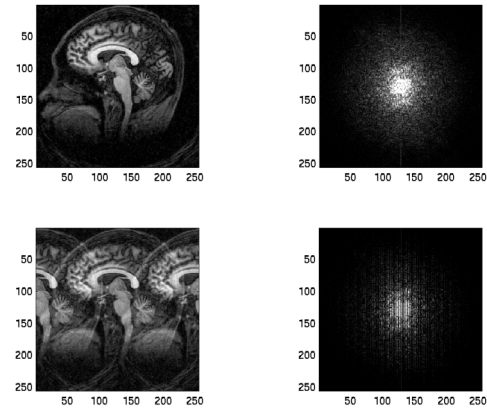


Bioengineering 280A
Principles of Biomedical Imaging

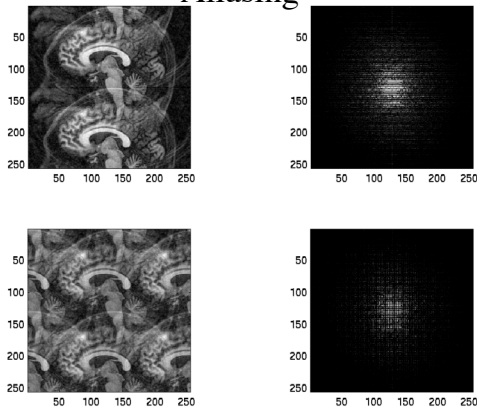
Fall Quarter 2008
MRI Lecture 3

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Sampling in k-space

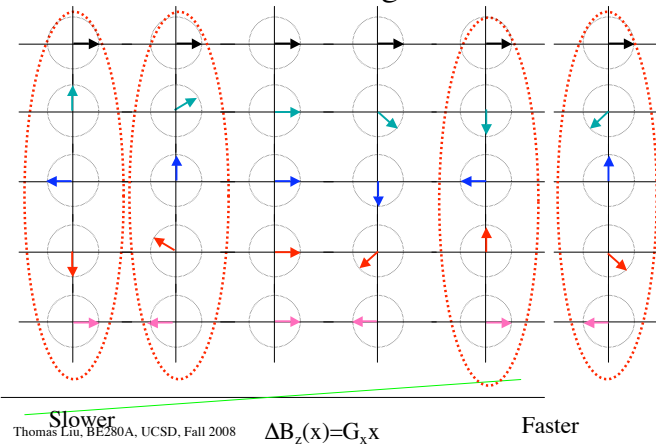


Aliasing

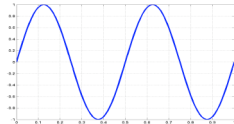


T1

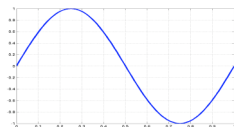
Aliasing



Intuitive view of Aliasing



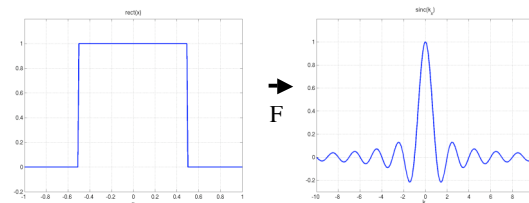
$$k_x = 2/FOV$$



$$k_x = 1/FOV$$

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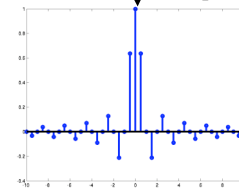
Fourier Sampling



F

Instead of sampling the signal, we sample its Fourier Transform

Sample

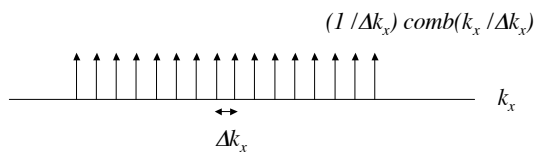


???

