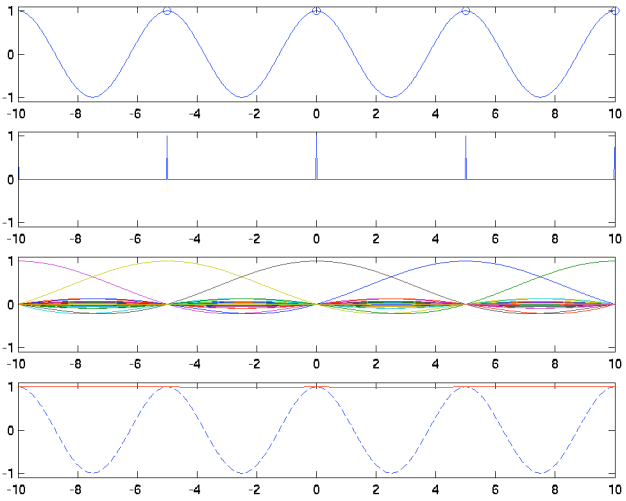
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2006
CT/Fourier Lecture 4

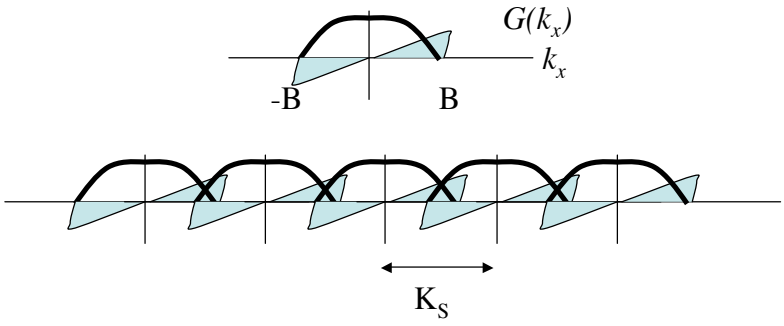
Topics

- Aliasing
- Sampling Requirements in CT
- Fanbeam and Spiral

Aliasing Example

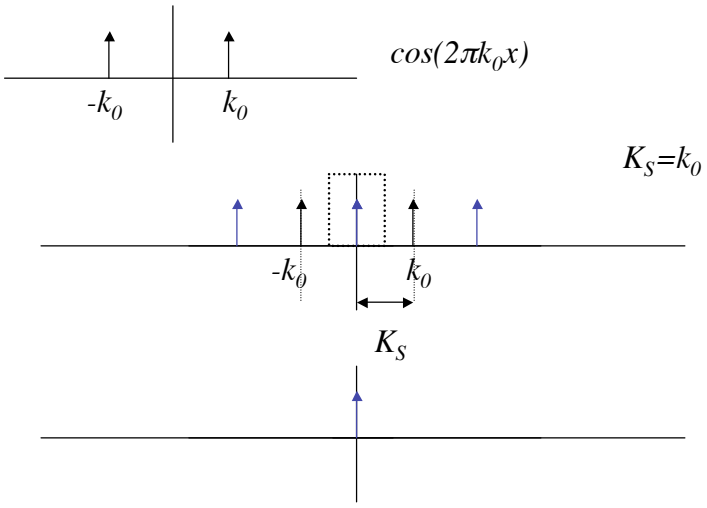


Aliasing

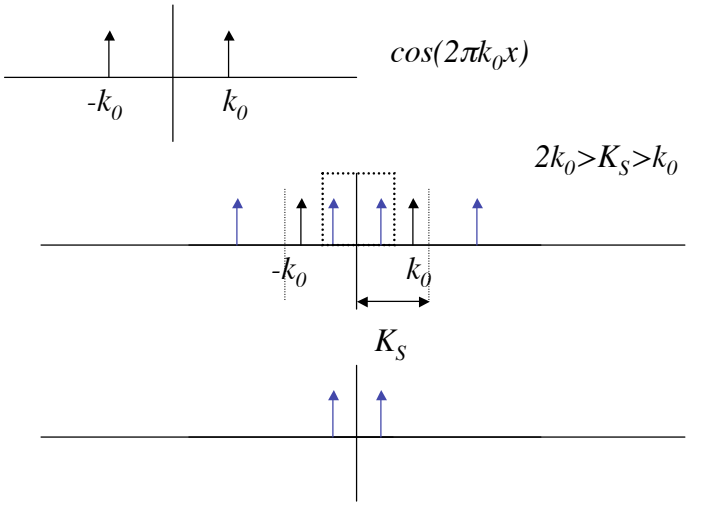


Aliasing occurs when the Nyquist condition is not satisfied.
This occurs for $K_S \leq 2B$

Aliasing Example



Aliasing Example

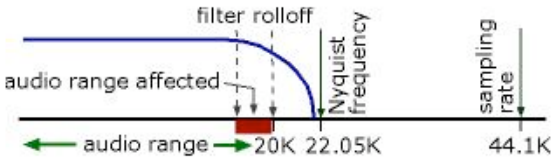


Anti-aliasing Filter

To minimize aliasing, filter the signal before sampling. Typically, choice of highest sampling rate is determined by technical considerations and cost.

Once the sampling rate has been determined, a low-pass filter is used to set the bandwidth of the signal.

Example: CD sampling rate = 44.1 KHz; But real-world music content has frequencies above 22 KHz. Apply an anti-aliasing filter that starts rolling off around 20 KHz.



http://www.indiana.edu/~emusic/etext/digital_audio/chapter5_rate.shtml

