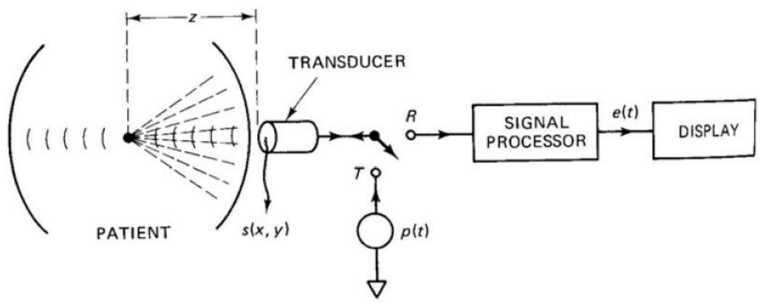
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2006
Ultrasound Lecture 1



## Basic System




Echo occurs at $t=2z/c$ where $c$ is approximately 1500 m/s or 1.5 mm/μs

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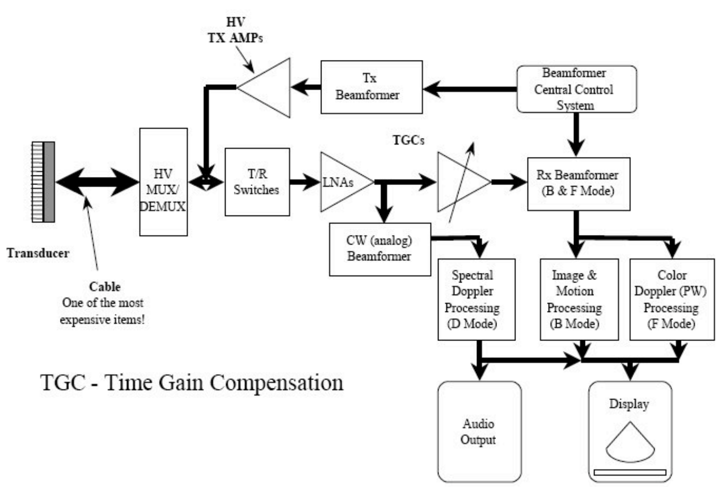
Macovski 1983

## Basic System




TGC - Time Gain Compensation

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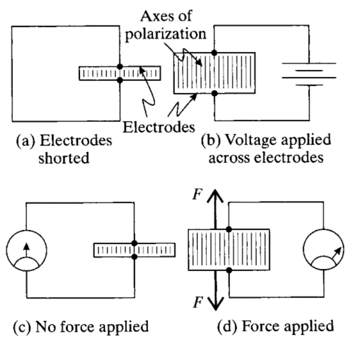
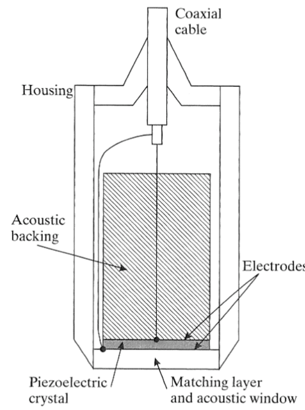
# Transducer

