

Bioengineering 280A
Principles of Biomedical Imaging

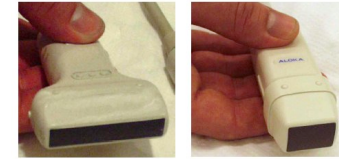
Fall Quarter 2007
Ultrasound Lecture 1

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Acuson Sequoia

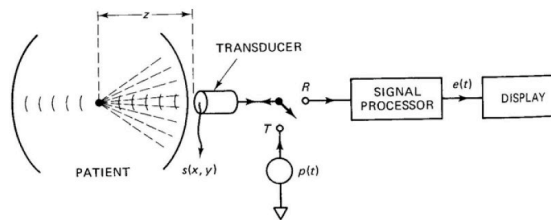
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Sonosite 180 From Sutens 2002



Basic System

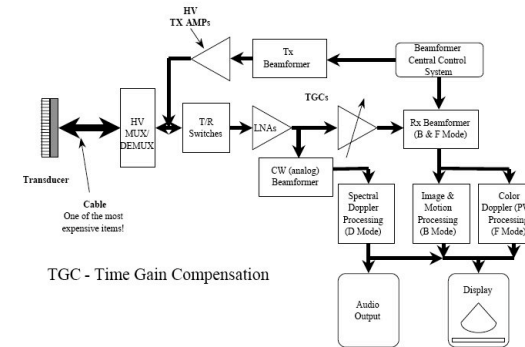


Echo occurs at $t=2z/c$ where c is approximately
1500 m/s or 1.5 mm/ μ s

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Macovski 1983

Basic System

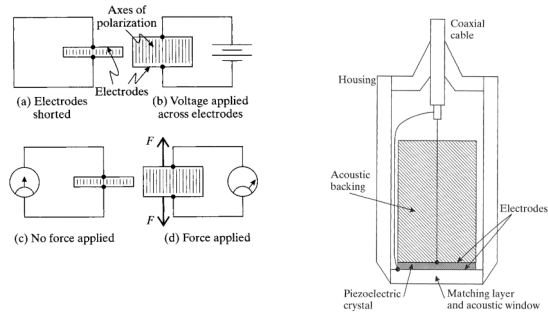


TGC - Time Gain Compensation

Brunner 2002

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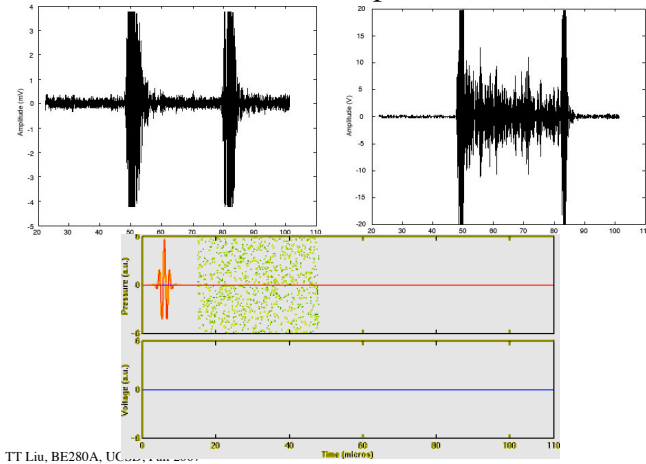
Transducer



Prince and Links 2006

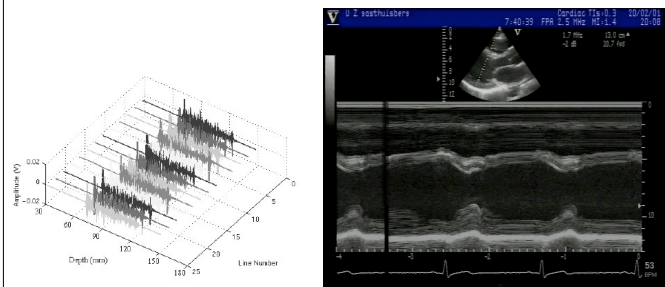
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A-Mode (Amplitude)