Oversampling

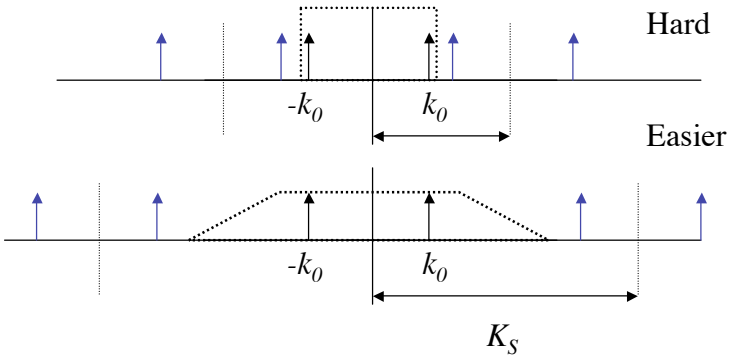
Why sample higher than the Nyquist frequency?

-- the requirements on the analog low-pass filter are reduced.

-- final filtering and downsampling can be done in the digital domain where it is easier to get nearly ideal filters.

rate	use(s)
32K	older DATs, voice quality
44.1K	CD, DAT, digital recording software/hardware
48K	DAT, digital recording software/hardware
96K	digital recording software/hardware
192K	digital recording software/hardware

Oversampling



Example

1. Consider the function $g(x) = \cos^2(2\pi k_0 x)$. Sketch this function. You sample this signal in the spatial domain with a sampling rate $K_S = 1/\Delta x$ (e.g. samples spaced at intervals of Δx). What is the minimum sampling rate that you can use without aliasing? Give an intuitive explanation for your answer.

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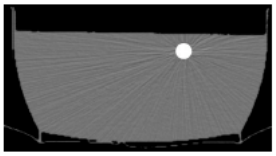
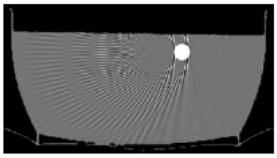
Artifacts

