Bioengineering 280A
Principles of Biomedical Imaging

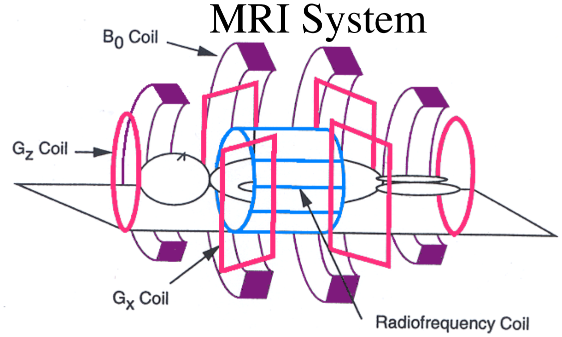
Fall Quarter 2007
MRI Lecture 2

Gradients

Spins precess at the Larmor frequency, which is proportional to the local magnetic field. In a constant magnetic field $B_z = B_0$, all the spins precess at the same frequency (ignoring chemical shift).

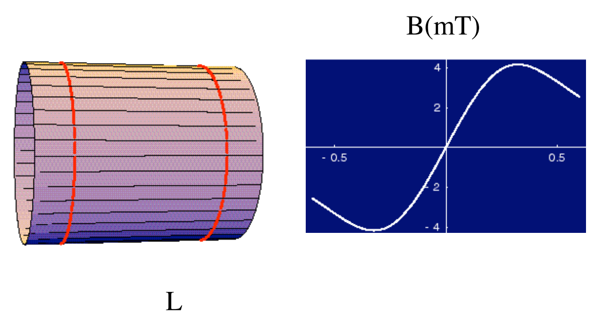
Gradient coils are used to add a spatial variation to B_z such that $B_z(x,y,z) = B_0 + \Delta B_z(x,y,z)$. Thus, spins at different physical locations will precess at different frequencies.

MRI System



Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

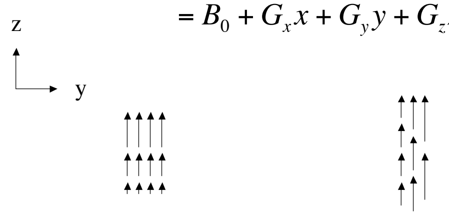
Z Gradient Coil



Credit: Buxton 2002

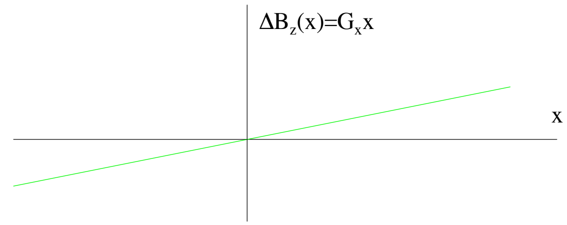
Gradient Fields

$$B_z(x,y,z) = B_0 + \frac{\partial B_z}{\partial x}x + \frac{\partial B_z}{\partial y}y + \frac{\partial B_z}{\partial z}z$$
$$= B_0 + G_x x + G_y y + G_z z$$



$$G_z = \frac{\partial B_z}{\partial z} > 0 \qquad G_y = \frac{\partial B_z}{\partial y} > 0$$

Interpretation



Spins Precess at $\gamma B_0 - \gamma G_x x$ (slower)

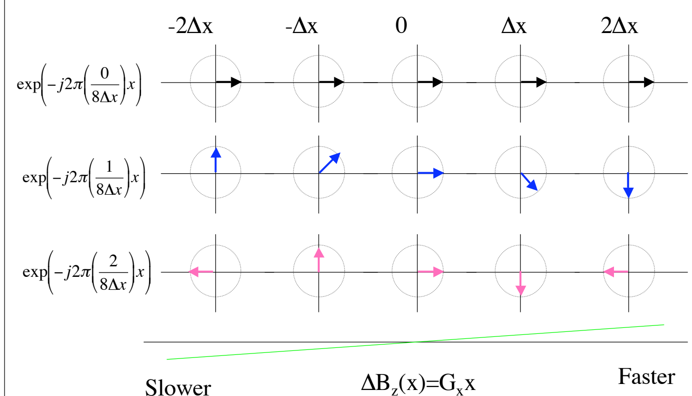
Spins Precess at γB_0

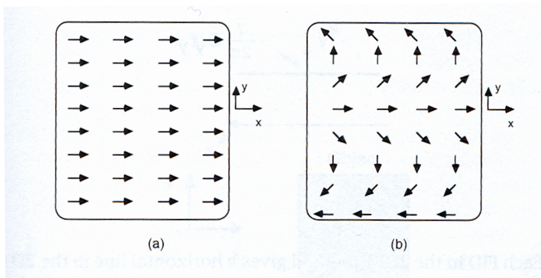
Spins Precess at $\gamma B_0 + \gamma G_x x$ (faster)

Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

Interpretation

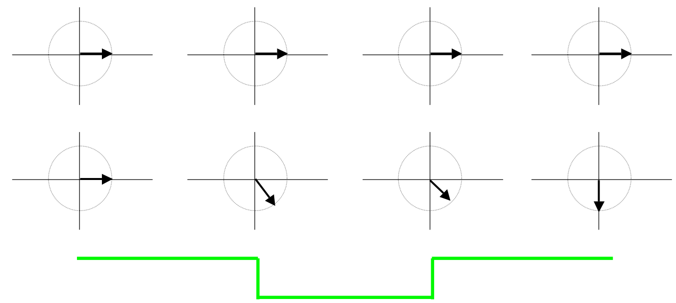




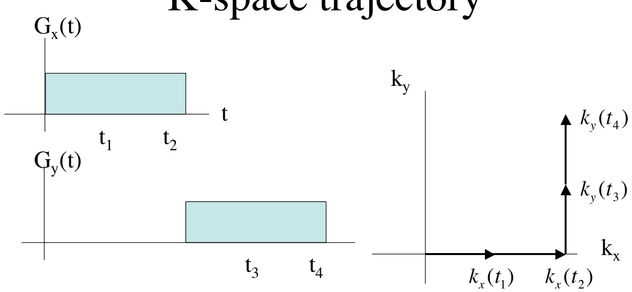
$k_x=0; k_y=0$ $\quad$ $k_x=0; k_y\neq 0$

