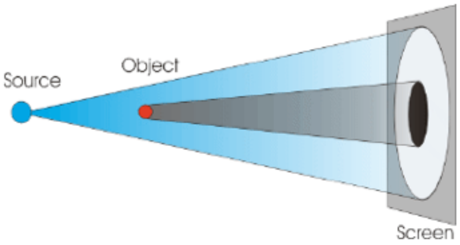
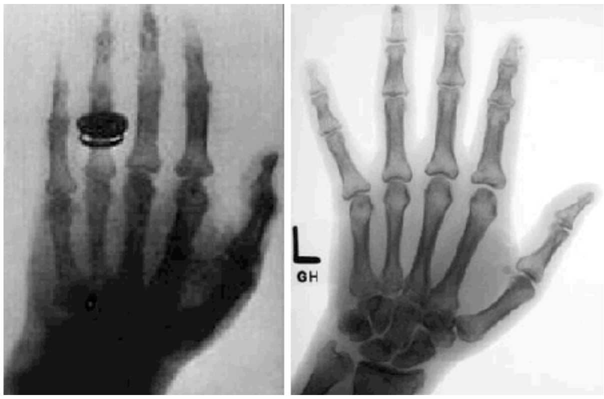
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2006
X-Rays Lecture 2

Topics

- Review topics from last lecture
- Attenuation
- Contrast
- Noise
- Image Equation

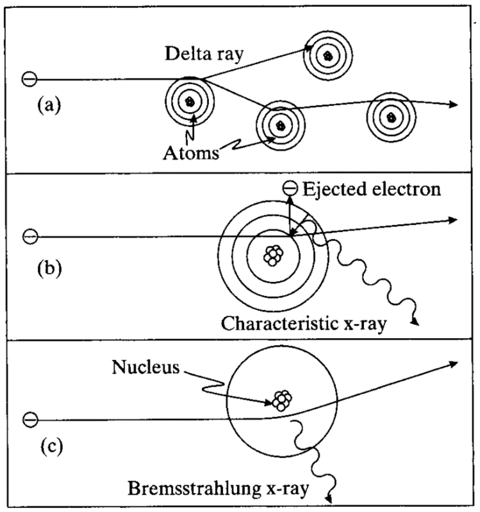




X-Ray Production

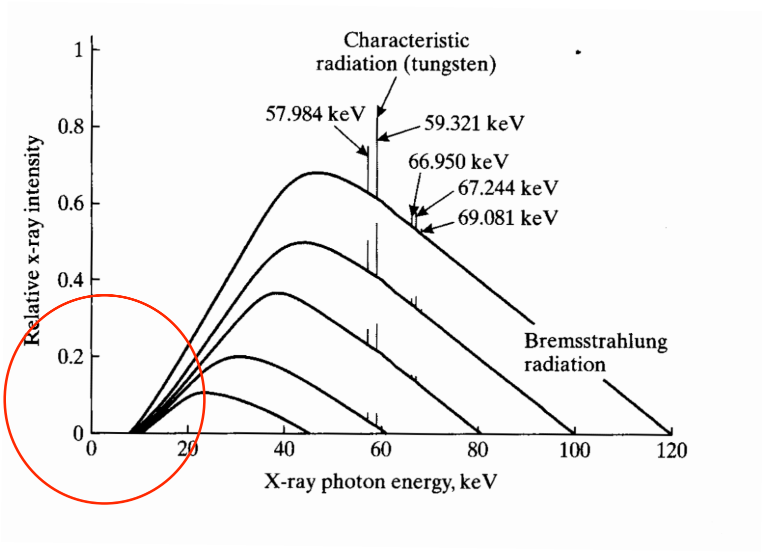
Collisional transfers

Radiative transfers



Prince and Links 2005

X-Ray Spectrum



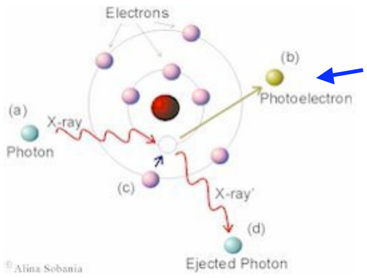
Lower energy photons are absorbed by anode, tube, and other filters

Prince and Links 2005

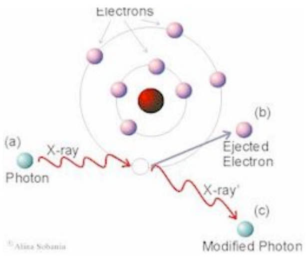
Interaction with Matter

Typical energy range for diagnostic x-rays is below 200 keV.