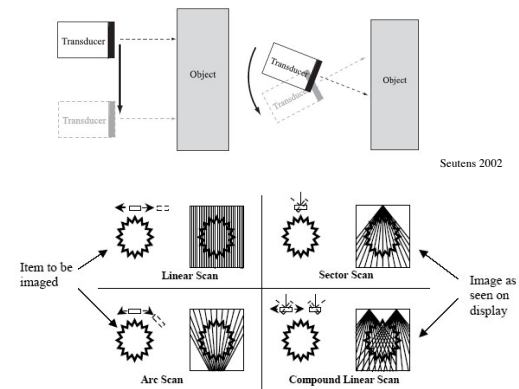
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M-Mode (Motion)



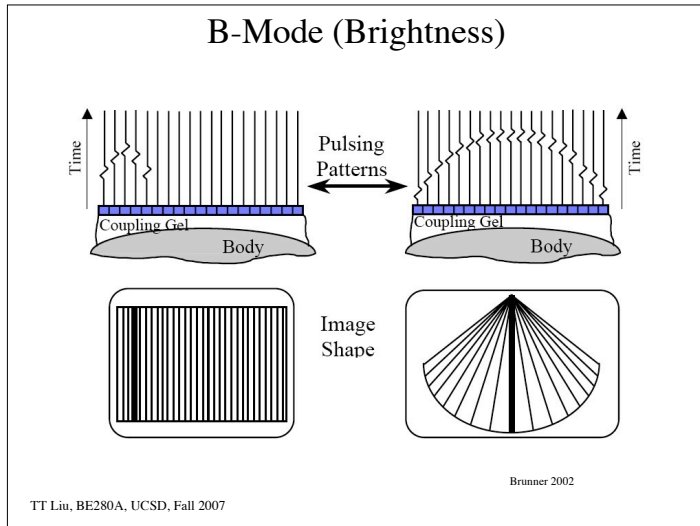
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B-Mode (Brightness)



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Brunner 2002



B-Mode

Seutens 2002

Mayo Clinic

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B-Mode

Aorta Wall

Atherosclerotic Plaque

Credit: Mayo Clinic

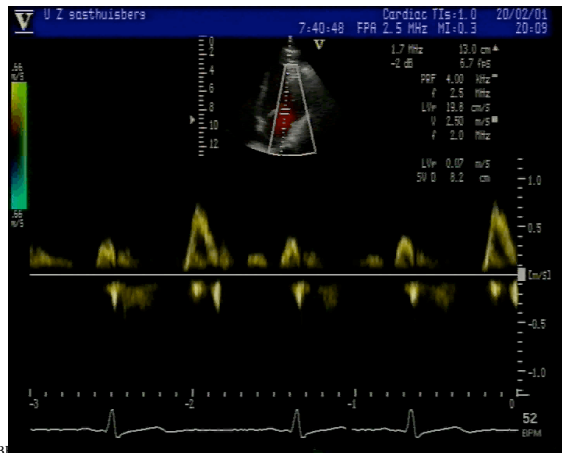
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CW Doppler Imaging

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Seutens 2002

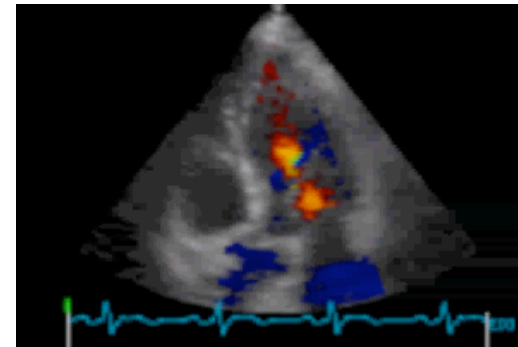
PW Doppler Imaging



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Seutens 2002

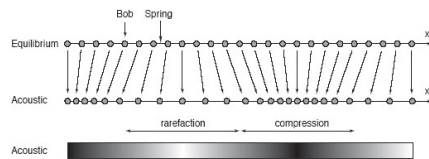
Color Doppler Imaging



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Seutens 2002

Acoustic Waves



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Suetens 2002

Speed of Sound

$$c = \sqrt{\frac{1}{\kappa\rho}} \quad [\text{m s}^{-1}]$$

κ = compressibility $[\text{m s}^2 \text{kg}^{-1}] = [1/\text{Pascal}]$

ρ = density $[\text{kg m}^{-3}]$

Material	Density	Speed m/s
Air	1.2	330
Water	1000	1480
Bone	1380-1810	4080
Fat	920	1450
Liver	1060	1570