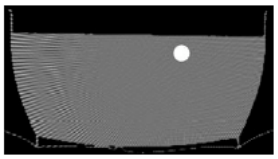
Object



Effect of Noise



Aliasing due to insufficient number of detectors



Aliasing due to insufficient number of views

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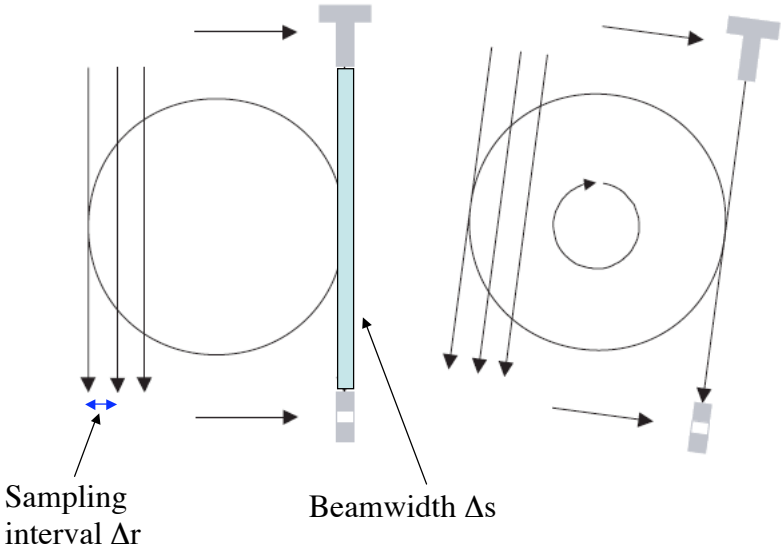
Detector Sampling Requirements



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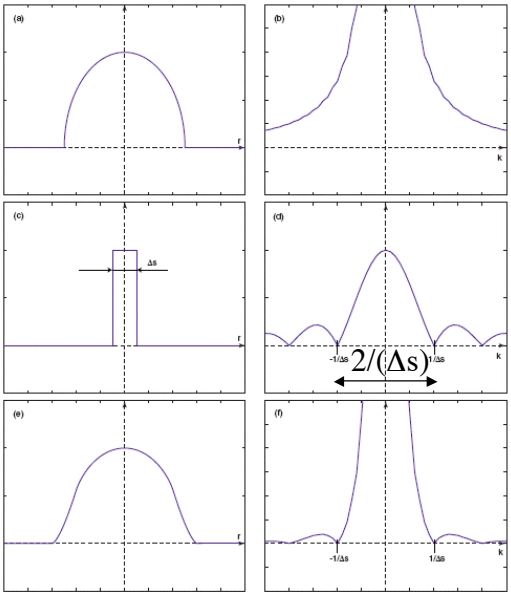
Smoothing of Projection

Projection

Beam Width

Smoothed Projection



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Smoothing of Projection

$$g_s(l,\theta) = rect(l/\Delta s) * g(l,\theta)$$

$$G_s(k_x,\theta) = \Delta s \sin c(k_x \Delta s) G(k_x,\theta)$$

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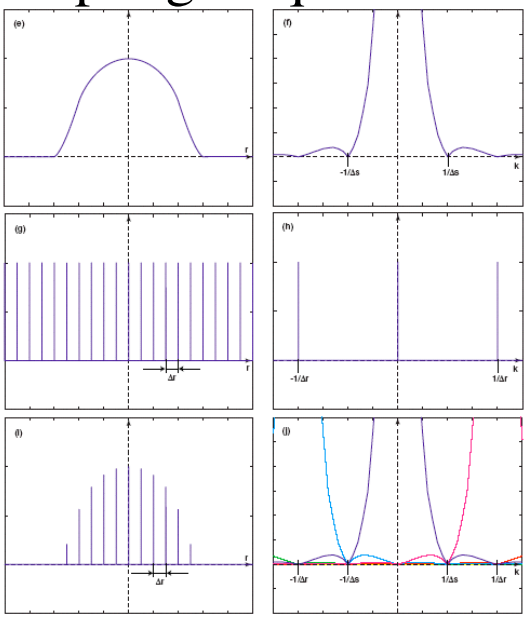
Sampling Requirements