The two most important types of interaction are photoeletric absorption and Compton scattering.



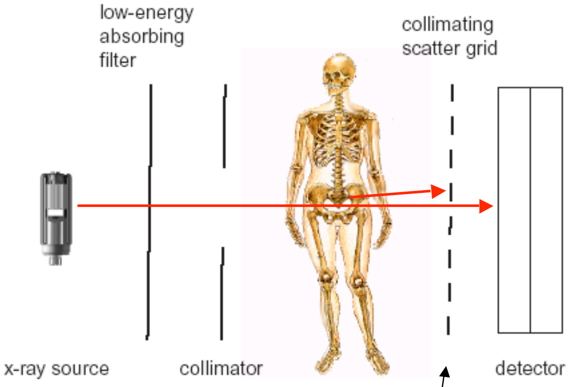
Photoelectric effect dominates at low x-ray energies and high atomic numbers. $E_{e^-} = \hbar\nu - E_B$



Compton scattering dominates at high x-ray energies and low atomic numbers, not much contrast

http://www.eee.ntu.ac.uk/research/vision/asobania

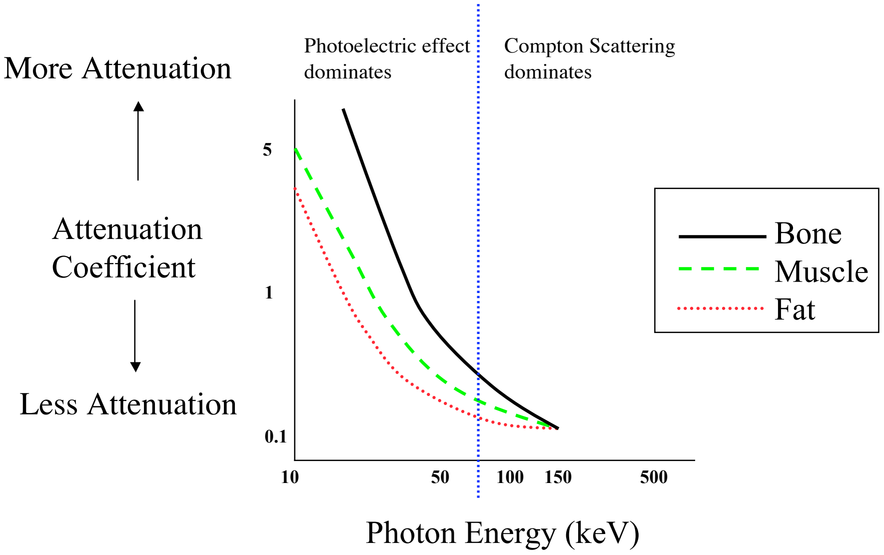
X-Ray Imaging Chain



Reduces effects of Compton scattering

Suetens 2002

Attenuation



Adapted from www.cis.rit.edu/class/simg215/xrays.ppt

Intensity

$$I = E\phi$$

Energy

Photon flux rate

Number of photons

$$\phi = \frac{N}{A\Delta t}$$

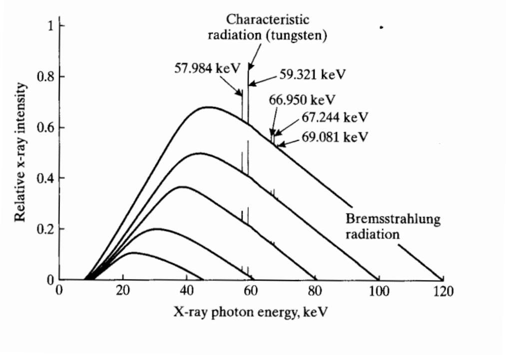
Unit Area

Unit Time

Intensity

$$\phi = \int_0^\infty S(E')dE'$$

X-ray spectrum



$$I = \int_0^\infty S(E')E'dE'$$

Attenuation

$n = \mu N \Delta x$ photons lost per unit length

$\mu = \dfrac{n/N}{\Delta x}$ fraction of photons lost per unit length