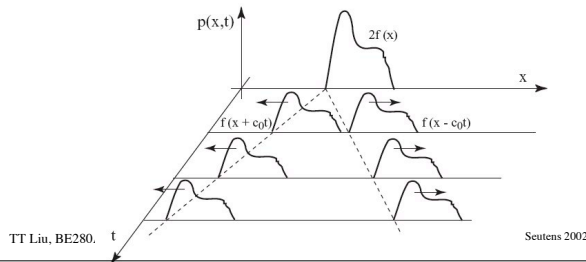
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Acoustic Wave Equation

$$\nabla^2 p = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

Solutions are of the form

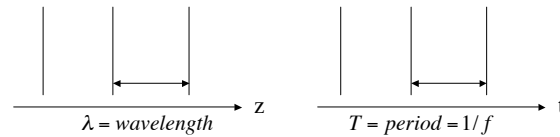
$$p(x, t) = A_1 f_1(x - ct) + A_2 f_2(x + ct)$$



Plane Waves

$$\begin{aligned} p(z, t) &= \cos(k(z - ct)) \\ &= \cos\left(\frac{2\pi}{\lambda}(z - ct)\right) \\ &= \cos\left(\frac{2\pi f}{c}(z - ct)\right) \\ &= \cos(2\pi f(z/c - t)) \end{aligned}$$

$$\begin{aligned} p(z, t) &= \exp(jk(z - ct)) \\ k &= \text{wavenumber} = \frac{2\pi}{\lambda} = 2\pi k_z \\ \lambda &= \text{wavelength} = \frac{c}{f} \\ f &= \text{frequency [cycles/sec]} \\ T &= \text{period} = \frac{1}{f} \end{aligned}$$



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Spherical Waves

Outward wave Inward wave

$$p(r, t) = \frac{1}{r} \phi(t - r/c) + \frac{1}{r} \phi(t + r/c)$$

Outward wave

$$p(r, t) = \frac{1}{r} \exp(j2\pi f(t - r/c))$$



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Impedance

$$\text{Impedance } Z = \frac{\text{Pressure}}{\text{Velocity}} = \frac{P}{v} = \rho c = \sqrt{\frac{\rho}{\kappa}}$$

density kg/m³ speed of sound

- Brain 1541 m/s
- Liver 1549
- Skull bone 4080 m/s
- Water 1480 m/s

Note: particle velocity and speed of sound are not the same!

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Impedance

$$Z = \rho c = \sqrt{\frac{\rho}{\kappa}}$$

Material	Density	Speed m/s	Z (kg/m ² /s)
Air	1.2	330	0.0004
Water	1000	1480	1.5
Bone	1380-1810	4080	3.75-7.38
Fat	920	1450	1.35
Liver	1060	1570	1.64-1.68

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Acoustic Intensity

$$I = pv$$

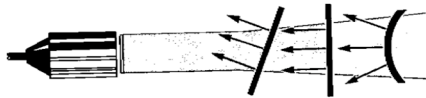
$$= \frac{p^2}{Z}$$

Also called acoustic energy flux.
Analogous to electric power

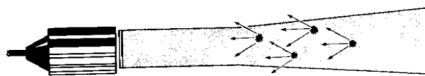
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Echos