F⁻¹

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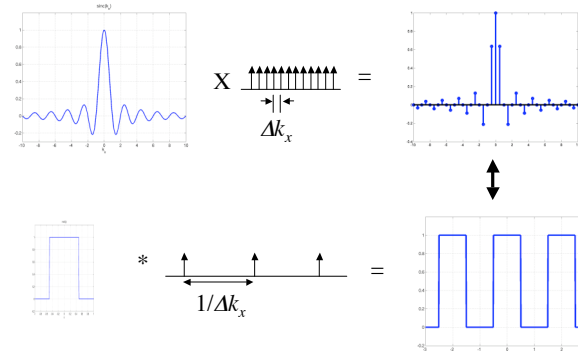
Fourier Sampling



$$\begin{aligned} G_S(k_x) &= G(k_x) \frac{1}{\Delta k_x} \text{comb}\left(\frac{k_x}{\Delta k_x}\right) \\ &= G(k_x) \sum_{n=-\infty}^{\infty} \delta(k_x - n\Delta k_x) \\ &= \sum_{n=-\infty}^{\infty} G(n\Delta k_x) \delta(k_x - n\Delta k_x) \end{aligned}$$

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Fourier Sampling -- Inverse Transform



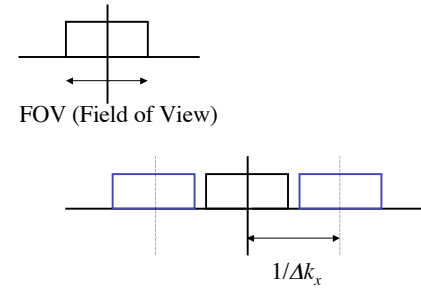
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Fourier Sampling -- Inverse Transform

$$\begin{aligned}
 g_s(x) &= F^{-1}[G_s(k_x)] \\
 &= F^{-1}\left[G(k_x) \frac{1}{\Delta k_x} \text{comb}\left(\frac{k_x}{\Delta k_x}\right)\right] \\
 &= F^{-1}[G(k_x)] * F^{-1}\left[\frac{1}{\Delta k_x} \text{comb}\left(\frac{k_x}{\Delta k_x}\right)\right] \\
 &= g(x) * \text{comb}(x\Delta k_x) \\
 &= g(x) * \sum_{n=-\infty}^{\infty} \delta(x\Delta k_x - n) \\
 &= g(x) * \frac{1}{\Delta k_x} \sum_{n=-\infty}^{\infty} \delta\left(x - \frac{n}{\Delta k_x}\right) \\
 &= \frac{1}{\Delta k_x} \sum_{n=-\infty}^{\infty} g\left(x - \frac{n}{\Delta k_x}\right)
 \end{aligned}$$

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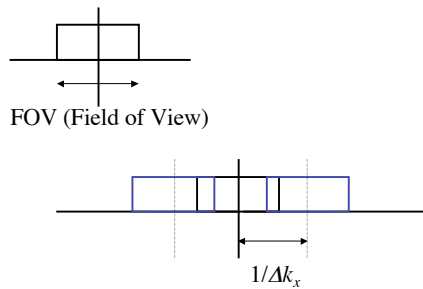
Nyquist Condition



To avoid overlap, $1/\Delta k_x > \text{FOV}$, or equivalently, $\Delta k_x < 1/\text{FOV}$

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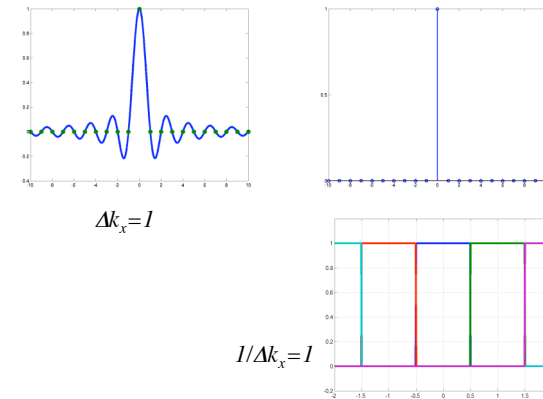
Aliasing



Aliasing occurs when $1/\Delta k_x < \text{FOV}$

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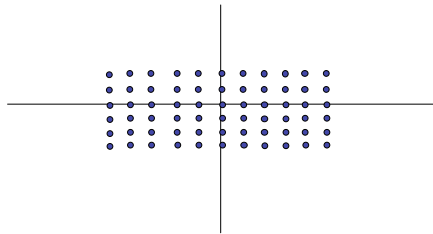
Aliasing Example