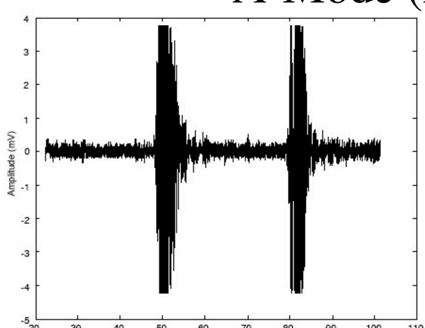


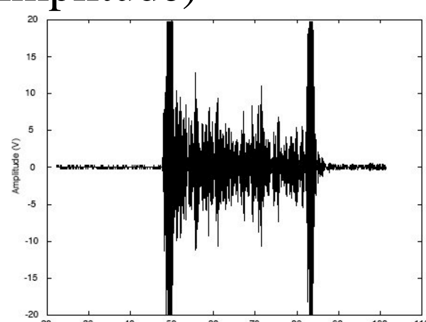



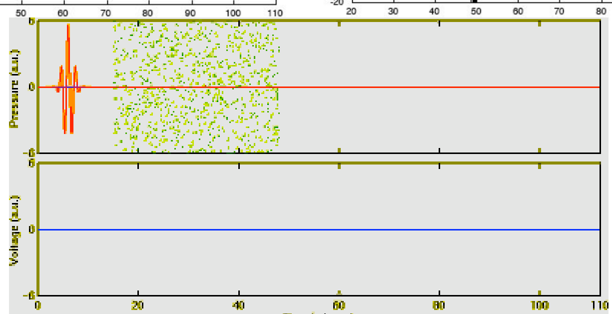

Prince and Links 2006



# A-Mode (Amplitude)



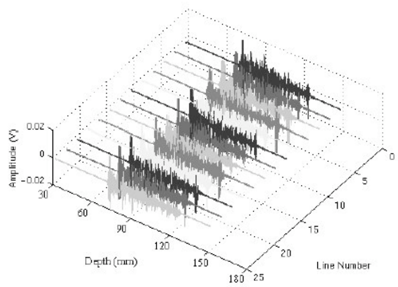



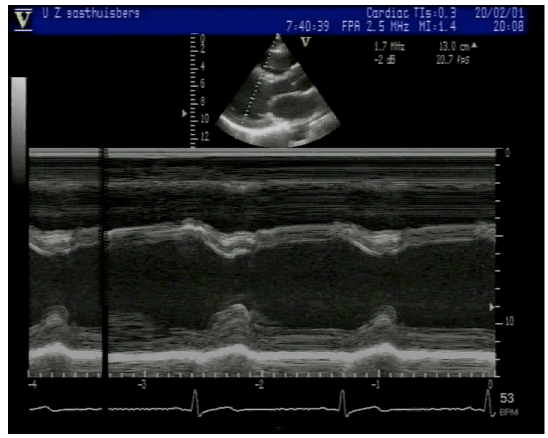






Seutens 2002

# M-Mode (Motion)

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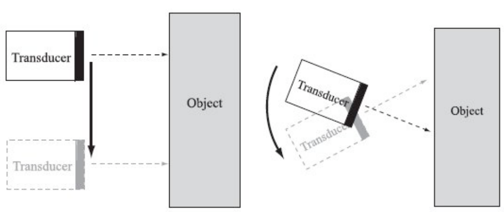
# B-Mode (Brightness)

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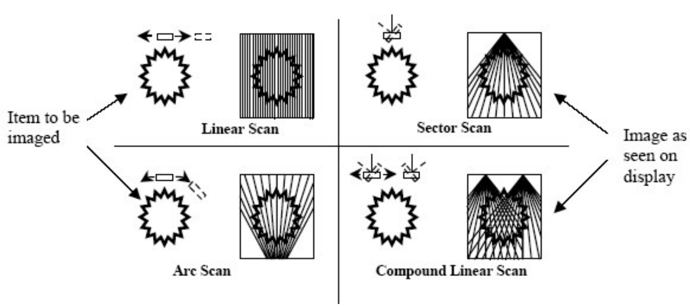


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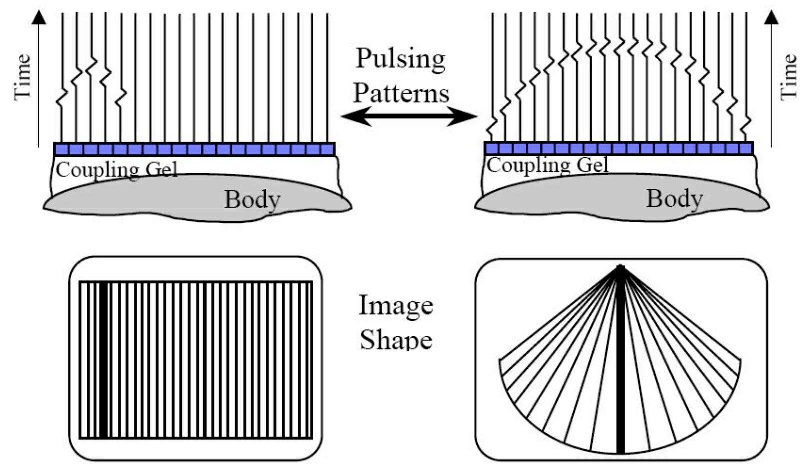
# B-Mode (Brightness)