SPECULAR ECHOES

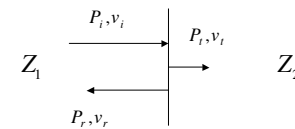


SCATTERED ECHOES



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Specular Reflection



Material	Reflectivity
Brain-skull	0.66
Fat-muscle	0.10
Muscle-blood	0.03
Soft-tissue-air	.9995

$$v_i - v_r = v_t \quad (\text{velocity boundary condition})$$

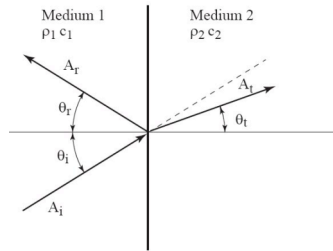
$$\frac{P_i}{Z_1} - \frac{P_r}{Z_1} = \frac{P_t}{Z_2}$$

$$P_i + P_r = P_t \quad (\text{pressure boundary condition})$$

$$R = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \approx \frac{\Delta Z}{Z_0}$$

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Reflection and Refraction



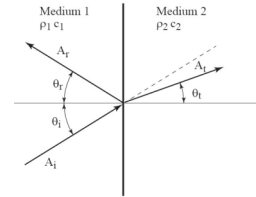
Snell's Law

$$\frac{\sin \theta_i}{c_1} = \frac{\sin \theta_r}{c_1} = \frac{\sin \theta_t}{c_2}$$

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Reflection and Refraction



$$v_i \cos \theta_i = v_r \cos \theta_r + v_t \cos \theta_t$$

$$\frac{p_i}{Z_1} \cos \theta_i = \frac{p_r}{Z_1} \cos \theta_r + \frac{p_t}{Z_2} \cos \theta_t$$

$$p_i + p_r = p_t$$

Pressure Reflectivity

$$R = \frac{p_r}{p_i} = \frac{Z_2 \cos \theta_i - Z_1 \cos \theta_t}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$

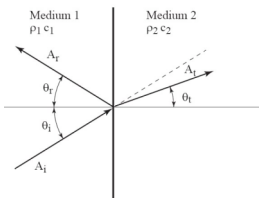
Pressure Transmittivity

$$T = \frac{p_t}{p_i} = \frac{2Z_2 \cos \theta_i}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$

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Reflection and Refraction



Intensity Reflectivity $R_I = \frac{I_r}{I_i} = \frac{p_r^2}{p_i^2} = \left(\frac{Z_2 \cos \theta_i - Z_1 \cos \theta_t}{Z_2 \cos \theta_i + Z_1 \cos \theta_t} \right)^2$

Intensity Transmittivity $T_I = \frac{I_t}{I_i} = \frac{p_t^2 Z_1}{p_i^2 Z_2} = \frac{4Z_1 Z_2 \cos^2 \theta_i}{(Z_2 \cos \theta_i + Z_1 \cos \theta_t)^2}$

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Example

Example : Fat/liver interface at normal incidence

$$Z_{fat} = 1.35 \times 10^{-6} \text{ kg m}^{-2} \text{ s}^{-1}$$

$$Z_{liver} = 1.66 \times 10^{-6} \text{ kg m}^{-2} \text{ s}^{-1}$$

$$R_I = \left(\frac{Z_{liver} - Z_{fat}}{Z_{liver} + Z_{fat}} \right)^2 = 0.103$$

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Scattering

Point scatterers retransmit the incident wave equally in all direction (e.g. isotropic scattering).

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Attenuation

Loss of acoustic energy during propagation.
Conversion of acoustic energy into heat.

$$p(z,t) = A_z f(t - c/z)$$

$$= A_0 \exp(-\mu_a z) f(t - c/z)$$

Amplitude attenuation factor

$$\mu_a = -\frac{1}{z} \ln \frac{A_z}{A_0} \quad : \text{ units = nepers/cm}$$

$$\alpha = -20 \frac{1}{z} \log_{10} \frac{A_z}{A_0} = 20 \mu_a \log_{10}(e) \approx 8.7 \mu_a \quad : \text{ dB/cm}$$

↑
Attenuation coefficient

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Attenuation

$$\alpha(f) = \alpha_0 f^n$$

For frequencies used in medical ultrasound, $n \approx 1$.

