Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2006
Ultrasound Lecture 2



# Depth of Penetration

Assume system can handle L dB of loss, then

$$L = 20\log_{10}\left(\frac{A_z}{A_0}\right)$$

We also have the definition

$$\alpha = -\frac{1}{z}20\log_{10}\left(\frac{A_z}{A_0}\right)$$

and the approximation

$$\alpha = \alpha_0 f$$

Total range a wave can travel before attenuation L is

$$z = \frac{L}{\alpha_0 f}$$

Depth of penetration is

$$d_p = \frac{L}{2\alpha_0 f}$$

# Depth of Penetration

Assume L = 80 dB; $\alpha_0$= 1dB/cm/MHz

| Frequency (MHz) | Depth of Penetration(cm) |
|---|---|
| 1 | 40 |
| 2 | 20 |
| 3 | 13 |
| 5 | 8 |
| 10 | 4 |
| 20 | 2 |



# Pulse Repetition and Frame Rate

Need to wait for echoes to return before transmitting new pulse

Pulse repetition interval is

$$T_R \geq \frac{2d_p}{c}$$

Pulse repetition rate is

$$f_R = \frac{1}{T_R}$$

If N pulses are required to form an image, then the frame rate is

$$F = \frac{1}{NT_R}$$

# Example

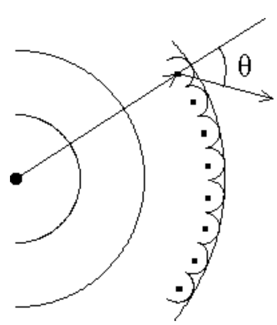
$N = 256,\ L = 80dB,\ c = 1540m/s,\ \alpha_0 = 1dB/cm/MHz$

What frequency should be used to achieve a frame rate of 15 frame/sec?

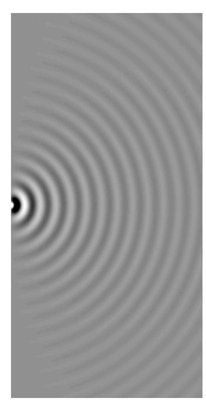
$$T_R = \frac{1}{FN} = 0.26ms$$
$$T_R \geq \frac{2d_p}{c} = \frac{L}{\alpha_0 fc}$$
$$f \geq \frac{L}{acT_R} = 1.99MHz$$



# Huygen's Principle

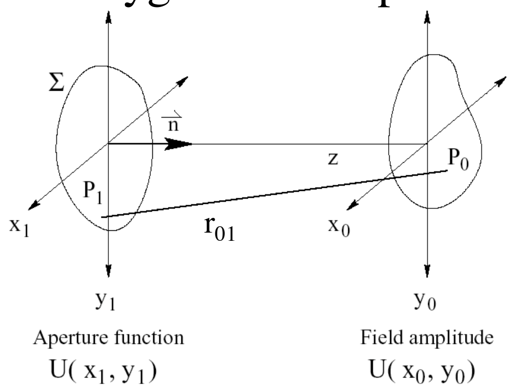



http://www.fink.com/thesis/chapter2.html



http://www.cbem.imperial.ac.uk/ardan/diff/hfw.html

# Huygen's Principle




*Wavenumber*

$$k = \frac{2\pi}{\lambda}$$

$$U(P_0) = \iint_\Sigma h(P_0, P_1) U(P_1) ds$$

$$\text{where } h(P_0, P_1) = \frac{1}{j\lambda} \frac{\exp(jkr_{01})}{r_{01}} \cos(\vec{n}, \vec{r}_{01})$$

Oliquity Factor

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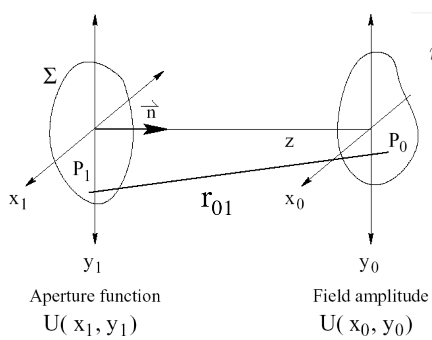
# Small-Angle (paraxial) Approximation




$$r_{01} = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$cos(\vec{n}, \vec{r}_{01}) \approx 1$$

$$r_{01} \approx z$$

$$h(x_0, y_0; x_1, y_1) \approx \frac{1}{j\lambda z} \exp(jkr_{01})$$

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# Fresnel Approximation

$$r_{01} = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$
$$= z\sqrt{1 + \left(\frac{(x_1 - x_0)}{z}\right)^2 + \left(\frac{(y_1 - y_0)^2}{z}\right)}$$
$$\approx z\left[1 + \frac{1}{2}\left(\frac{(x_1 - x_0)}{z}\right)^2 + \frac{1}{2}\left(\frac{(y_1 - y_0)}{z}\right)^2\right]$$

