Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2008
MRI Lecture 1



# Topics

- The concept of spin
- Precession of magnetic spin
- Relaxation



# Spin

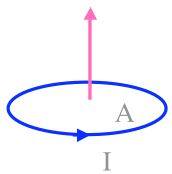
- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.



# The History of Spin

- 1921 Stern and Gerlach observed quantization of magnetic moments of silver atoms
- 1925 Uhlenbeck and Goudsmit introduce the concept of spin for electrons.
- 1933 Stern and Gerlach measure the effect of nuclear spin.
- 1937 Rabi predicts and observes nuclear magnetic resonance.

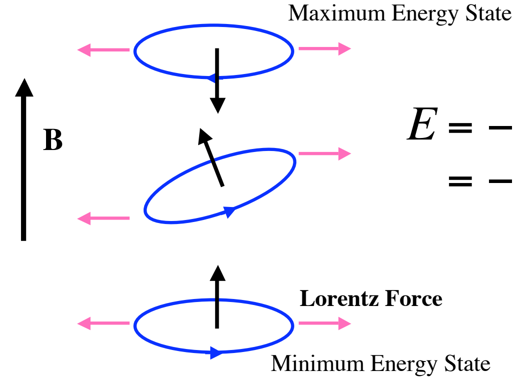
## Classical Magnetic Moment




$$\vec{\mu} = \mathrm{I}\mathrm{A}\hat{\mathrm{n}}$$



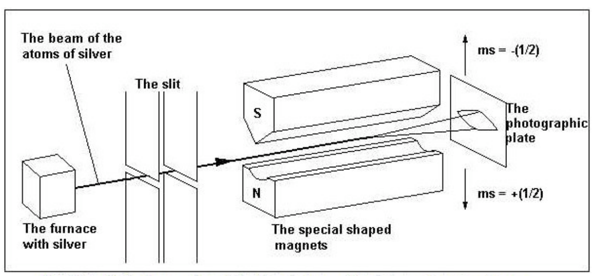
## Energy in a Magnetic Field




$$E = -\vec{\mu} \bullet \mathbf{B}$$
$$= -\mu_z B$$



## Stern-Gerlach Experiment



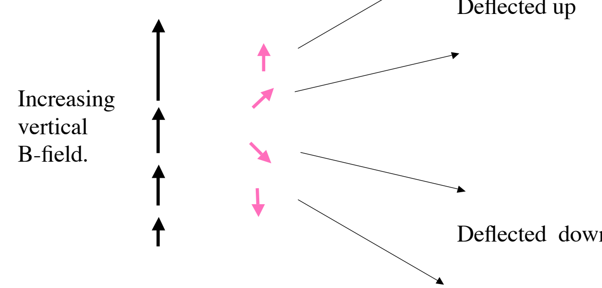

The Stern-Gerlach experiment. On the photographic plate are two clear tracks.

Image from http://library.thinkquest.org/19662/high/eng/exp-stern-gerlach.html?tqskip=1



## Force in a Field Gradient

$$\mathbf{F} = -\nabla E = \mu_z \frac{\partial B_z}{\partial z}$$



## Stern-Gerlach Experiment

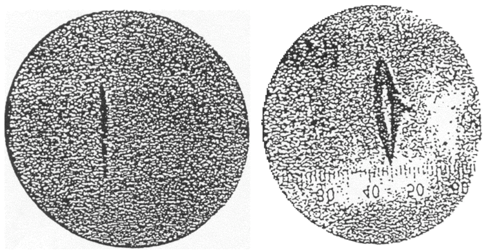



Image from http://library.thinkquest.org/19662/high/eng/exp-stern-gerlach.html?tqskip=1

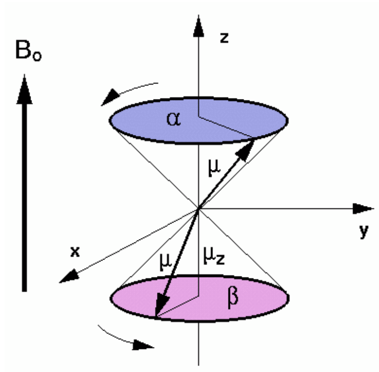


## Quantization of Magnetic Moment

The key finding of the Stern-Gerlach experiment is that the magnetic moment is quantized. That is, it can only take on discrete values.