$$\alpha(f) \approx \alpha_0 f$$

Material	α_0 [dB/cm/MHz]
fat	0.63
liver	0.94
Cardiac muscle	1.8
bone	20.0

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Example

Example : Fat at 5 MHz

$$\text{Attenuation coefficient} = 5 \text{ MHz} \times 0.63 \text{ dB/cm/MHz}$$

$$= 3.15 \text{ dB/cm}$$

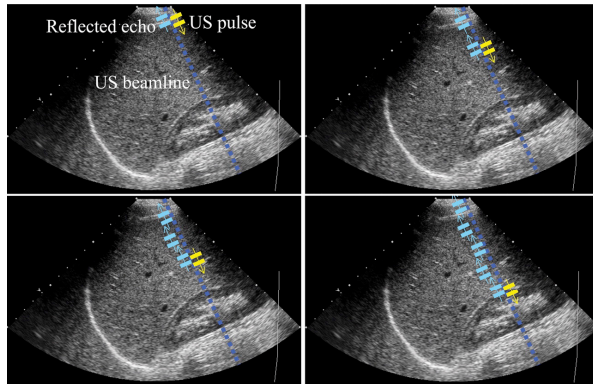
After 4 cm, attenuation = $4 * 3.15 = 12.6 \text{ dB}$

Relative amplitude is $10^{(-12.6/20)} = 0.2344$

Recall $\text{dB} = 20 \log_{10}(A_z / A_0)$

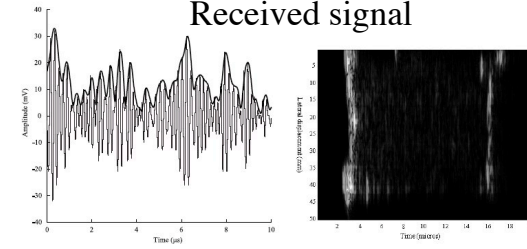
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Received signal



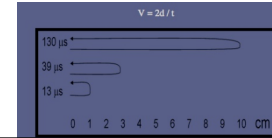
TT Liu, BE280A, UCSD, Fall 2007 <http://radiographics.rsna.org/content/vol23/issue4/images/large/g03j25c1x.jpeg>

Received signal



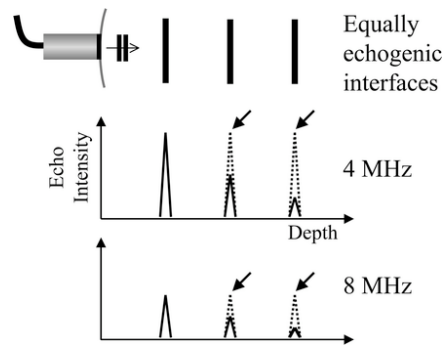
$$e(t) = K \int \int \int \frac{e^{-2\alpha z}}{z} R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

Attenuation (points to $e^{-2\alpha z}$)
 Reflection/Scattering (points to $R(x, y, z)$)
 Beam width (points to $s(x, y)$)
 Pulse (points to $p(t - 2z/c)$)



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Attenuation Correction



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Attenuation Correction and Signal Equation

$$e(t) = K \int \int \int \frac{e^{-2\alpha z}}{z} R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

$$\approx K \frac{e^{-\alpha ct}}{ct/2} \int \int \int R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

$$e_c(t) = ct e^{\alpha ct} e(t)$$

$$\approx K \int \int \int R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

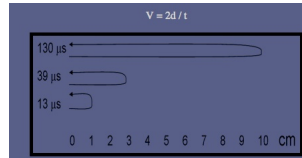
$$= \frac{c}{2} \int \int \int R(x, y, c\tau/2) s(x, y) p(t - \tau) dx dy d\tau$$