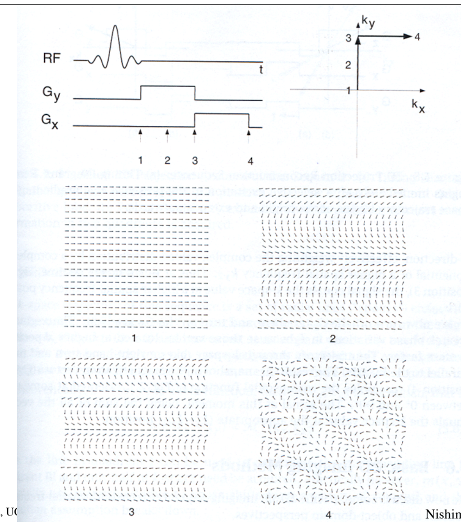
Fig 3.12 from Nishimura

Phase with time-varying gradient



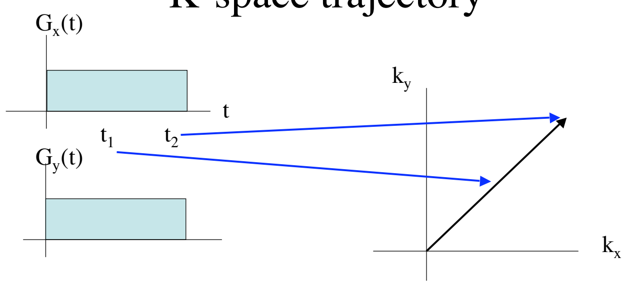
K-space trajectory



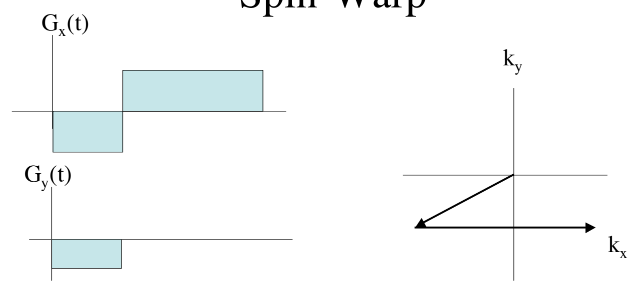


Nishimura 1996

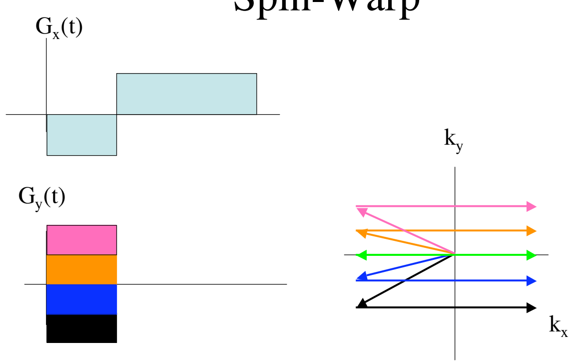
K-space trajectory



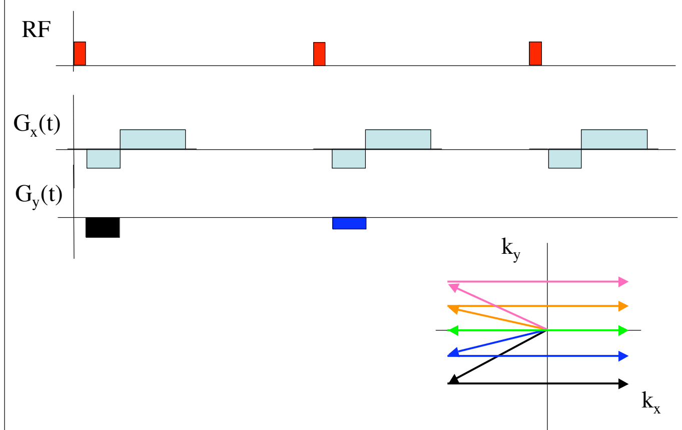
Spin-Warp



Spin-Warp



Spin-Warp Pulse Sequence



Gradient Fields

Define

$$\vec{G} \equiv G_x\hat{i} + G_y\hat{j} + G_z\hat{k} \qquad \vec{r} \equiv x\hat{i} + y\hat{j} + z\hat{k}$$

So that

$$G_x x + G_y y + G_z z = \vec{G}\cdot\vec{r}$$

Also, let the gradient fields be a function of time. Then the z-directed magnetic field at each point in the volume is given by :

$$B_z(\vec{r},t) = B_0 + \vec{G}(t)\cdot\vec{r}$$

Static Gradient Fields

In a uniform magnetic field, the transverse magnetization is given by:

$$M(t) = M(0)e^{-j\omega_0 t}e^{-t/T_2}$$

In the presence of non time-varying gradients we have

$$\begin{aligned} M(\vec{r}) &= M(\vec{r},0)e^{-j\gamma B_z(\vec{r})t}e^{-t/T_2(\vec{r})} \\ &= M(\vec{r},0)e^{-j\gamma(B_0+\vec{G}\cdot\vec{r})t}e^{-t/T_2(\vec{r})} \\ &= M(\vec{r},0)e^{-j\omega_0 t}e^{-j\gamma\vec{G}\cdot\vec{r}t}e^{-t/T_2(\vec{r})} \end{aligned}$$

Time-Varying Gradient Fields

In the presence of time-varying gradients the frequency as a function of space and time is:

$$\begin{aligned} \omega(\vec{r},t) &= \gamma B_z(\vec{r},t) \\ &= \gamma B_0 + \gamma\vec{G}(t)\cdot\vec{r} \\ &= \omega_0 + \Delta\omega(\vec{r},t) \end{aligned}$$

Phase

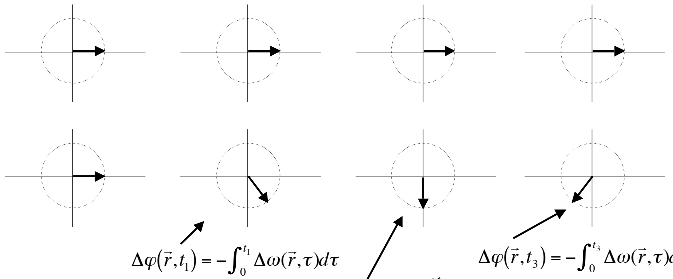
Phase = angle of the magnetization phasor
Frequency = rate of change of angle (e.g. radians/sec)
Phase = time integral of frequency

$$\begin{aligned} \varphi(\vec{r},t) &= -\int_0^t \omega(\vec{r},\tau)d\tau \\ &= -\omega_0 t + \Delta\varphi(\vec{r},t) \end{aligned}$$

Where the incremental phase due to the gradients is

$$\begin{aligned} \Delta\varphi(\vec{r},t) &= -\int_0^t \Delta\omega(\vec{r},\tau)d\tau \\ &= -\int_0^t \gamma\vec{G}(\vec{r},\tau)\cdot\vec{r}\,d\tau \end{aligned}$$

Phase with constant gradient



$\Delta\varphi(\vec{r},t_1) = -\int_0^{t_1} \Delta\omega(\vec{r},\tau)d\tau$

$\Delta\varphi(\vec{r},t_2) = -\int_0^{t_2} \Delta\omega(\vec{r},\tau)d\tau$
$= -\Delta\omega(\vec{r})t_2$
if $\Delta\omega$ is non - time varying.