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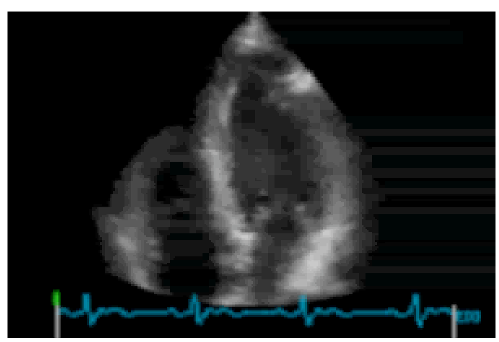
# B-Mode



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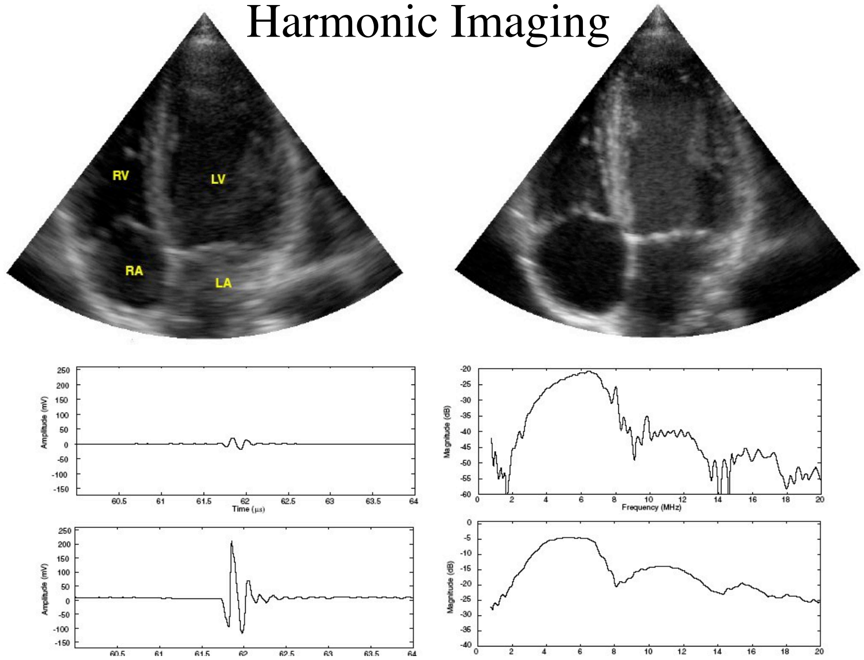
# Harmonic Imaging

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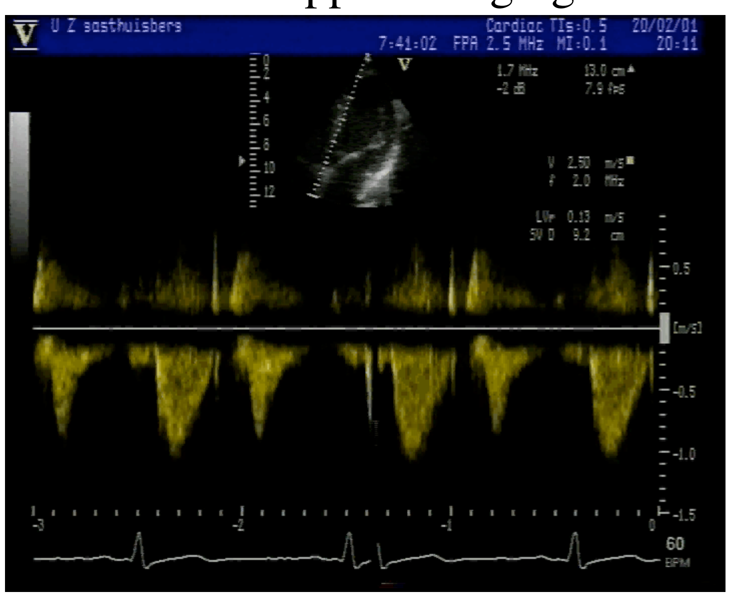
# CW Doppler Imaging