Approximates spherical wavefront with a parabolic phase profile

$$h(x_0, y_0; x_1, y_1) \approx \frac{\exp(jkz)}{j\lambda z} \exp\left[\frac{jk}{2z}\left[(x_1 - x_0)^2 + (y_1 - y_0)^2\right]\right]$$



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# Fresnel Approximation

$$U(x_0,y_0) = \iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}\left((x_1 - x_0)^2 + (y_1 - y_0)^2\right)\right) s(x_1,y_1)\,dx_1 dy_1$$
$$= \frac{\exp(jkz)}{j\lambda z}\left(s(x_0,y_0) ** \exp\left(\frac{jk}{2z}\left(x_0^2 + y_0^2\right)\right)\right)$$

## Fraunhofer Approximation

$$kr_{01} \approx kz\left(1 + \frac{1}{2}\left(\frac{x_1 - x_0}{z}\right)^2 + \frac{1}{2}\left(\frac{y_1 - y_0}{z}\right)^2\right)$$

$$= kz\left(1 + \frac{1}{2z^2}\left(x_1^2 - 2x_1x_0 + x_0^2\right) + \frac{1}{2z^2}\left(y_1^2 - 2y_1y_0 + y_0^2\right)\right)$$

$$= kz + \frac{k}{2z}\left(x_1^2 + y_1^2\right) + \frac{k}{2z}\left(x_0^2 + y_0^2\right) - \frac{k}{z}\left(x_1x_0 + y_1y_0\right)$$

$$\approx kz + + \frac{k}{2z}\left(x_0^2 + y_0^2\right) - \frac{k}{z}\left(x_1x_0 + y_1y_0\right)$$

Assume this term is negligible.

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## Fraunhofer Condition

Phase term due to position on transducer is $\frac{k}{2z}\left(x_1^2 + y_1^2\right)$

Far-field condition is

$$\frac{k}{2z}\left(x_1^2 + y_1^2\right) << 1$$

$$z >> \frac{k}{2}\left(x_1^2 + y_1^2\right) = \frac{\pi}{\lambda}\left(x_1^2 + y_1^2\right)$$

For a square DxD transducer, $x_1^2 + y_1^2 = D^2/4$

$$z >> \frac{\pi D^2}{4\lambda} \approx \frac{D^2}{\lambda}$$

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# Fraunhofer Approximation

$$U(x_0, y_0) \approx \frac{\exp(jkz)\exp\left[\frac{jk}{2z}(x_0^2 + y_0^2)\right]}{j\lambda z} \iint_{-\infty}^{\infty} U(x_1, y_1) \exp\left[-\frac{j2\pi}{\lambda z}(x_0 x_1 + y_0 y_1)\right] dx_1 dy_1$$

Quadratic phase term

Fourier transform of the source with

$$k_x = \frac{x_0}{\lambda z} \quad k_y = \frac{y_0}{\lambda z}$$

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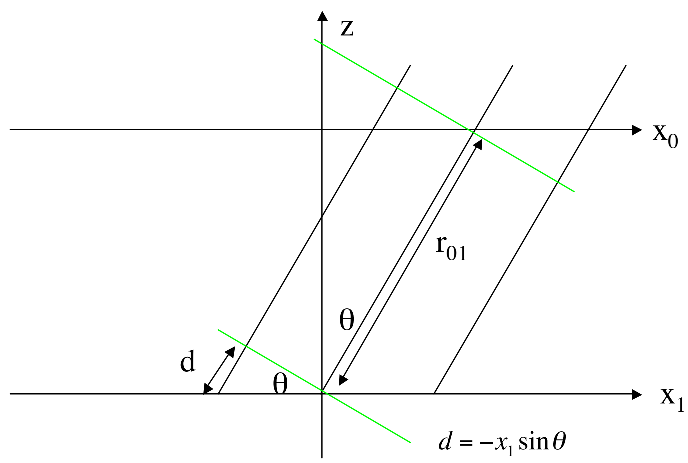
# Plane Wave (Fraunhofer) Approximation




