
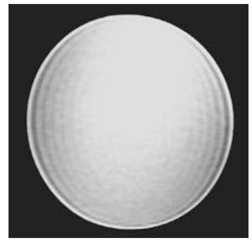
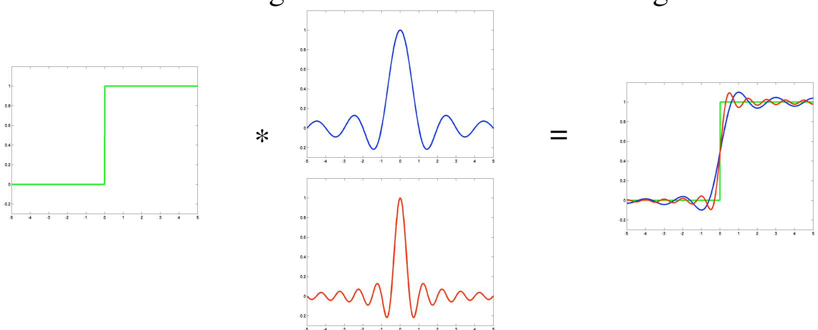
Gibbs Artifact



256x256 image

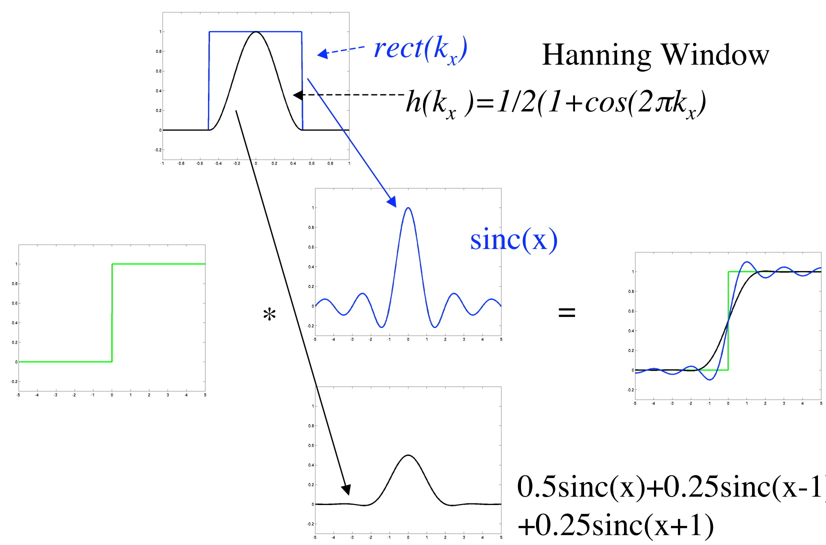


256x128 image



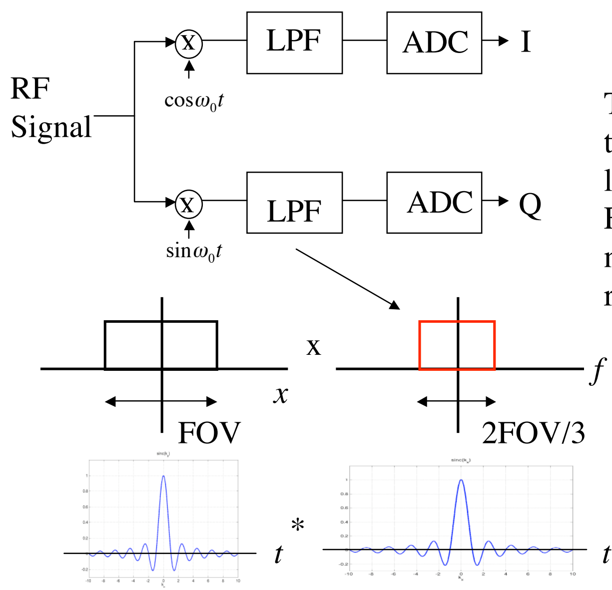
Images from http://www.mritutor.org/mritutor/gibbs.htm