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Depth Response

Depth response

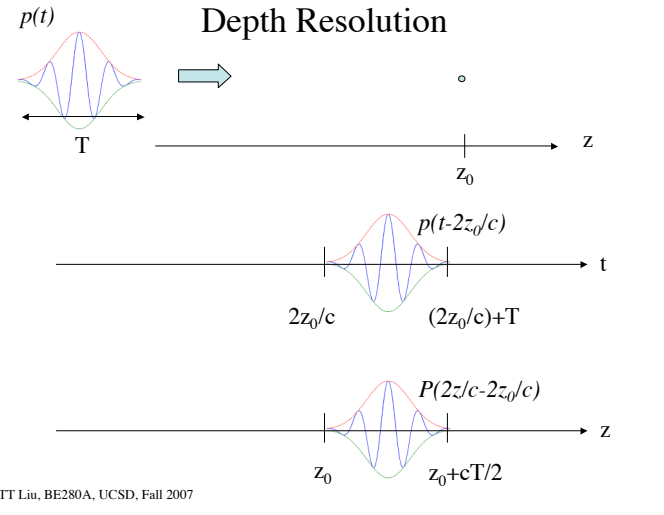
$$p(t - 2z_0/c) = p(2z/c - 2z_0/c) \\ = p\left(\frac{2(z - z_0)}{c}\right)$$



Therefore impulse response is simply $p(t)$ in the time domain or $p(2z/c)$ in the spatial domain

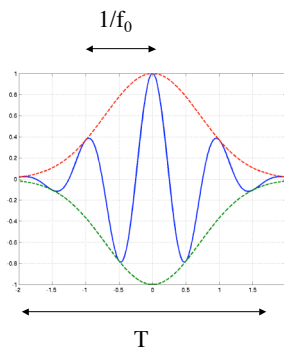
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Depth Resolution



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Depth Resolution



$p(t) = p(2z/c)$ determines the depth resolution

Pulses are of the form $a(t)\cos(2\pi f_0 t + \theta)$ where $a(t)$ is the envelope function and f_0 is the resonant frequency of the transducer.

The duration of T of $a(t)$ is typically chosen to be about 2 or 3 periods (e.g. $T = 3/f_0$). If the duration is too short, the bandwidth of the pulse will be very large and much of its power will be attenuated.

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c/f_0 = 1.5\lambda$.

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Depth Resolution

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c/f_0 = 1.5\lambda$.

Example :

For $f_0 = 5$ MHz, $\lambda = c/f = (1500\text{ m/s})/(5 \times 10^6\text{ Hz}) = 0.3\text{ mm}$

$\Delta z = 1.5\lambda = 0.45$ mm

Trade - off

Higher $f_0 \Rightarrow$ Smaller $\Delta z \Rightarrow$ but more attenuation

Example : Assume 1dB/cm/MHz

For 10 cm depth, 20 cm roundtrip path length.

At 1 MHz 20 dB of attenuation \Rightarrow Attenuation = 0.1

At 10 MHz 200 dB of attenuation \Rightarrow Attenuation = 1×10^{-10}

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Depth of Penetration

Assume system can handle L dB of loss, then

$$L = 20 \log_{10} \left(\frac{A_z}{A_0} \right)$$

We also have the definition

$$\alpha = -\frac{1}{z} 20 \log_{10} \left(\frac{A_z}{A_0} \right)$$

and the approximation

$$\alpha = \alpha_0 f$$

Total range a wave can travel before attenuation L is

$$z = \frac{L}{\alpha_0 f}$$

Depth of penetration is

$$d_p = \frac{L}{2\alpha_0 f}$$

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Depth of Penetration

Assume L = 80 dB; $\alpha_0 = 1 \text{ dB/cm/MHz}$

Frequency (MHz)	Depth of Penetration (cm)
1	40
2	20
3	13
5	8
10	4
20	2

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Pulse Repetition and Frame Rate

Need to wait for echoes to return before transmitting new pulse

Pulse repetition interval is

$$T_R \geq \frac{2d_p}{c}$$

Pulse repetition rate is

$$f_R = \frac{1}{T_R}$$

If N pulses are required to form an image, then the frame rate is

$$F = \frac{1}{NT_R}$$

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Example

$N = 256$, $L = 80 \text{ dB}$, $c = 1540 \text{ m/s}$, $\alpha_0 = 1 \text{ dB/cm/MHz}$

What frequency should be used to achieve a frame rate of 15 frame/sec?