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2D Comb Function

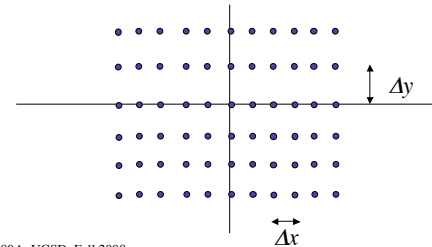
$$\begin{aligned} \text{comb}(x,y) &= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x-m,y-n) \\ &= \sum_{m=-\infty}^{\infty} \delta(x-m) \sum_{n=-\infty}^{\infty} \delta(y-n) \\ &= \text{comb}(x)\text{comb}(y) \end{aligned}$$



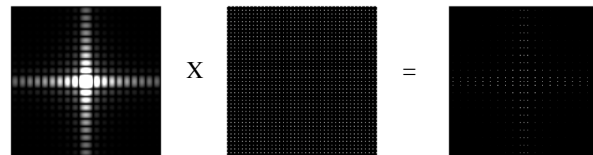
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Scaled 2D Comb Function

$$\begin{aligned} \text{comb}(x/\Delta x, y/\Delta y) &= \text{comb}(x/\Delta x)\text{comb}(y/\Delta y) \\ &= \Delta x \Delta y \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x-m\Delta x)\delta(y-n\Delta y) \end{aligned}$$



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2D k-space sampling

$$\begin{aligned} G_S(k_x, k_y) &= G(k_x, k_y) \frac{1}{\Delta k_x \Delta k_y} \text{comb}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right) \\ &= G(k_x, k_y) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(k_x - m\Delta k_x, k_y - n\Delta k_y) \\ &= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} G(m\Delta k_x, n\Delta k_y) \delta(k_x - m\Delta k_x, k_y - n\Delta k_y) \end{aligned}$$

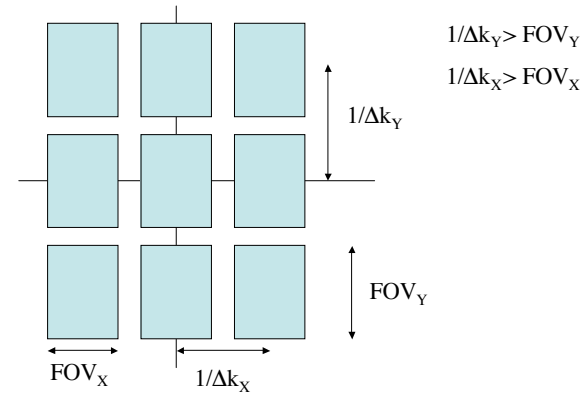
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2D k-space sampling

$$\begin{aligned}
 g_S(x, y) &= F^{-1} \left[G_S(k_x, k_y) \right] \\
 &= F^{-1} \left[G(k_x, k_y) \frac{1}{\Delta k_x \Delta k_y} \text{comb} \left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y} \right) \right] \\
 &= F^{-1} \left[G(k_x, k_y) \right] * F^{-1} \left[\frac{1}{\Delta k_x \Delta k_y} \text{comb} \left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y} \right) \right] \\
 &= g(x, y) * * \text{comb}(x \Delta k_x) \text{comb}(y \Delta k_y) \\
 &= g(x) * * \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x \Delta k_x - m) \delta(y \Delta k_y - n) \\
 &= g(x) * * \frac{1}{\Delta k_x \Delta k_y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta \left(x - \frac{m}{\Delta k_x} \right) \delta \left(y - \frac{n}{\Delta k_y} \right) \\
 &= \frac{1}{\Delta k_x \Delta k_y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g \left(x - \frac{m}{\Delta k_x}, y - \frac{n}{\Delta k_y} \right)
 \end{aligned}$$