Smoothed Projection

Detectors $\Delta r \leq \Delta s/2$

Sampled Smooth Projection



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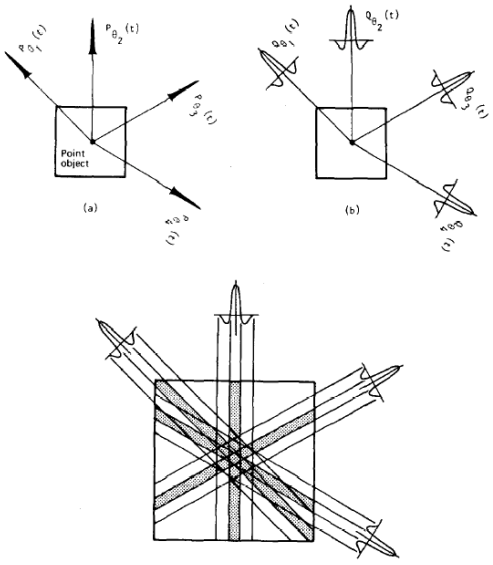
View Aliasing



Kak and Slaney

View Sampling Requirements

View Sampling -- how many views?

Basic idea is that to make the maximum angular sampling the same as the projection sampling.

$$\frac{\pi FOV}{N_{views}} = \Delta r$$

$$N_{views,360} = \frac{\pi FOV}{\Delta r} = \pi N_{proj} \quad \text{(for 360 degrees)}$$

$$N_{views,180} = \frac{\pi N_{proj}}{2} \quad \text{(for 180 degrees)}$$

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Example

beamwidth Δs = 1 mm

Field of View (FOV) = 50 cm

$\Delta r = \Delta s/2$ = 0.5 mm

500 mm/ 0.5 mm = N = 1000 detector samples

$\pi * N$ = 3146 views per 360 degrees

$\approx$ 1500 views per 180 degrees

CT "Rule of Thumb"

$N_{view} = N_{detectors} = N_{pixels}$

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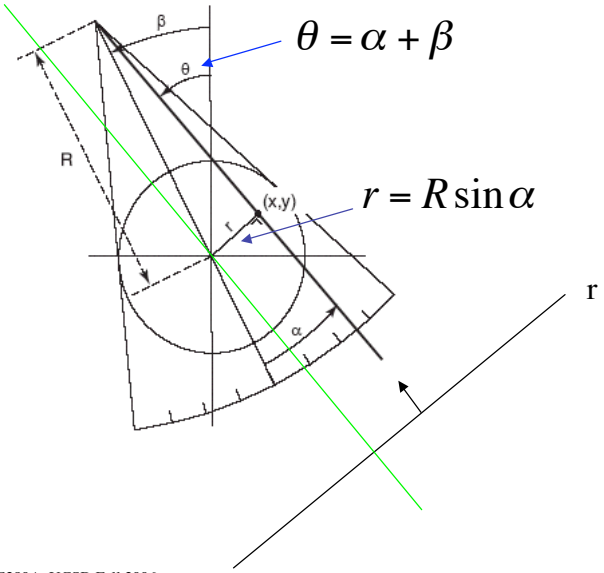
Fan Beam



$\theta = \alpha + \beta$

$r = R\sin\alpha$

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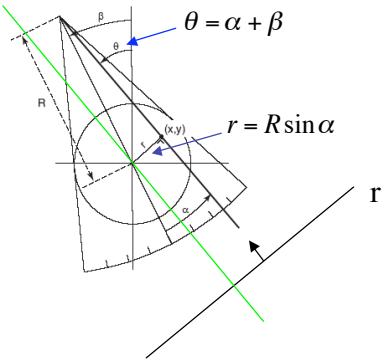
Fan Beam



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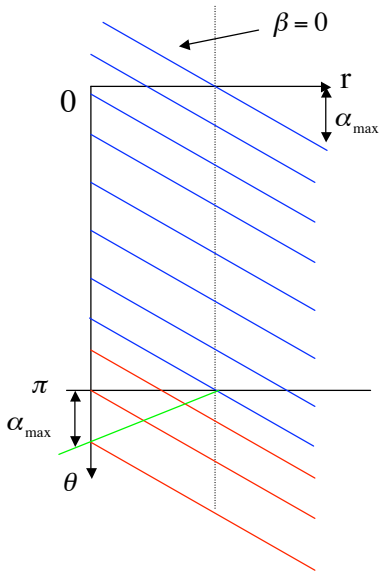