$$\Delta N = -n \longrightarrow \frac{dN}{dx} = -\mu N \longrightarrow N(x) = N_0 e^{-\mu x}$$

For mono-energetic case, intensity is

$$I(\Delta x) = I_0 e^{-\mu \Delta x}$$

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Attenuation

Inhomogeneous Slab

$$\frac{dN}{dx} = -\mu(x)N \longrightarrow N(x) = N_0 \exp\left(-\int_0^x \mu(x')dx'\right)$$

$$I(x) = I_0 \exp\left(-\int_0^x \mu(x')dx'\right)$$

Attenuation depends on energy, so also need to integrate over energies

$$I(x) = \int_0^\infty S_0(E')E' \exp\left(-\int_0^x \mu(x';E')dx'\right)dE'$$

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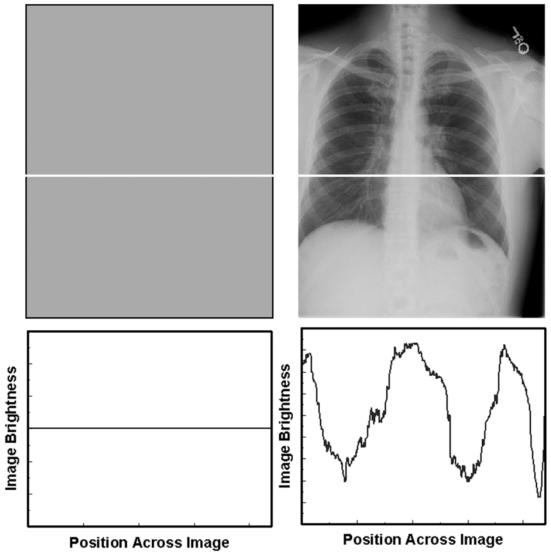
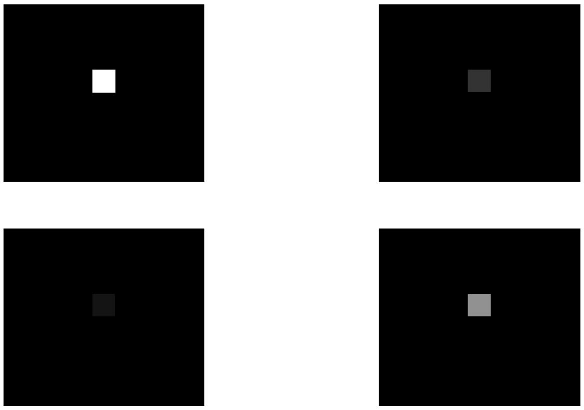
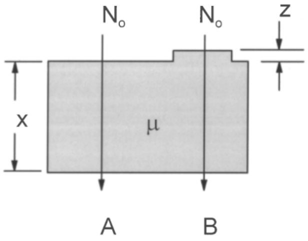
Contrast



Bushberg et al 2001

Contrast





(A) X-ray Imaging

Background intensity → $A = N_0 \exp(-\mu x)$ Bushberg et al 2001

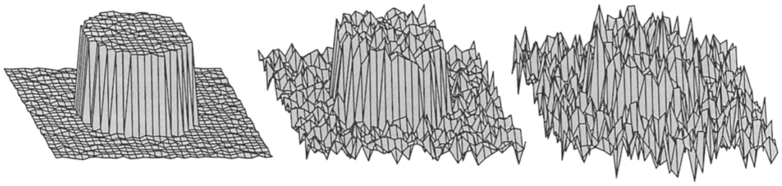
Object intensity → $B = N_0 \exp(-\mu(x+z))$