$$d = -x_1 \sin\theta$$

$$\sin\theta = \frac{x_0}{r_{01}} \approx \frac{x_0}{z}$$

$$d \approx -\frac{x_0 x_1}{z}$$

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# Plane Wave Approximation

$$\frac{1}{r}\exp(jkr) \approx \frac{1}{z}\exp\big(jk(z+d)\big) = \frac{1}{z}\exp\left(j\frac{2\pi}{\lambda}\left(z - \frac{x_0 x_1}{z}\right)\right)$$

$$U(x_0) = \int_{-\infty}^{\infty} s(x_1)\frac{1}{r}\exp(jkr)dx_1$$

$$\approx \int_{-\infty}^{\infty} s(x_1)\frac{1}{z}\exp\left(j\frac{2\pi}{\lambda}\left(z - \frac{x_0 x_1}{z}\right)\right)dx_1$$

$$= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)\int_{-\infty}^{\infty} s(x_1)\exp\left(-\frac{j2\pi x_0 x_1}{\lambda z}\right)dx_1$$

$$= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)\int_{-\infty}^{\infty} s(x_1)\exp\left(-j2\pi k_x x_1\right)dx_1$$

$$= \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)F\left[s(x)\right]\Big|_{k_x = \frac{x_0}{\lambda z}}$$



# Plane Wave Approximation

In general

$$U(x_0, y_0) = \frac{1}{z}\exp\left(j\frac{2\pi z}{\lambda}\right)F\left[s(x,y)\right]\Big|_{k_x = \frac{x_0}{\lambda z}, k_y = \frac{y_0}{\lambda z},}$$

*Example*

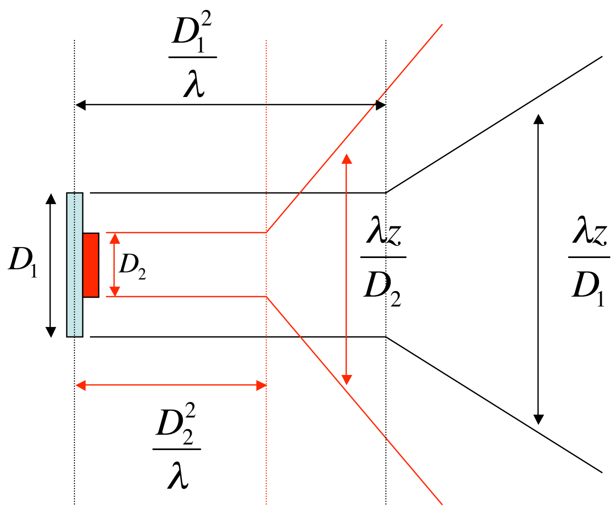
$$s(x,y) = rect(x/D)rect(y/D)$$

$$U(x_0, y_0) = \frac{1}{z}\exp(jkz)D^2 \operatorname{sinc}\left(Dk_x\right)\operatorname{sinc}\left(Dk_y\right)$$

$$= \frac{1}{z}\exp(jkz)D^2 \operatorname{sinc}\left(D\frac{x_0}{\lambda z}\right)\operatorname{sinc}\left(Dk_y\frac{y_0}{\lambda z}\right)$$

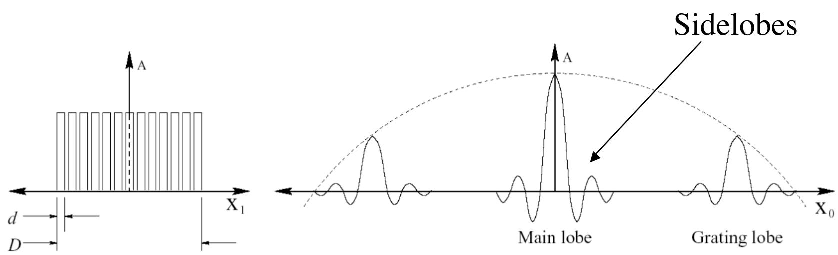
Zeros occur at $x_0 = \frac{n\lambda z}{D}$ and $y_0 = \frac{n\lambda z}{D}$

Beamwidth of the sinc function is $\frac{\lambda z}{D}$






# Example




$$rect\left(\frac{x}{D}\right)\left[rect\left(\frac{x}{d}\right)*\frac{1}{d}comb\left(\frac{x}{d}\right)\right] \Leftrightarrow D\,\mathrm{sinc}(Dk_x)*\left[d\,\mathrm{sinc}(dk_x)comb(dk_x)\right]$$

Question: What should we do to reduce the sidelobes?