In the experiment, the finding was that the component of magnetization along the direction of the applied field was quantized:

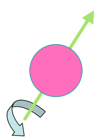
$\mu_z = +\mu_0$ OR $-\mu_0$




http://www.le.ac.uk/biochem/mp84/teaching



## Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where $\gamma$ is the gyromagnetic ratio and $\mathbf{S}$ is the spin angular momentum.



## Quantization of Angular Momentum

Because the magnetic moment is quantized, so is the angular momentum.

In particular, the z-component of the angular momentum Is quantized as follows:

$$S_z = m_s \hbar$$

$$m_s \in \{-s, -(s-1), \ldots s\}$$

$s$ is an integer or half intege

# Nuclear Spin Rules

| Number of Protons | Number of Neutrons | Spin | Examples |
|---|---|---|---|
| Even | Even | 0 | $^{12}C$, $^{16}O$ |
| Even | Odd | j/2 | $^{17}O$ |
| Odd | Even | j/2 | $^{1}H$, $^{23}Na$, $^{31}P$ |
| Odd | Odd | j | $^{2}H$ |



# Hydrogen Proton

Spin 1/2

$$S_z = \begin{cases} +\hbar/2 \\ -\hbar/2 \end{cases}$$

$$\mu_z = \begin{cases} +\gamma\hbar/2 \\ -\gamma\hbar/2 \end{cases}$$

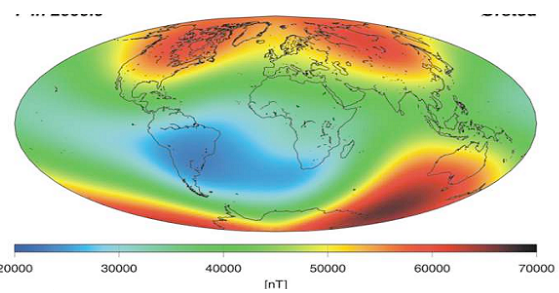


# Magnetic Field Units

1 Tesla = 10,000 Gauss

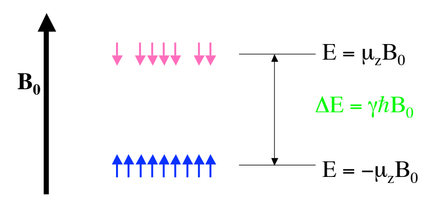
Earth's field is about 0.5 Gauss

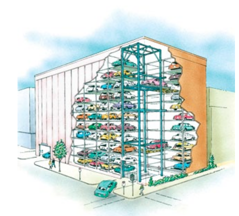
0.5 Gauss = $0.5\times10^{-4}$ T = 50 $\mu$T






# Boltzmann Distribution



$$\frac{\text{Number Spins Up}}{\text{Number Spins Down}} = \exp(-\Delta E/kT)$$

Ratio = 0.999990 at 1.5T !!!

Corresponds to an excess of about 10 up spins per million

# Equilibrium Magnetization

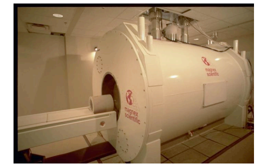
$$\mathbf{M}_0 = N\langle \mu_z \rangle = N\left(\frac{n_{up}(\mu_z) - n_{down}(\mu_z)}{N}\right)$$
$$= N\mu \frac{e^{\mu_z B/(kT)} - e^{-\mu_z B/(kT)}}{e^{\mu_z B/(kT)} + e^{-\mu_z B/(kT)}}$$
$$\approx N\mu_z^2 B/(kT)$$
$$= N\gamma^2 \hbar^2 B/(4kT)$$

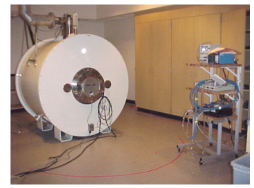
N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.



# Bigger is better


3T Human imager at UCSD.


7T Human imager at U. Minn.


7T Rodent Imager at UCSD


9.4T Human imager at UIC

TT. Liu, BE280A, UCSD Fall 2008

# Gyromagnetic Ratios

| Nucleus | Spin | Magnetic Moment | $\gamma/(2\pi)$ (MHz/Tesla) | Abundance |
|---|---|---|---|---|
| $^{1}$H | 1/2 | 2.793 | 42.58 | 88 M |
| $^{23}$Na | 3/2 | 2.216 | 11.27 | 80 mM |
| $^{31}$P | 1/2 | 1.131 | 17.25 | 75 mM |