Subject/Local Contrast → $C_S = \dfrac{B-A}{A}$

$$= \frac{N_0 \exp(-\mu(x+z)) - N_0 \exp(-\mu x)}{N_0 \exp(-\mu x)}$$

$$= \exp(-\mu z) - 1$$

Noise and Image Quality



Low Noise Medium Noise High Noise

Bushberg et al 2001

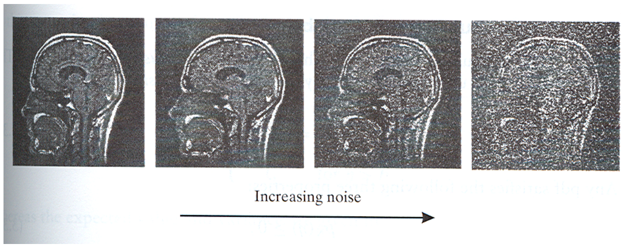


Figure 3.10
The effect of noise on image quality: image quality decreases rapidly with increasing noise contamination.

Prince and Links 2005

What is Noise?

Fluctuations in either the imaging system or the object being imaged.

Quantization Noise: Due to conversion from analog waveform to digital number.

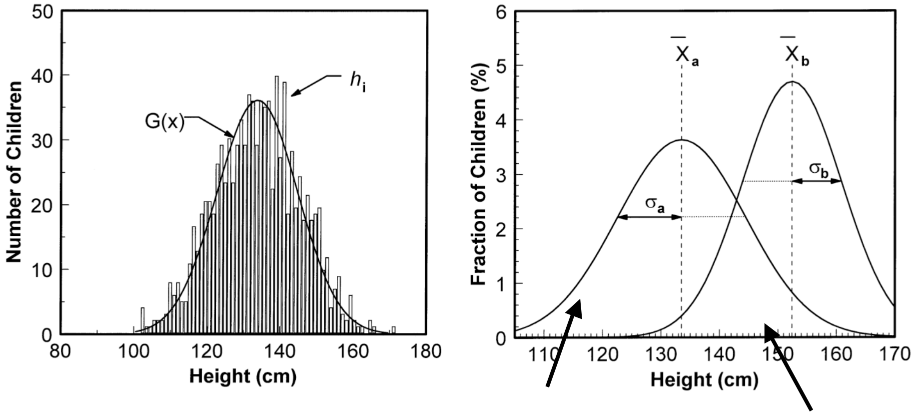
Quantum Noise: Random fluctuation in the number of photons emitted and recorded.

Thermal Noise: Random fluctuations present in all electronic systems. Also, sample noise in MRI

Other types: flicker, burst, avalanche - observed in semiconductor devices.

Structured Noise: physiological sources, interference

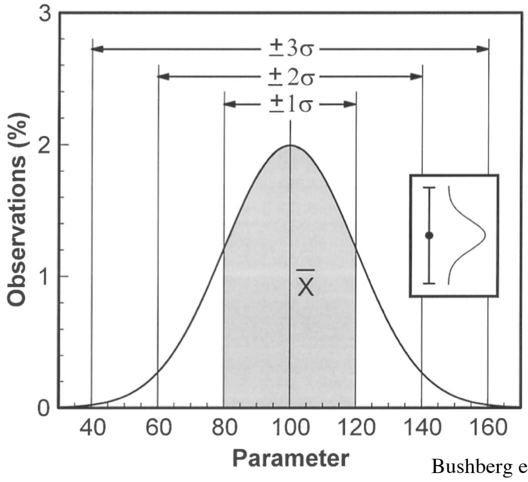
Histograms and Distributions



3rd grade heights   6th grade heights

Bushberg et al 2001

Gaussian Distribution



Bushberg et al 2001

1, 2, and 3 standard deviation intervals correspond to 68%, 95%, and 99% of the observations

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Poisson Process

Events occur at random instants of time at an average rate of λ events per second.