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Nyquist Conditions



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Windowing

Windowing the data in Fourier space

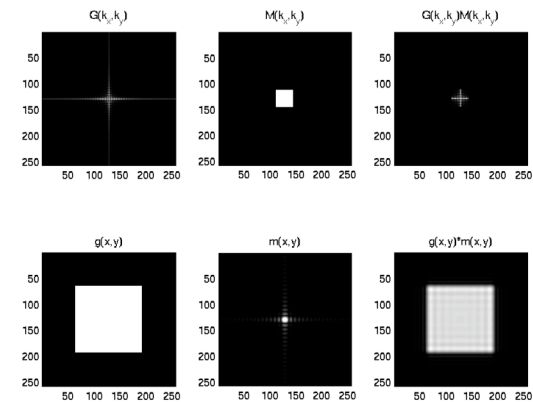
$$G_W(k_x, k_y) = G(k_x, k_y) W(k_x, k_y)$$

Results in convolution of the object with the inverse transform of the window

$$g_W(x, y) = g(x, y) * w(x, y)$$

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Resolution



1

Windowing Example

$$W(k_x, k_y) = \text{rect}\left(\frac{k_x}{W_{k_x}}\right) \text{rect}\left(\frac{k_y}{W_{k_y}}\right)$$

$$w(x, y) = F^{-1} \left[\text{rect}\left(\frac{k_x}{W_{k_x}}\right) \text{rect}\left(\frac{k_y}{W_{k_y}}\right) \right]$$

$$= W_{k_x} W_{k_y} \text{sinc}(W_{k_x} x) \text{sinc}(W_{k_y} y)$$

$$g_w(x, y) = g(x, y) * W_{k_x} W_{k_y} \text{sinc}(W_{k_x} x) \text{sinc}(W_{k_y} y)$$

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Effective Width

$$w_E = \frac{1}{w(0)} \int_{-\infty}^{\infty} w(x) dx$$

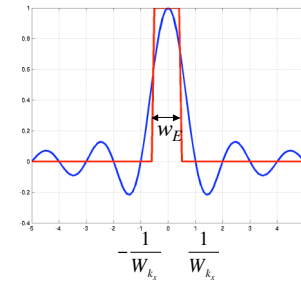
Example

$$w_E = \frac{1}{1} \int_{-\infty}^{\infty} \text{sinc}(W_{k_x} x) dx$$

$$= F[\text{sinc}(W_{k_x} x)] \Big|_{k_x=0}$$

$$= \frac{1}{W_{k_x}} \text{rect}\left(\frac{k_x}{W_{k_x}}\right) \Big|_{k_x=0}$$

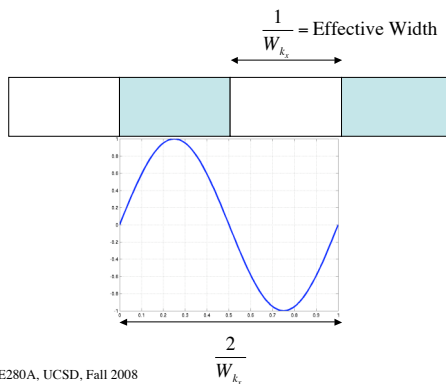
$$= \frac{1}{W_{k_x}}$$



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Resolution and spatial frequency

With a window of width W_{k_x} , the highest spatial frequency is $W_{k_x}/2$.
This corresponds to a spatial period of $2/W_{k_x}$.



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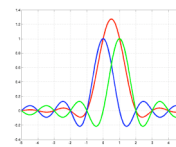
Windowing Example

$$g(x, y) = [\delta(x) + \delta(x-1)]\delta(y)$$

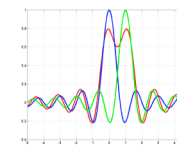
$$g_w(x, y) = [\delta(x) + \delta(x-1)]\delta(y) * W_{k_x} W_{k_y} \text{sinc}(W_{k_x} x) \text{sinc}(W_{k_y} y)$$

$$= W_{k_x} W_{k_y} \left([\delta(x) + \delta(x-1)] * \text{sinc}(W_{k_x} x) \right) \text{sinc}(W_{k_y} y)$$

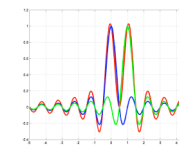
$$= W_{k_x} W_{k_y} \left(\text{sinc}(W_{k_x} x) + \text{sinc}(W_{k_x} (x-1)) \right) \text{sinc}(W_{k_y} y)$$