Source: Haacke et al., p. 27

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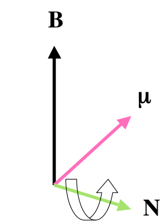
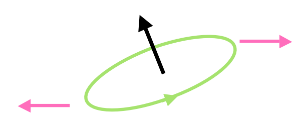
# Torque




$\mathbf{N} = \mu \times \mathbf{B}$

Torque

For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)



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# Precession

Torque

$\mathbf{N} = \mu \times \mathbf{B}$

$\frac{d\mathbf{S}}{dt} = \mathbf{N}$

Change in Angular momentum

$\frac{d\mathbf{S}}{dt} = \mu \times \mathbf{B}$

$\mu = \gamma \mathbf{S}$

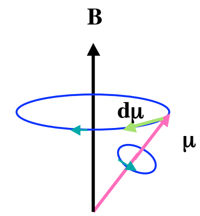
Relation between magnetic moment and angular momentum

$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$



# Precession

$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$

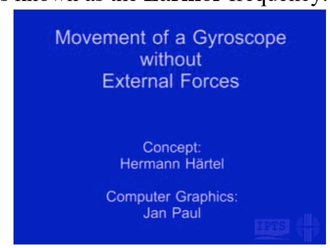



Analogous to motion of a gyroscope

Precesses at an angular frequency of

$\omega = \gamma \mathbf{B}$

This is known as the **Larmor** frequency.




http://www.astrophysik.uni-kiel.de/~hhaertel/mpg_e/gyros_free.mpg



# Larmor Frequency

| | |
|---|---|
| $\omega = \gamma \mathbf{B}$ | Angular frequency in rad/sec |
| $f = \gamma \mathbf{B} / (2\pi)$ | Frequency in cycles/sec or Hertz, Abbreviated Hz |

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.



# Notation and Units

1 Tesla = 10,000 Gauss

Earth's field is about 0.5 Gauss

0.5 Gauss = $0.5\times10^{-4}$ T = 50 $\mu$T

$\gamma = 26752$ radians/second/Gauss

$\gamma = \gamma / 2\pi = 4258$ Hz/Gauss

$= 42.58$ MHz/Tesla

# Recap

- Spins: angular momentum and magnetic moment are quantized.
- Spins precess about a static field at the Larmor frequency.
- In MRI we work with the net magnetic moment.
- In the presence of a static field and non-zero temperature, the equilibirum net magnetic moment is aligned with the field (longitudinal), since transverse components cancel out.
- We will use an radiofrequency pulse to tip this longitudinal component into the transverse plane.



# Magnetization Vector

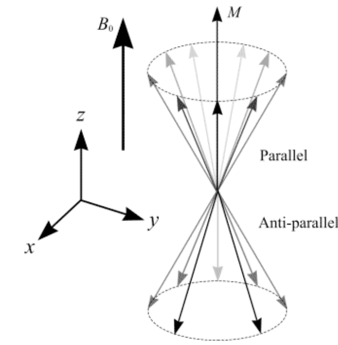
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

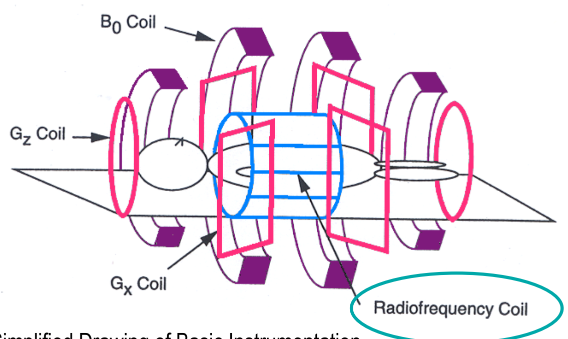
$$\mathbf{M} = \frac{1}{V} \sum_{\substack{\text{protons} \\ \text{in V}}} \mu_i$$

$$\boxed{\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}}$$




http://www.easymeasure.co.uk/principlesmri.aspx



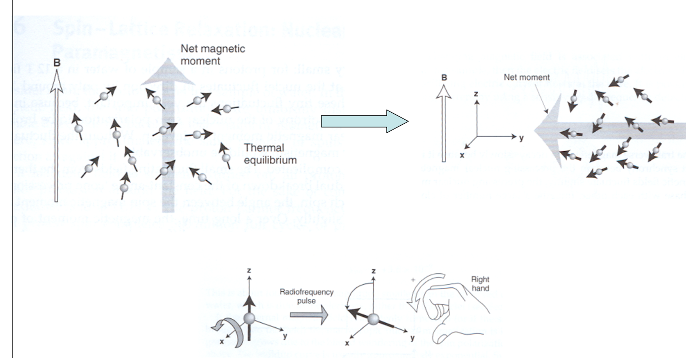



Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field $B_o$,
gradient fields (two of three shown),
and radiofrequency field $B_1$.