Examples: arrival of customers to an ATM, emission of photons from an x-ray source, lightning strikes in a thunderstorm.

Assumptions:

1) Probability of more than 1 event in an small time interval is small.
2) Probability of event occurring in a given small time interval is independent of another event occuring in other small time intervals.

Poisson Process

$$P\left[N(t) = k\right] = \frac{\left(\lambda t\right)^k}{k!}\exp(-\lambda t)$$

λ = Average rate of events per second
λt = Average number of events at time *t*
λt = Variance in number of events

Probability of interarrival times
$P\left[T > t\right] = e^{-\lambda t}$

Example

A service center receives an average of 15 inquiries per minute. Find the probability that 3 inquiries arrive in the first 10 seconds.

$\lambda = 15/60 = 0.25$
$\lambda t = 0.25(10) = 2.5$

$$P[N(t=10) = 3) = \frac{(2.5)^3}{3!}\exp(-2.5) = .2138$$

Quantum Noise

Fluctuation in the number of photons emitted by the x-ray source and recorded by the detector.

$$P_k = \frac{N_0^k \exp(-N_0)}{k!}$$

P_k : Probability of emitting k photons in a given time interval.

N_0 : Average number of photons emitted in that time interval = λt

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Transmitted Photons

$$Q_k = \frac{(tN_0)^k \exp(-tN_0)}{k!}$$

Q_k : Probability of k photons making it through object

N_0 : Average number of photons emitted in that time interval = λt

$t = \exp(-\int \mu dz) =$ fraction of photons transmitted

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Mean and Variance

For a Poisson process, the mean = variance, i.e. $\overline{X} = \sigma^2$

Therefore, the standard deviation is given by $\sigma = \sqrt{\overline{X}}$

For X-ray systems, if the mean number of counts is N, then the standard deviation in the number of counts is $\sigma = \sqrt{N}$.

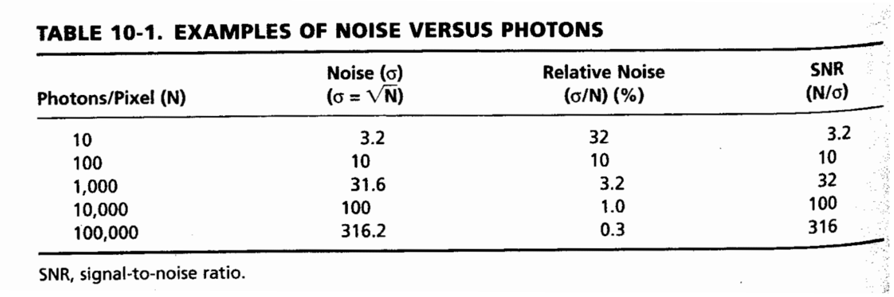
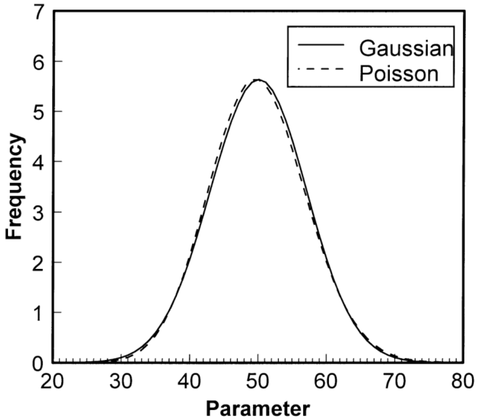


TABLE 10-1. EXAMPLES OF NOISE VERSUS PHOTONS

Photons/Pixel (N)	Noise (σ) ($\sigma = \sqrt{N}$)	Relative Noise (σ/N) (%)	SNR (N/σ)
10	3.2	32	3.2
100	10	10	10
1,000	31.6	3.2	32
10,000	100	1.0	100
100,000	316.2	0.3	316