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# Transducer Dimension

Goal: Operate in the Fresnel Zone

$z < D^2 / \lambda$

$D_{opt} \approx \sqrt{\lambda z_{\max}}$

*Example*

$z_{\max} = 20\ cm$

$\lambda = 0.5\ mm$

$D_{opt} = 1\ cm$



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# Focusing in Fresnel Zone

$$U(x_0,y_0) = \iint \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left((x_1-x_0)^2+(y_1-y_0)^2\right)\right)s(x_1,y_1)dx_1dy_1$$

$$= \iint \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left((x_1^2+y_1^2)+(x_0^2+y_0^2)-2(x_1x_0+y_1y_0)\right)\right)s(x_1,y_1)dx_1dy_1$$

$$= \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}(x_0^2+y_0^2)\right)\iint \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}(x_1^2+y_1^2)\right)\exp\left(-\frac{jk}{z}(x_1x_0+y_1y_0)\right)s(x_1,y_1)dx_1dy_1$$

Use time delays to compensate for this phase term

$$U(x_0,y_0) = \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}(x_0^2+y_0^2)\right)F\left[\exp\left(\frac{jk}{2z}(x_1^2+y_1^2)\right)s(x_1,y_1)\right]$$

# Focusing in Fresnel Zone

$$U(x_0,y_0) = \frac{\exp(jkz)}{j\lambda z}\exp\left(\frac{jk}{2z}\left(x_0^2+y_0^2\right)\right)F\left[\exp\left(\frac{jk}{2z}\left(x_1^2+y_1^2\right)\right)s(x_1,y_1)\right]$$

Make $s(x_1,y_1) = s_0(x_1,y_1)\exp\left(-\frac{jk}{2z_0}\left(x_1^2+y_1^2\right)\right)$

At the focal depth $z = z_0$

$$U(x_0,y_0) = \frac{\exp(jkz_0)}{j\lambda z_0}\exp\left(\frac{jk}{2z_0}\left(x_0^2+y_0^2\right)\right)F\left[s(x_1,y_1)\right]$$

Beamwidth at the focal depth is: $\frac{\lambda z_0}{D}$

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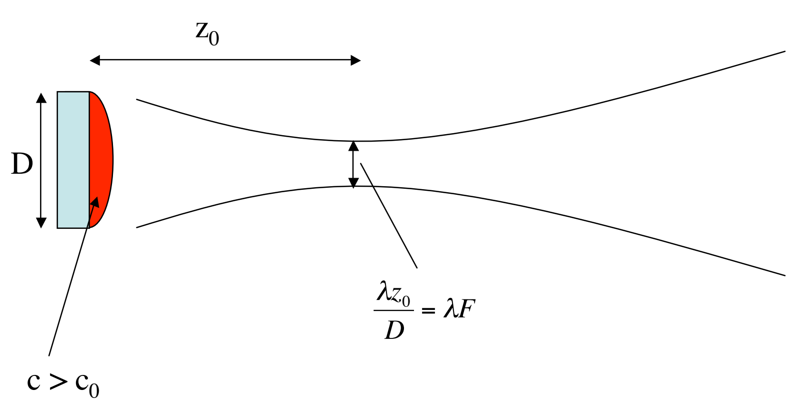
# Acoustic Lens

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# Depth of Focus

When $z \neq z_0$, the phase term is $\Delta\Phi = \exp\left(-\frac{jk}{2z_0}\left(x_1^2 + y_1^2\right)\right)\exp\left(-\frac{jk}{2z}\left(x_1^2 + y_1^2\right)\right)$

and the lens is not perfectly focused.

Consider variation in the x - direction.

$$\Delta\Phi = \frac{kx^2}{2}\left(\frac{1}{z} - \frac{1}{z_0}\right)$$

For transducer of size D, $\frac{x^2}{2} = \frac{D^2}{4}$

If we want $\left|\Delta\Phi\right| = \left|\frac{\pi D^2}{2\lambda}\left(\frac{1}{z} - \frac{1}{z_0}\right)\right| < 1$ radian then

$$\left|\frac{1}{z} - \frac{1}{z_0}\right| < \frac{2\lambda}{\pi D^2}$$

The larger the D, the smaller the depth of focus.