# RF Excitation




From Levitt, Spin Dynamics, 2001

# RF Excitation

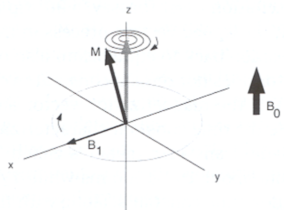



Image & caption: Nishimura, Fig. 3.2

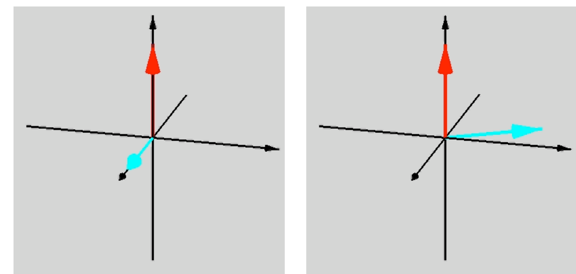
At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

$B_1$ radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

http://www.eecs.umich.edu/%7EdnollBME516/

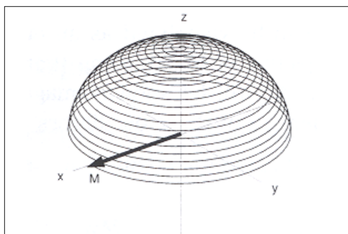


# RF Excitation



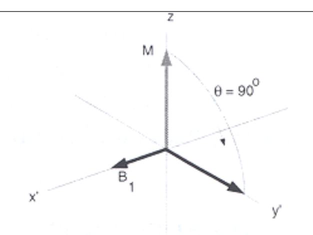


http://www.eecs.umich.edu/%7EdnollBME516/

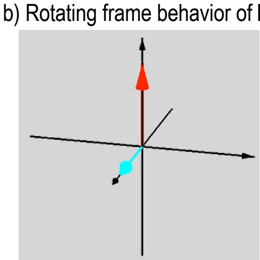



a) Laboratory frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3



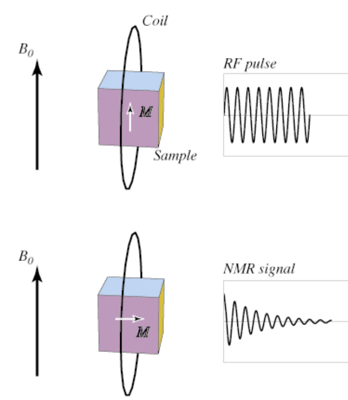

b) Rotating frame behavior of **M**





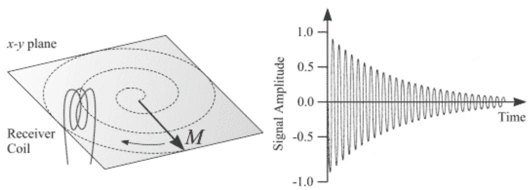
http://www.eecs.umich.edu/%7EdnollBME516/

# RF Excitation




From Buxton 2002

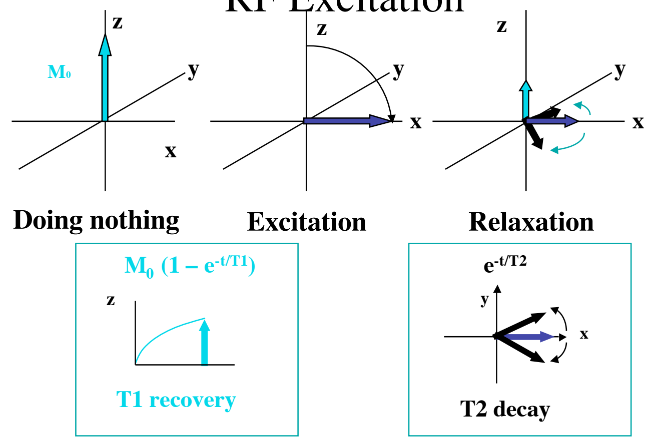
# Free Induction Decay (FID)




http://www.easymeasure.co.uk/principlesmri.aspx



# RF Excitation




Credit: Larry Frank



# Relaxation

An excitation pulse rotates the magnetization vector away from its equilibrium state (purely longitudinal). The resulting vector has both longitudinal $\mathbf{M}_z$ and tranverse $\mathbf{M}_{xy}$ components.