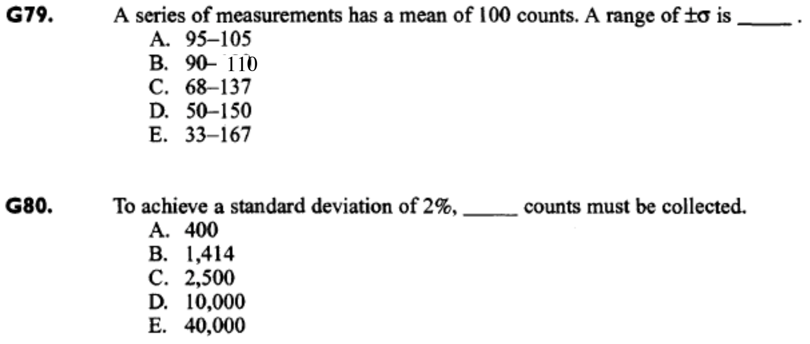
SNR, signal-to-noise ratio.



Bushberg et al 2001

Poisson Distribution describes x-ray counting statistics.

Gaussian distribution is good approximation to Poisson when $\sigma = \sqrt{\overline{X}}$



G79. A series of measurements has a mean of 100 counts. A range of ±σ is _____ .

A. 95–105
B. 90– 110
C. 68–137
D. 50–150
E. 33–167

G80. To achieve a standard deviation of 2%, _____ counts must be collected.

A. 400
B. 1,414
C. 2,500
D. 10,000
E. 40,000

G73. A radioactive sample is counted many times, and the mean is 2500 counts. 96% of the readings will lie between _____ and _____ counts.

- A. 2300 2500
- B. 2400 2500
- C. 2400 2600
- D. 2450 2550
- E. 2500 2700

D70. How many counts must be collected in an instrument with zero background to obtain an error limit of 1% with a confidence interval of 95%?

- A. 1000
- B. 3162
- C. 10,000
- D. 40,000
- E. 100,000

Contrast and SNR for X-Rays

$$Contrast = C = \frac{I_t - I_b}{I_b} \qquad SNR = \frac{I_t - I_b}{\sigma_b}$$

$$I_b = \frac{N_b \cdot E}{A\Delta t} \rightarrow \mathrm{var}(I_b) = \mathrm{var}(N_b)\left(\frac{E}{A\Delta t}\right)^2 = N_b\left(\frac{E}{A\Delta t}\right)^2$$

$$\sigma_b = std(I_b) = \sqrt{N_b}\left(\frac{E}{A\Delta t}\right)$$

$$SNR = \frac{CI_b}{\sigma_b} = C\sqrt{N_b} = C\sqrt{\Phi A R t \eta}$$

Detector efficiency

Fraction transmitted

Photons/Roentegen/cm^2

Area

Exposure in Roentgens

Example

$\Phi = 637 \times 10^{6}$ photons $\mathrm{R}^{-1} cm^{-2}$

$R = 50$ mR

t = 0.05

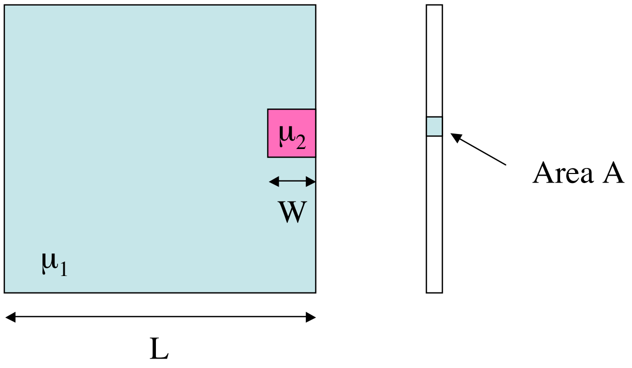
$\eta =$ 0.25

A = $1\mathrm{mm}^2$

$C = 0.1$ (10% contrast)

$\mathrm{SNR} = 0.1\sqrt{6.37 \times 10^{8} \cdot .05 \cdot .25 \cdot .01} = 6.3$