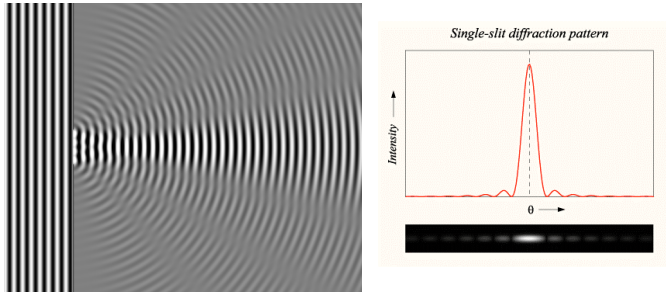
$$T_R = \frac{1}{FN} = 0.26 \text{ ms}$$

$$T_R \geq \frac{2d_p}{c} = \frac{L}{\alpha_0 f c}$$

$$f \geq \frac{L}{acT_R} = 1.99 \text{ MHz}$$

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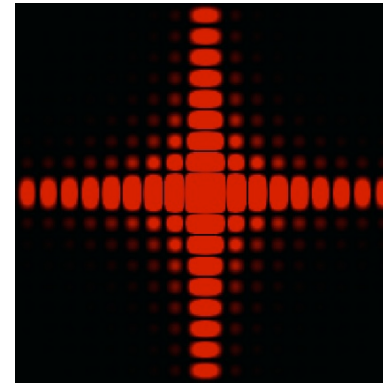
Single-slit Diffraction



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Source:wikipedia

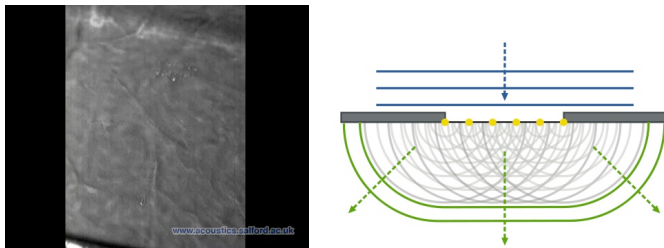
Square Aperture Diffraction



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Source:wikipedia

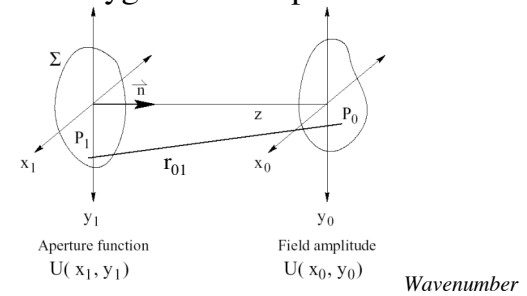
Huygen's Principle



Source:wikipedia

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Huygen's Principle



$$U(P_0) = \iint_{\Sigma} h(P_0, P_1) U(P_1) ds$$

where $h(P_0, P_1) = \frac{1}{j\lambda} \frac{\exp(jk\vec{r}_{01})}{r_{01}} \cos(\vec{n}, \vec{r}_{01})$