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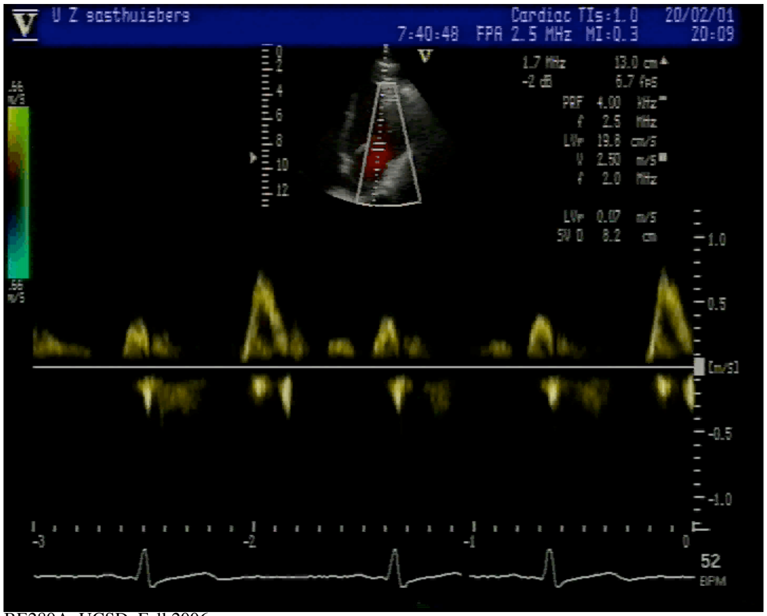
# PW Doppler Imaging

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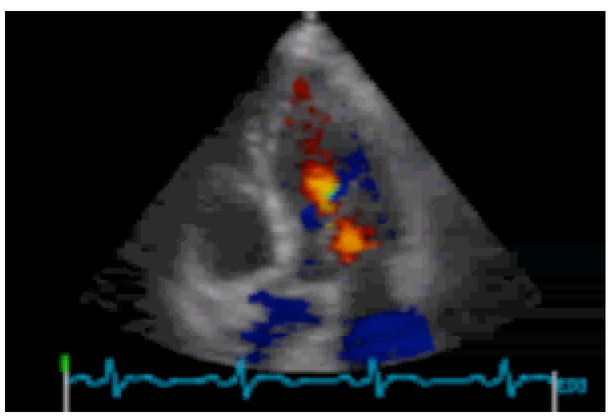
# Color Doppler Imaging



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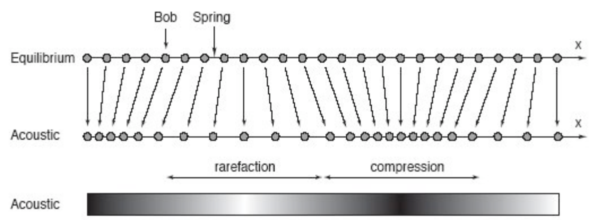
# Acoustic Waves

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# Speed of Sound

$$c = \sqrt{\frac{1}{\kappa\rho}} \quad [\text{m s}^{-1}]$$

$$\kappa = \text{ compressibility} \quad [\text{m s}^2\text{ kg}^{-1}] = [1/\text{Pascal}]$$

$$\rho = \text{density} \quad [\text{kg m}^{-3}]$$

| Material | Density | Speed m/s |
|---|---|---|
| Air | 1.2 | 330 |
| Water | 1000 | 1480 |
| Bone | 1380-1810 | 4080 |
| Fat | 920 | 1450 |
| Liver | 1060 | 1570 |

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# Impedance

$$\text{Impedance } Z = \frac{\text{Pressure}}{\text{Velocity}} = \frac{P}{v} = \rho c = \sqrt{\frac{\rho}{\kappa}}$$