$\Delta\varphi(\vec{r},t_3) = -\int_0^{t_3} \Delta\omega(\vec{r},\tau)d\tau$

Time-Varying Gradient Fields

The transverse magnetization is then given by

$$\begin{aligned} M(\vec{r},t) &= M(\vec{r},0)e^{-t/T_2(\vec{r})}e^{\varphi(\vec{r},t)} \\ &= M(\vec{r},0)e^{-t/T_2(\vec{r})}e^{-j\omega_0 t}\exp\left(-j\int_o^t \Delta\omega(\vec{r},t)d\tau\right) \\ &= M(\vec{r},0)e^{-t/T_2(\vec{r})}e^{-j\omega_0 t}\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right) \end{aligned}$$

Signal Equation

Signal from a volume

$$\begin{aligned} s_r(t) &= \int_V M(\vec{r},t)dV \\ &= \int_x\int_y\int_z M(x,y,z,0)e^{-t/T_2(\vec{r})}e^{-j\omega_0 t}\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdydz \end{aligned}$$

For now, consider signal from a slice along z and drop the T_2 term. Define $m(x,y) \equiv \int_{z_0-\Delta z/2}^{z_0+\Delta z/2} M(\vec{r},t)dz$

To obtain

$$s_r(t) = \int_x\int_y m(x,y)e^{-j\omega_0 t}\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdy$$

Signal Equation

Demodulate the signal to obtain

$$\begin{aligned} s(t) &= e^{j\omega_0 t}s_r(t) \\ &= \int_x\int_y m(x,y)\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdy \\ &= \int_x\int_y m(x,y)\exp\left(-j\gamma\int_o^t \left[G_x(\tau)x + G_y(\tau)y\right]d\tau\right)dxdy \\ &= \int_x\int_y m(x,y)\exp\left(-j2\pi\left(k_x(t)x + k_y(t)y\right)\right)dxdy \end{aligned}$$

Where

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

MR signal is Fourier Transform

$$
\begin{aligned}
s(t) &= \int_x \int_y m(x,y)\exp\left(-j2\pi\left(k_x(t)x + k_y(t)y\right)\right)dxdy \\
&= M\left(k_x(t),k_y(t)\right) \\
&= F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}
\end{aligned}
$$

Recap

- Frequency = rate of change of phase.
- Higher magnetic field -> higher Larmor frequency -> phase changes more rapidly with time.
- With a constant gradient G_x, spins at different x locations precess at different frequencies -> spins at greater x-values change phase more rapidly.
- With a constant gradient, distribution of phases across x locations changes with time. (phase modulation)
- More rapid change of phase with x -> higher spatial frequency k_x

K-space

At each point in time, the received signal is the Fourier transform of the object

$$s(t) = M\left(k_x(t),k_y(t)\right) = F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}$$

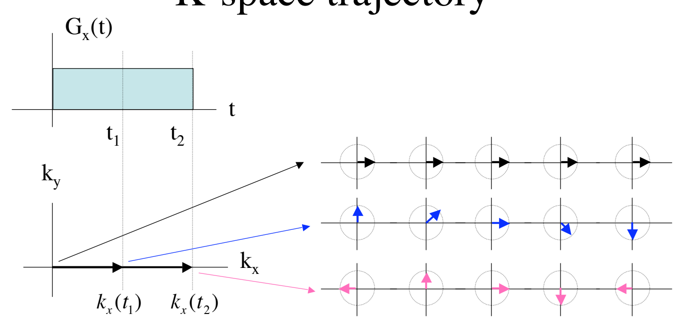
evaluated at the spatial frequencies:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Thus, the gradients control our position in k-space. The design of an MRI pulse sequence requires us to efficiently cover enough of k-space to form our image.

K-space trajectory



$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

Units

Spatial frequencies (k_x, k_y) have units of 1/distance. Most commonly, 1/cm

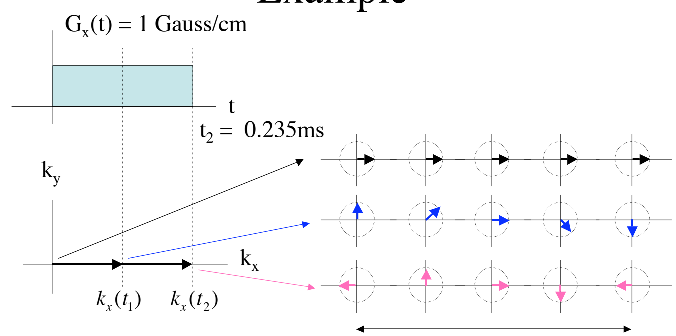
Gradient strengths have units of (magnetic field)/distance. Most commonly G/cm or mT/m

$\gamma/(2\pi)$ has units of Hz/G or Hz/Tesla.

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$= [Hz/Gauss][Gauss/cm][\sec]$$
$$= [1/cm]$$

Example



$$k_x(t_2) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$
$$= 4257 Hz/G \cdot 1G/cm \cdot 0.235\times10^{-3} s$$
$$= 1\ cm^{-1}$$