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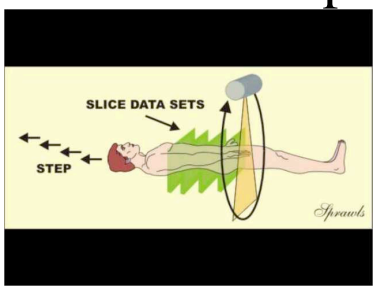
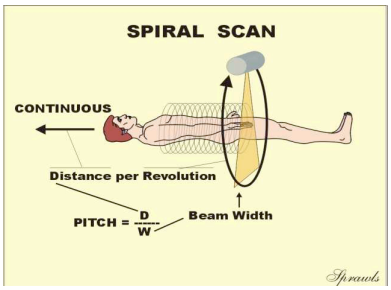
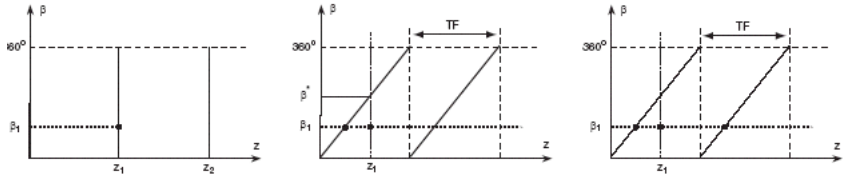
Spiral CT







Nearest Neighbor Interpolation

Linear Interpolation

TT Liu, BE280A, UCSD Fall 2006

From http://www.sprawls.org/resources/CTIMG/classroom.htm
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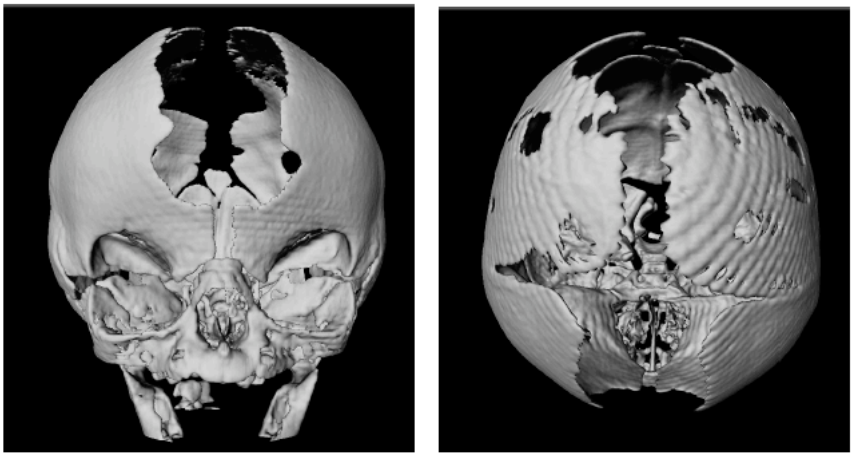
Longitudinal Aliasing in Spiral CT



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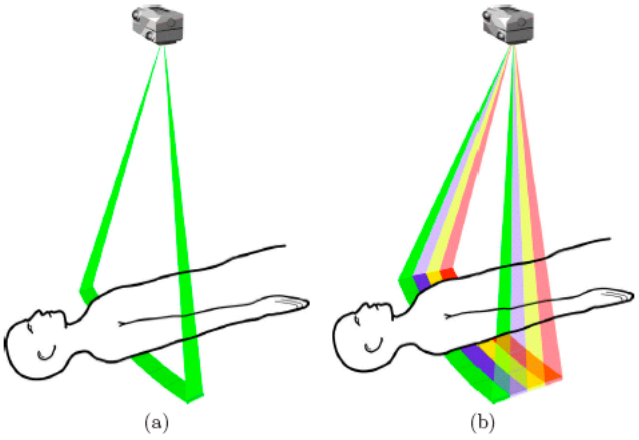
Multislice CT



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