$W_{k_x} = 1$



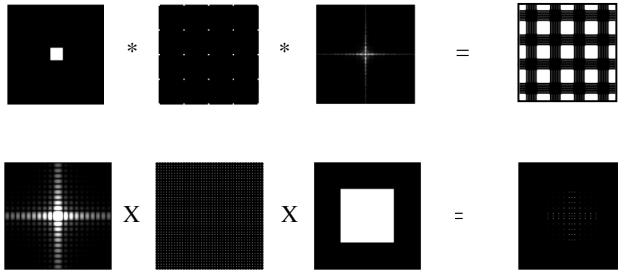
$W_{k_x} = 1.5$



$W_{k_x} = 2$

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Sampling and Windowing



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Sampling and Windowing

Sampling and windowing the data in Fourier space

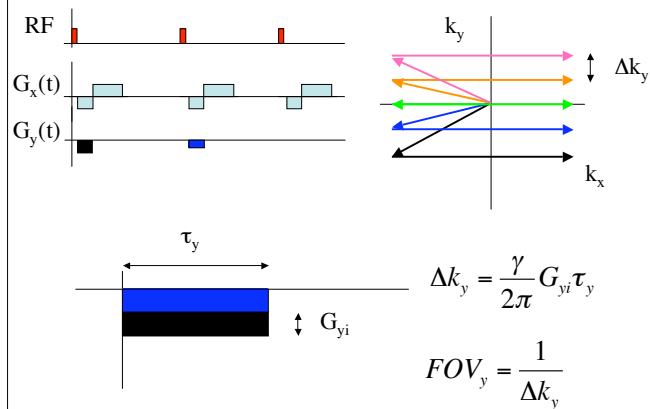
$$G_{SW}(k_x, k_y) = G(k_x, k_y) \frac{1}{\Delta k_x \Delta k_y} \text{comb}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right) \text{rect}\left(\frac{k_x}{W_{k_x}}, \frac{k_y}{W_{k_y}}\right)$$

Results in replication and convolution in object space.

$$g_{SW}(x, y) = W_{k_x} W_{k_y} g(x, y) ** \text{comb}(\Delta k_x x, \Delta k_y y) ** \text{sinc}(W_{k_x} x) \text{sinc}(W_{k_y} y)$$

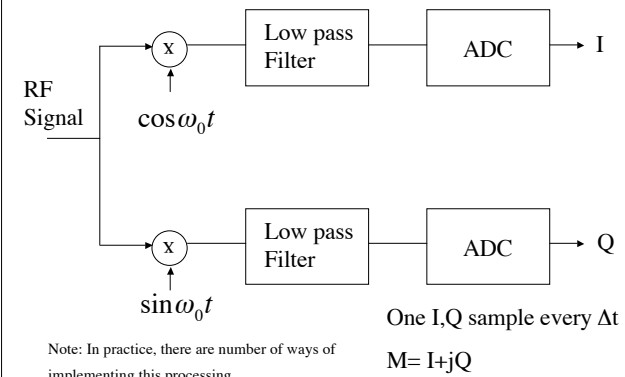
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Sampling in k_y