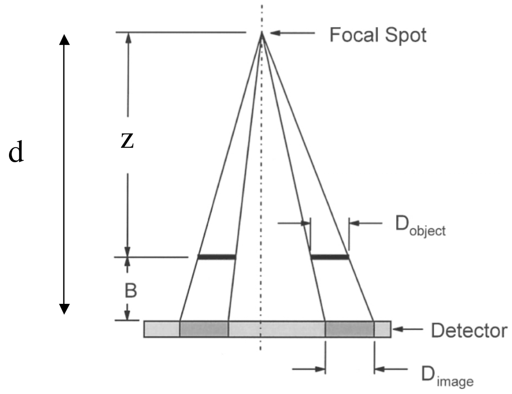
$20\log_{10}(6.3) = 16\ dB$



$$C = \frac{I_t - I_b}{I_b} = \frac{N_0\left(\exp\left(-\left(\mu_1(L-W)+\mu_2 W\right)\right) - \exp(-\mu_1 L)\right)}{N_0 \exp(-\mu_1 L)}$$

$$SNR = C\sqrt{N_0 A \exp(-\mu_1 L)}$$

Magnification of Object

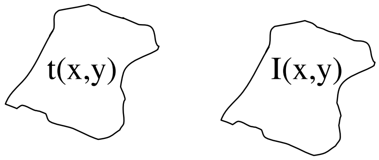


$$M(z) = \frac{d}{z}$$

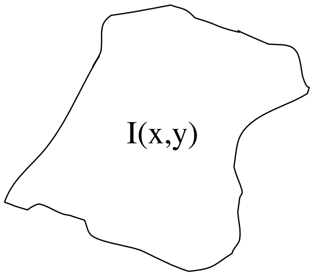
$$= \frac{\text{Source to Image Distance (SID)}}{\text{Source to Object Distance (SOD)}}$$

Bushberg et al 2001

Magnification of Object



M = 1: I(x,y) = t(x,y)



M = 2: I(x,y) = t(x/2,y/2)

In general, I(x,y) = t(x/M(z),y/M(z))

Bushberg et al 2001

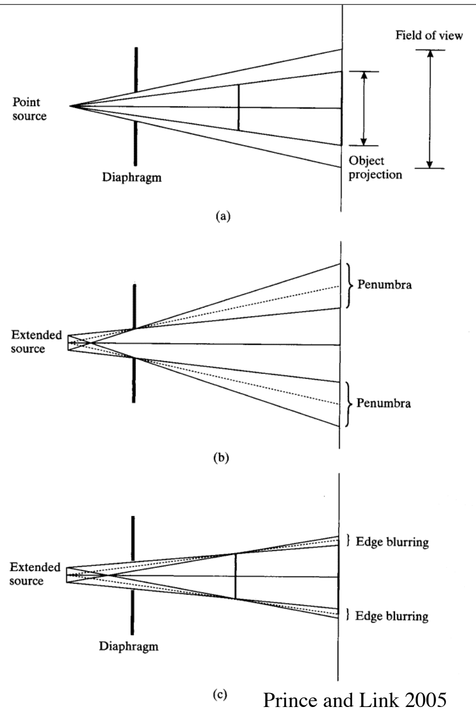
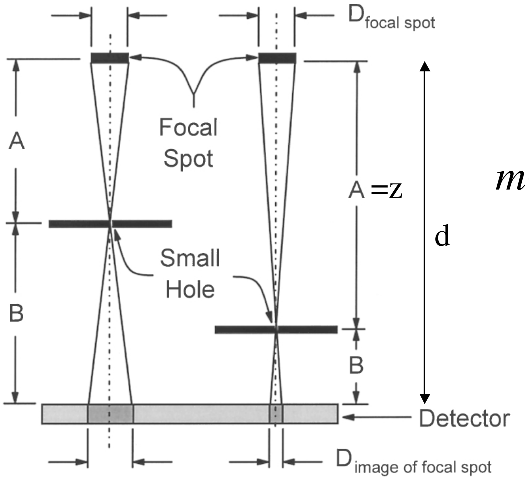


Figure 5.18
Effects of extended source. (a) Ideal field of view and object projection (with magnification). (b) Penumbra at edges of field of view due to extended source. (c) Blurred object edges due to extended source.