density kg/m$^3$

speed of sound
Brain 1541 m/s
Liver 1549
Skull bone 4080 m/s
Water 1480 m/s

**Note: particle velocity and speed of sound are not the same!**

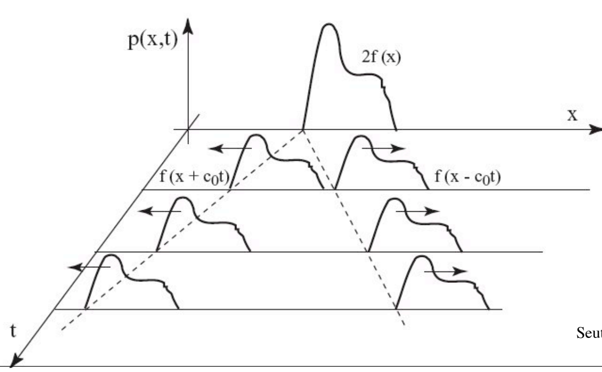


# Acoustic Wave Equation

$$\nabla^2 p = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

Solutions are of the form

$$p(x,t) = A_1 f_1(x - ct) + A_2 f_2(x + ct)$$



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# Plane Waves

$$p(z,t) = \exp\left(jk(z-ct)\right)$$

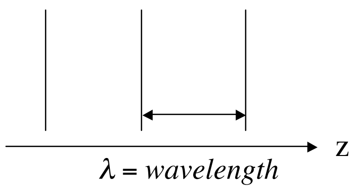
$$\begin{aligned} p(z,t) &= \cos\left(k(z-ct)\right) \\ &= \cos\left(\frac{2\pi}{\lambda}(z-ct)\right) \\ &= \cos\left(\frac{2\pi f}{c}(z-ct)\right) \\ &= \cos\left(2\pi f\,(z/c-t)\right) \end{aligned}$$

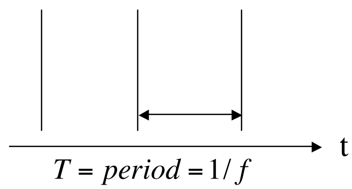
$$k = wavenumber = \frac{2\pi}{\lambda} = 2\pi k_z$$

$$\lambda = wavelength = \frac{c}{f}$$

$$f = frequency \text{ [cycles/sec]}$$

$$T = period = \frac{1}{f}$$






# Spherical Waves

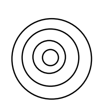
Outward wave

Inward wave

$$p(r,t) = \frac{1}{r}\phi(t - r/c) + \frac{1}{r}\phi(t + r/c)$$

Outward wave

$$p(r,t) = \frac{1}{r}\exp(j2\pi f\,(t - r/c))$$



# Acoustic Intensity

$$I = pv$$
$$= \frac{p^2}{Z}$$

Also called acoustic energy flux.
Analogous to electric power

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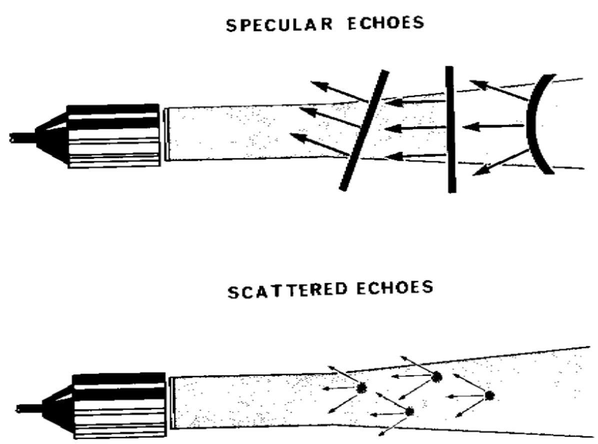
# Echos

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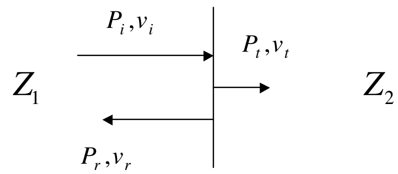
# Specular Reflection




| Material | Reflectivity |
| --- | --- |
| Brain-skull | 0.66 |
| Fat-muscle | 0.10 |
| Muscle-blood | 0.03 |
| Soft-tissue-air | .9995 |

$v_i - v_r = v_t$ (velocity boundary condition)

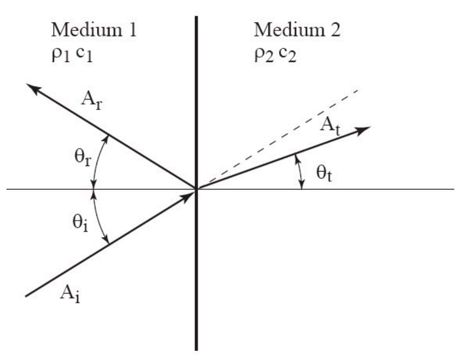
$$\frac{P_i}{Z_1} - \frac{P_r}{Z_1} = \frac{P_t}{Z_2}$$