Due to thermal interactions, the magnetization will return to its equilibrium state with characteristic time constants.

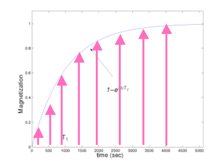
$T_1$ spin-lattice time constant, return to equilibrium of $\mathbf{M}_z$

$T_2$ spin-spin time constant, return to equilibrium of $\mathbf{M}_{xy}$



# Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$

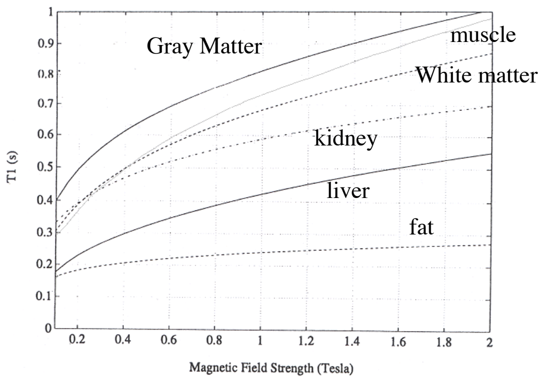


After a 90 degree pulse $\quad M_z(t) = M_0(1 - e^{-t/T_1})$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that $T_1$ increases with $\mathbf{B}$.

# T1 Values

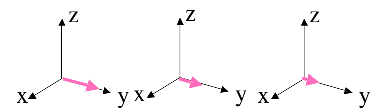



Image, caption: Nishimura, Fig. 4.2



# Transverse Relaxation

$$\frac{d\mathbf{M}_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$



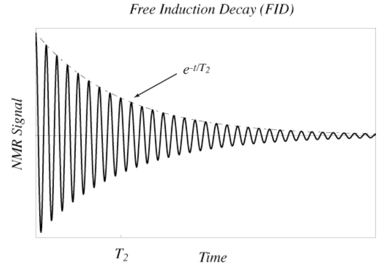
Each spin's local field is affected by the z-component of the field due to other spins. Thus, the Larmor frequency of each spin will be slightly different. This leads to a dephasing of the transverse magnetization, which is characterized by an exponential decay.

$T_2$ is largely independent of field. $T_2$ is short for low frequency fluctuations, such as those associated with slowly tumbling macromolecules.



# T2 Relaxation

*Free Induction Decay (FID)*

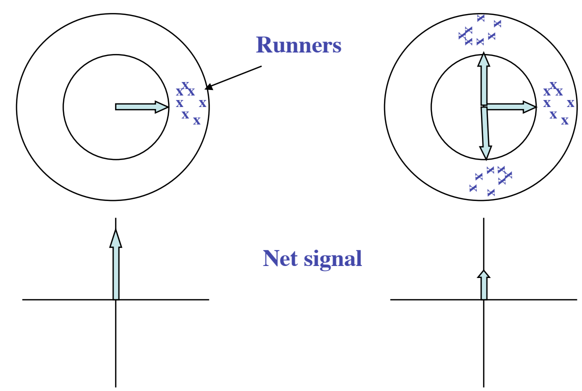



After a 90 degree excitation

$$M_{xy}(t) = M_0 e^{-t/T_2}$$



# T2 Relaxation




Credit: Larry Frank

# T2 Values

| Tissue | $T_2$ (ms) |
|---|---|
| gray matter | 100 |
| white matter | 92 |
| muscle | 47 |
| fat | 85 |
| kidney | 58 |
| liver | 43 |
| CSF | 4000 |

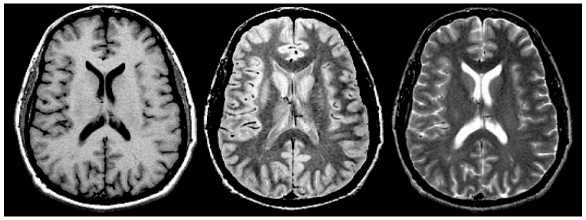
Table: adapted from Nishimura, Table 4.2

Solids exhibit very short $T_2$ relaxation times because there are many low frequency interactions between the immobile spins.

On the other hand, liquids show relatively long $T_2$ values, because the spins are highly mobile and net fields average out.



# Example



$T_1$-weighted    Density-weighted    $T_2$-weighted

Questions: How can one achieve T2 weighting? What are the relative T2's of the various tissues?