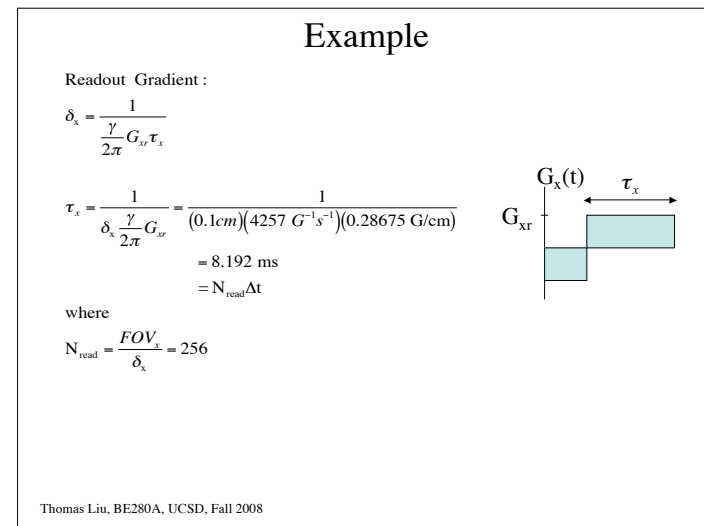
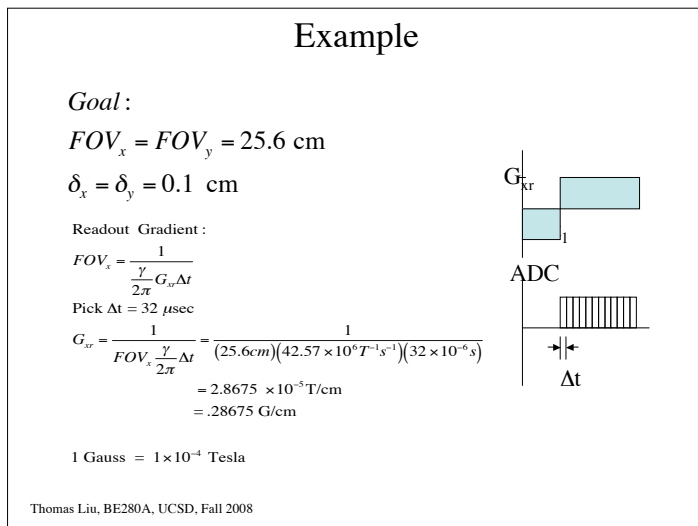
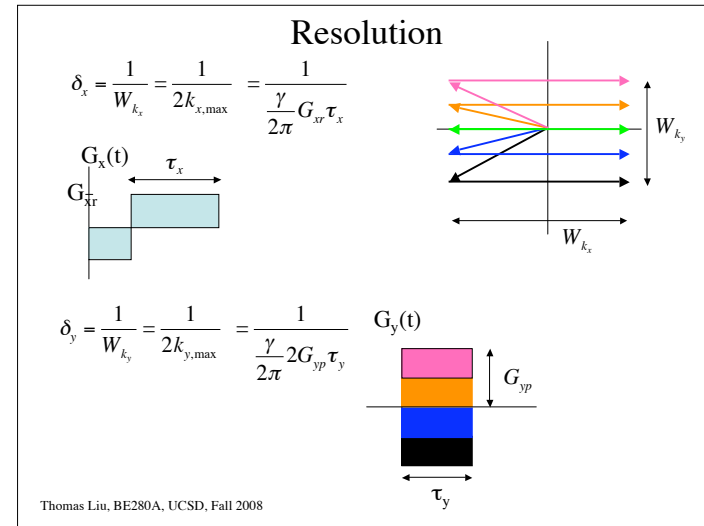
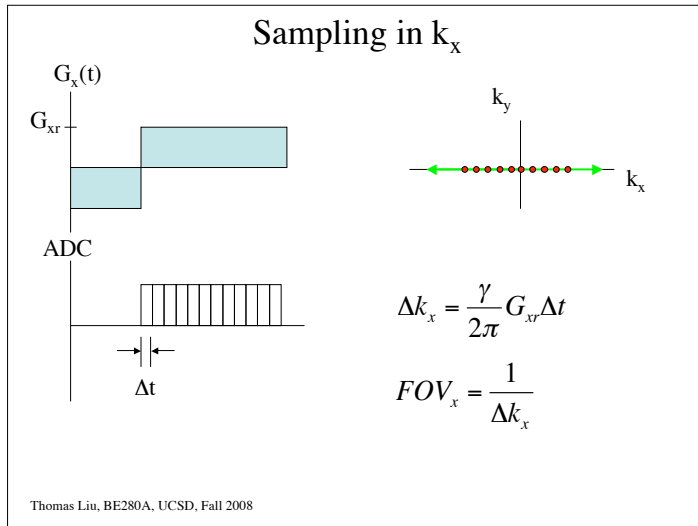


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Sampling in k_x



Note: In practice, there are number of ways of implementing this processing.
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Example

Phase - Encode Gradient :

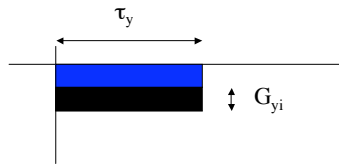
$$FOV_y = \frac{1}{\frac{\gamma}{2\pi} G_{pe} \tau_y}$$