$P_i + P_r = P_t$ (pressure boundary condition)

$$R = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \approx \frac{\Delta Z}{Z_0}$$



# Reflection and Refraction




Snell's Law

$$\frac{\sin\theta_i}{c_1} = \frac{\sin\theta_r}{c_1} = \frac{\sin\theta_t}{c_2}$$



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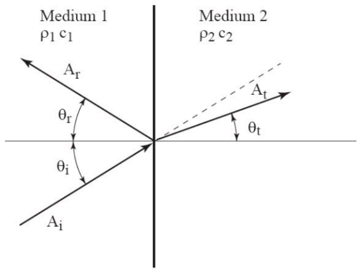
# Reflection and Refraction




$$v_i \cos\theta_i = v_r \cos\theta_r + v_t \cos\theta_t$$

$$\frac{p_i}{Z_1}\cos\theta_i = \frac{p_r}{Z_1}\cos\theta_r + \frac{p_t}{Z_2}\cos\theta_t$$

$$p_i + p_r = p_t$$

Pressure Reflectivity $$R = \frac{p_r}{p_i} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$

Pressure Transmittivity $$T = \frac{p_t}{p_i} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$

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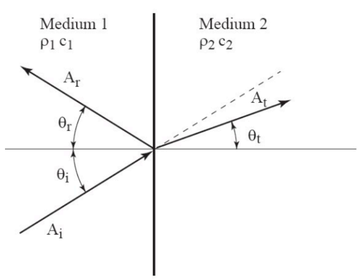
# Reflection and Refraction




Intensity Reflectivity $$R_I = \frac{I_r}{I_i} = \frac{p_r^2}{p_i^2} = \left(\frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}\right)^2$$

Intensity Transmittivity $$T_I = \frac{I_t}{I_i} = \frac{p_t^2 Z_1}{p_i^2 Z_2} = \frac{4Z_1Z_2\cos^2\theta_i}{\left(Z_2\cos\theta_i + Z_1\cos\theta_t\right)^2}$$

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# Example

Example : Fat/liver interface at normal incidence

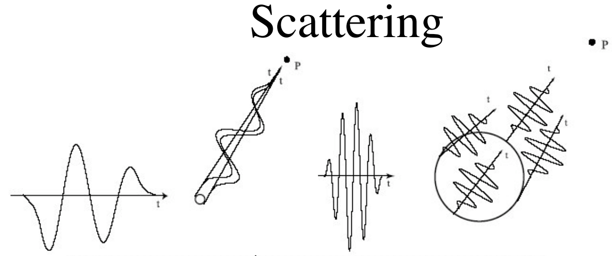
$Z_{fat} = 1.35 \times 10^{-6}$ kg m$^{-2}$ s$^{-1}$

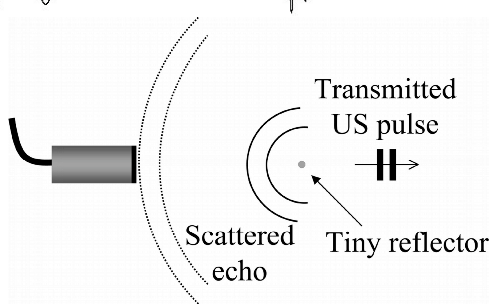
$Z_{liver} = 1.66 \times 10^{-6}$kg m$^{-2}$ s$^{-1}$

$$R_I = \left( \frac{Z_{liver} - Z_{fat}}{Z_{liver} + Z_{fat}} \right)^2 = 0.103$$



# Scattering






Point scatterers retransmit the incident wave equally in all direction (e.g. isotropic scattering).

## Attenuation

Loss of acoustic energy during propagation.