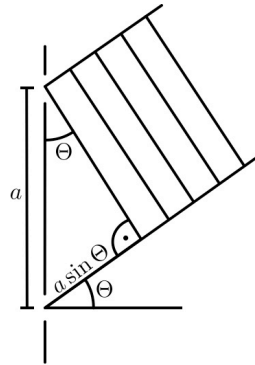
Wavenumber $k = \frac{2\pi}{\lambda}$

Oliquity Factor

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Anderson and Trabey 2000

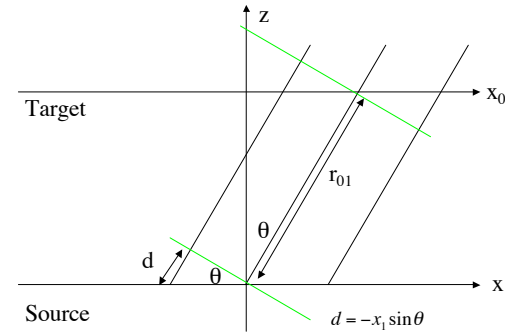
Two-Slit Interference



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Source:wikipedia

Plane Wave (Fraunhofer) Approximation



Assume overall distance for phase calculation is approximately $z+d$

$$d = -x_1 \sin \theta$$

$$\sin \theta = \frac{x_0}{r_{01}} \approx \frac{x_0}{z}$$

$$d \approx -\frac{x_0 x_1}{z}$$

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Plane Wave Approximation

$$\frac{1}{r} \exp(jkr) \approx \frac{1}{z} \exp(jk(z+d)) = \frac{1}{z} \exp\left(j \frac{2\pi}{\lambda} \left(z - \frac{x_0 x_1}{z}\right)\right)$$

$$U(x_0) = \int_{-\infty}^{\infty} s(x_1) \frac{1}{r} \exp(jkr) dx_1$$

$$\approx \int_{-\infty}^{\infty} s(x_1) \frac{1}{z} \exp\left(j \frac{2\pi}{\lambda} \left(z - \frac{x_0 x_1}{z}\right)\right) dx_1$$

$$= \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) \int_{-\infty}^{\infty} s(x_1) \exp\left(-\frac{j 2\pi x_0 x_1}{\lambda z}\right) dx_1$$

$$= \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) \int_{-\infty}^{\infty} s(x_1) \exp(-j 2\pi k_x x_1) dx_1$$

$$= \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) F[s(x)] \Big|_{k_x = \frac{x_0}{\lambda z}}$$

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Plane Wave Approximation

In general

$$U(x_0, y_0) = \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) F[s(x, y)] \Big|_{k_x = \frac{x_0}{\lambda z}, k_y = \frac{y_0}{\lambda z}}$$

Example

$$s(x, y) = \text{rect}(x/D) \text{rect}(y/D)$$

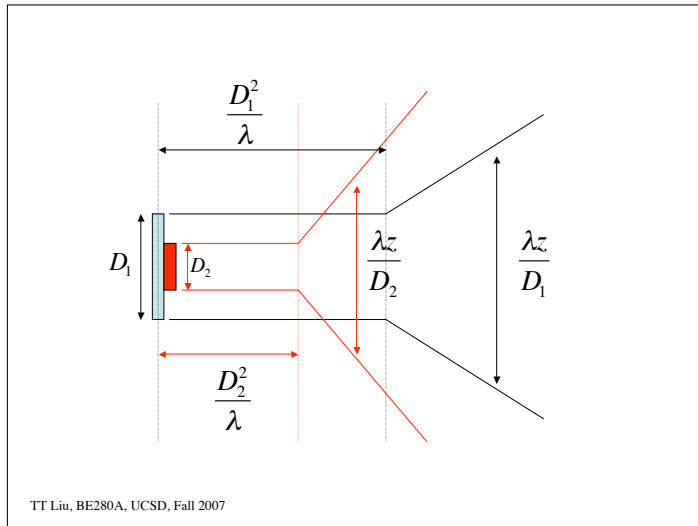
$$U(x_0, y_0) = \frac{1}{z} \exp(jkz) D^2 \text{sinc}(Dk_x) \text{sinc}(Dk_y)$$

$$= \frac{1}{z} \exp(jkz) D^2 \text{sinc}\left(D \frac{x_0}{\lambda z}\right) \text{sinc}\left(Dk_y \frac{y_0}{\lambda z}\right)$$

$$\text{Zeros occur at } x_0 = \frac{n\lambda z}{D} \text{ and } y_0 = \frac{n\lambda z}{D}$$

$$\text{Beamwidth of the sinc function is } \frac{\lambda z}{D}$$

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Example

$$\text{rect}\left(\frac{x}{D}\right) \left[\text{rect}\left(\frac{x}{d}\right) * \frac{1}{d} \text{comb}\left(\frac{x}{d}\right) \right] \Leftrightarrow D \text{sinc}(Dk_x) * [d \text{sinc}(dk_x) \text{comb}(dk_x)]$$

Question: What should we do to reduce the sidelobes?

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