Prince and Link 2005

Source magnification



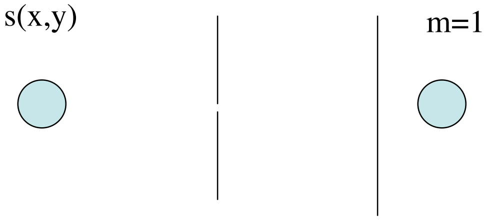
Bushberg et al 2001

$$m(z) = -\frac{d-z}{z} = -\frac{B}{A}$$
$$= 1 - M(z)$$

Image of a point object

$$I_d(x,y) = \lim_{m \to 0} ks(x/m, y/m)$$

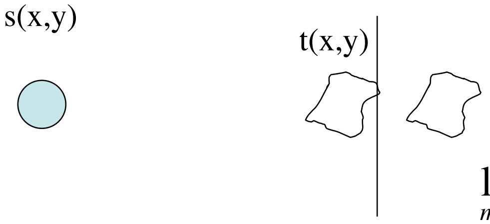
$$= \delta(x,y)$$



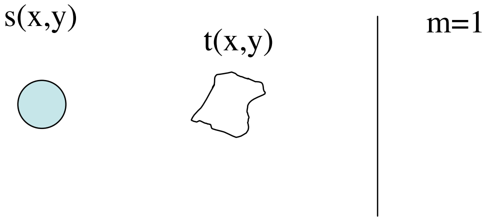
$$I_d(x,y) = ks(x,y)$$

In general, $I_d(x,y) = ks(x/m, y/m)$

Image of arbitrary object



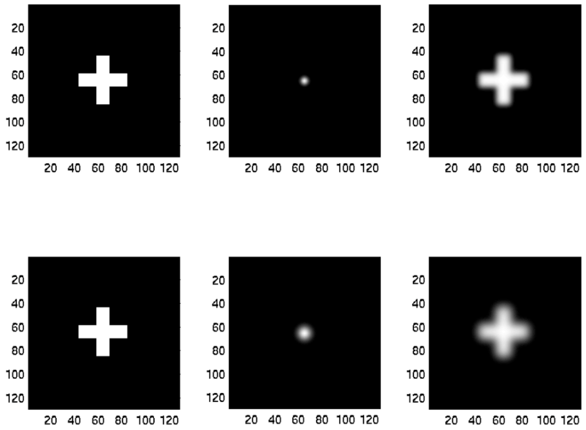
$$\lim_{m \to 0} I_d(x,y) = t(x,y)$$



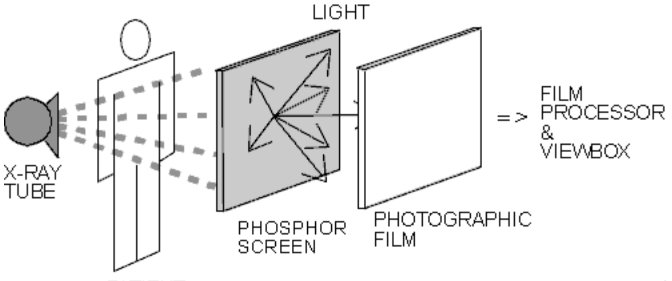
$$I_d(x,y) = ???$$

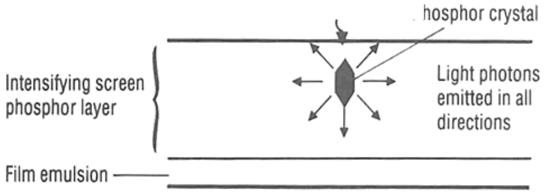
$$I_d(x,y) = ks(x/m, y/m) ** t(x/M, y/M)$$

Convolution



Film-screen blurring





$$I_d(x,y) = ks(x/m, y/m) ** t(x/M, y/M) ** h(x,y)$$

http://learntech.uwe.ac.uk/radiography/RScience/imaging_principles_d/diagimage11.htm
http://www.sunnybrook.utoronto.ca:8080/~selenium/xray.html#Film