Conversion of acoustic energy into heat.

$$\begin{aligned} p(z,t) &= A_z f(t - c/z) \\ &= A_0 \exp(-\mu_a z) f(t - c/z) \end{aligned}$$

Amplitude attenuation factor

$$\mu_a = -\frac{1}{z} \ln \frac{A_z}{A_0} \quad : \text{ units } = \text{ nepers/cm}$$

$$\alpha = -20 \frac{1}{z} \log_{10} \frac{A_z}{A_0} = 20 \mu_a \log_{10}(e) \approx 8.7 \mu_a \quad : \text{ dB/cm}$$

Attenuation coefficient



## Attenuation

$$\alpha(f) = \alpha_0 f^n$$

For frequencies used in medical ultrasound, $n \approx 1$.

$$\alpha(f) \approx \alpha_0 f$$

| Material | $\alpha_0$ [dB/cm/MHz] |
|---|---|
| fat | 0.63 |
| liver | 0.94 |
| Cardiac muscle | 1.8 |
| bone | 20.0 |

# Example

Example: Fat at 5 MHz

Attenuation coefficient $= 5\text{MHz} \times 0.63$ dB/cm/MHz

$= 3.15$dB/cm

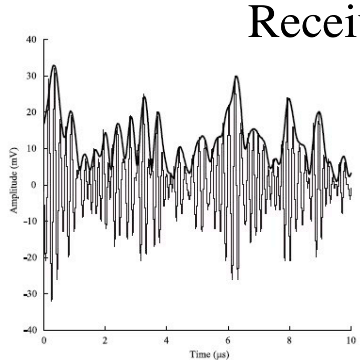
After 4 cm, attenuation $= 4 * 3.15 = 12.6$ dB

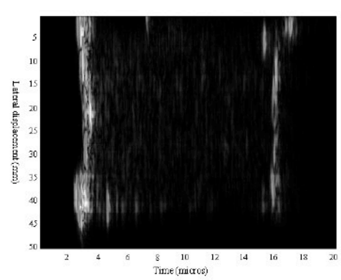
Relative amplitude is $10^{(-12.6/20)} = 0.2344$

Recall dB $\equiv 20\log_{10}\left(A_z / A_0\right)$



# Received signal




$$e(t) = K\int\int\int \frac{e^{-2\alpha z}}{z} R(x,y,z) s(x,y) p(t - 2z/c)\, dxdydz$$

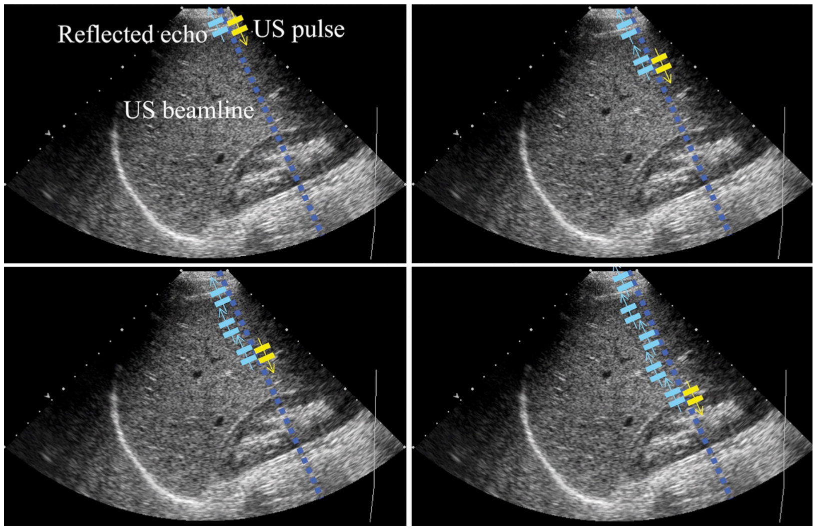
Attenuation

Reflection/Scattering

Beam width

Pulse

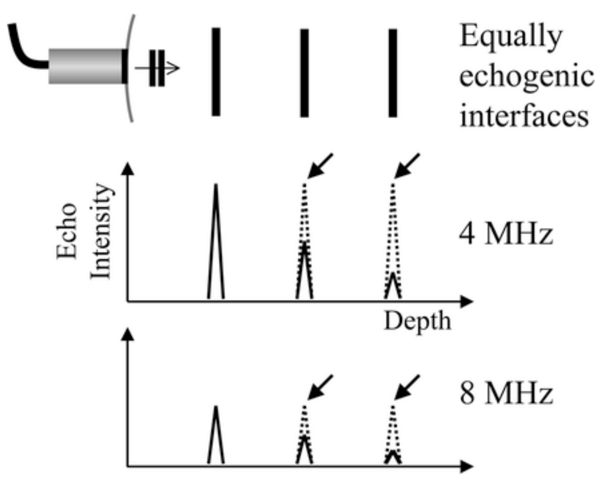
# Received signal




http://radiographics.rsnajnls.org/content/vol23/issue4/images/large/g03jl25c1x.jpeg