Apodization



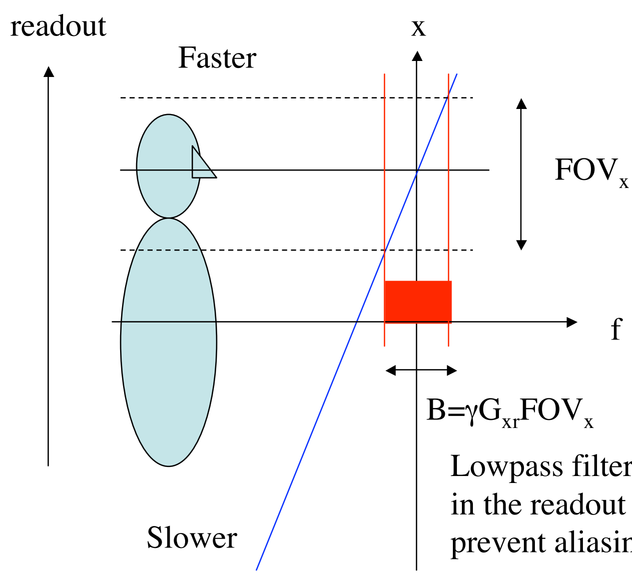
Images from http://www.mritutor.org/mritutor/gibbs.htm

Aliasing and Bandwidth



Temporal filtering in the readout direction limits the readout FOV. So there should never be aliasing in the readout direction.

Aliasing and Bandwidth



Lowpass filter
in the readout direction to
prevent aliasing.

Figure 7-31 Default Axial Directions

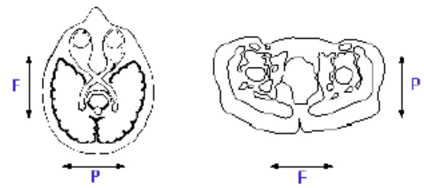
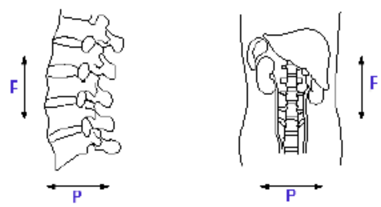


Figure 7-32 Default Sagittal and Coronal Directions



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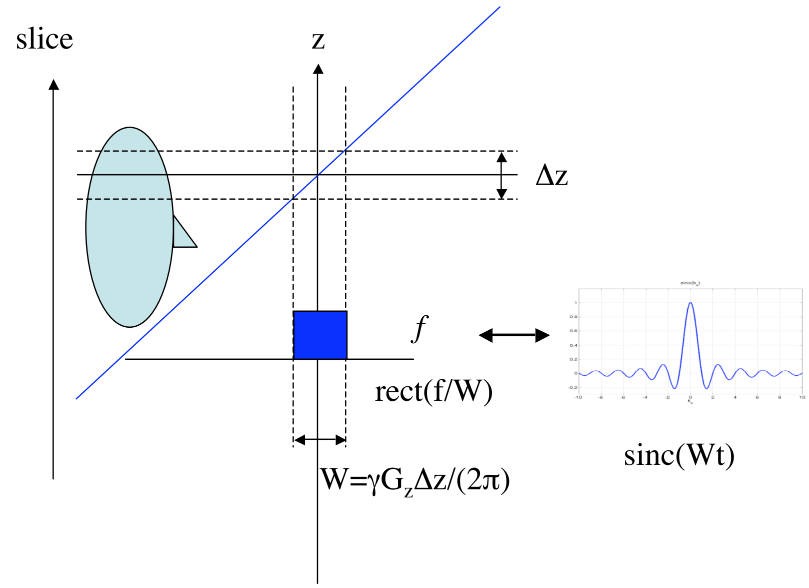
Slice Selection

Recall, that we can tip spins away from their equilibrium state by applying a radio-frequency pulse at the Larmor frequency.

In the presence of a spatial gradient G_z spins in an interval $-\Delta z/2$ to $-\Delta z/2$ have Larmor frequencies ranging from $\omega_0 - \gamma G_z \Delta z/2$ to $\omega_0 + \gamma G_z \Delta z/2$. In order to tip all the spins in this interval, we can apply an RF pulse with energy that is spaced over this frequency interval.

Slice Selection



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