# Attenuation Correction



Attenuation Correction and Signal Equation

$$e(t) = K\int\int\int \frac{e^{-2\alpha z}}{z} R(x,y,z)s(x,y)p(t-2z/c)dxdydz$$
$$\approx K\frac{e^{-\alpha ct}}{ct/2}\int\int\int R(x,y,z)s(x,y)p(t-2z/c)dxdydz$$

$$e_c(t) = cte^{\alpha ct}e(t)$$
$$\approx K\int\int\int R(x,y,z)s(x,y)p(t-2z/c)dxdydz$$
$$= \frac{c}{2}\int\int\int R(x,y,c\tau/2)s(x,y)p(t-\tau)dxdyd\tau$$



# Impulse Response

Transducer centered at 0,0 (defines image at 0,0)

$$e_c(t) = K\int\int\int R(x,y,z)s(x,y)p(t-2z/c)dxdydz$$
$$= K\int\int\int \delta(x,y,z-z_0)s(x,y)p(t-2z/c)dxdydz$$
$$= Ks(0,0)p(t-2z_0/c)$$

Transducer centered at $x_0$,$y_0$ (defines image at these coordinates)

$$e_c(t) = K\int\int\int R(x,y,z)s(x-x_0,y-y_0)p(t-2z/c)dxdydz$$
$$= K\int\int\int \delta(x,y,z-z_0)s(x-x_0,y-y_0)p(t-2z/c)dxdydz$$
$$= Ks(-x_0,-y_0)p(t-2z_0/c)$$

Lateral Response is therefore $s(-x,-y)$

# Impulse Response

Depth response

$$p(t-2z_0/c) = p(2z/c - 2z_0/c)$$
$$= p\left(\frac{2(z-z_0)}{c}\right)$$

Therefore impulse response is simply
$p(t)$ in the time domain or
$p(2z/c)$ in the spatial domain



## Signal Equation

In general, we can write

$$e_c(t,x_0,y_0) = K\int\int\int R(x,y,z)s(x-x_0,y-y_0)p(t-2z/c)dxdydz$$
$$= K\frac{c}{2}\left[R(x,y,ct/2) *** s(-x,-y)p(t)\right]\Big|_{x=x_0,y=y_0}$$

$$e_c(z',x_0,y_0) = K\int\int\int R(x,y,z)s(x-x_0,y-y_0)p(2(z'-z)/c)dxdydz$$
$$= \left[R(x,y,z') *** s(-x,-y)p(2z'/c)\right]\Big|_{x=x_0,y=y_0}$$

# Depth Resolution

$p(t) = p(2z/c)$ determines the depth resolution

Pulses are of the form $a(t)\cos(2\pi f_0 t + \theta)$ where $a(t)$ is the envelope function and $f_0$ is the resonant frequency of the transducer.

The duration of $T$ of $a(t)$ is typically chosen to be about 2 or 3 periods (e.g. $T = 3/f_0$). If the duration is too short, the bandwidth of the pulse will be very large and much of its power will be attenuated.

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c / f_0 = 1.5\lambda$.



# Depth Resolution

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c / f_0 = 1.5\lambda$.

Example:

For $f_0 = 5$ MHz, $\Delta z = (1.5)(1500 m/s)/(5 \times 10^6 Hz) = 0.45$ mm

Trade-off

Higher $f_0 \Rightarrow$ Smaller $\Delta z \Rightarrow$ but more attenuation

Example: Assume 1dB/cm/MHz

For 10 cm depth, 20 cm roundtrip path length.

At 1 MHz 20 dB of attenuation $\Rightarrow$ Attenuation = 0.1

At 10 MHz 200 dB of attenuation $\Rightarrow$ Attenuation = $1\text{x}10^{-10}$