Pick $\tau_y = 4.096 \text{ msec}$

$$G_{pe} = \frac{1}{FOV_y \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(25.6 \text{ cm})(42.57 \times 10^6 \text{ T}^{-1} \text{ s}^{-1})(4.096 \times 10^{-3} \text{ s})}$$

$$= 2.2402 \times 10^{-7} \text{ T/cm}$$

$$= .00224 \text{ G/cm}$$



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Example

Phase - Encode Gradient :

$$\delta_y = \frac{1}{\frac{\gamma}{2\pi} 2 G_{sp} \tau_y}$$

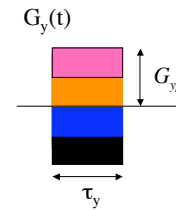
$$G_{sp} = \frac{1}{\delta_y 2 \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(0.1 \text{ cm})(4257 \text{ G}^{-1} \text{ s}^{-1})(4.096 \times 10^{-3} \text{ s})}$$

$$= 0.2868 \text{ G/cm}$$

$$= \frac{N_p}{2} G_{pe}$$

where

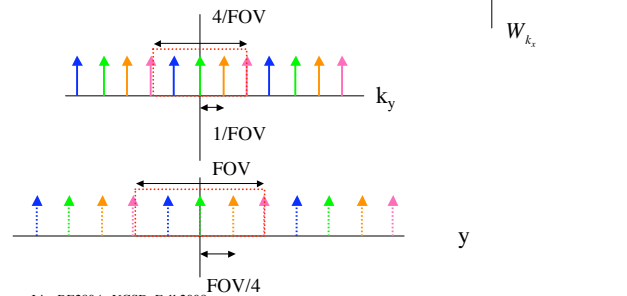
$$N_p = \frac{FOV_y}{\delta_y} = 256$$



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Sampling

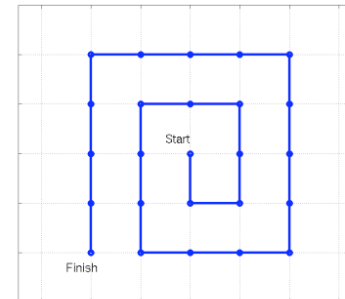
In practice, an even number (typically power of 2) sample is usually taken in each direction to take advantage of the Fast Fourier Transform (FFT) for reconstruction.



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Example

Consider the k-space trajectory shown below. ADC samples are acquired at the points shown with $\Delta t = 10 \mu\text{sec}$. The desired FOV (both x and y) is 10 cm and the desired resolution (both x and y) is 2.5 cm. Draw the gradient waveforms required to achieve the desired FOV and resolution. Label the waveform with the gradient amplitudes required to achieve the desired FOV and resolution. Also, make sure to label the time axis correctly.



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SCAN TIMING

of Echoes: 1, 2, 3, 4

TE: Min Full

TR: 750

Inv. Time

T12

Flip Angle

Echo Train Length

Bandwidth: 25

Bandwidth2

ACQUISITION TIMING

Freq: 352, Freq DIR: A/P

Phase: 192, Auto Center Freq: Water

NEX: 2.0, Flow Comp Direction

Phase FOV: 0.75, Autoshim, Phase Correct

of Acqs Before Pause, Agent

SCANNING RANGE

FDV: 22, Start, L/R Center, P/A Center


Slice Thickness: 5.0, End

Spacing: 2.0, # Slices, Table Delta

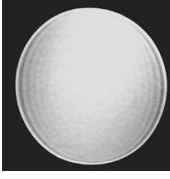
ACTUAL End

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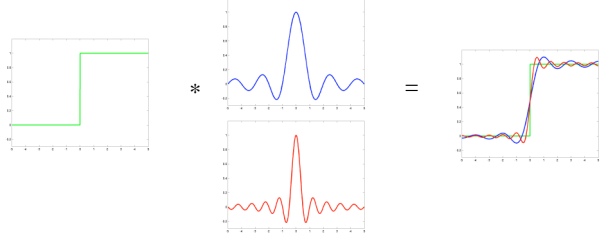
Gibbs Artifact



256x256 image

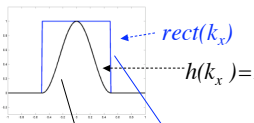


256x128 image



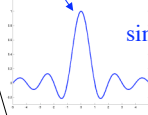
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Apodization

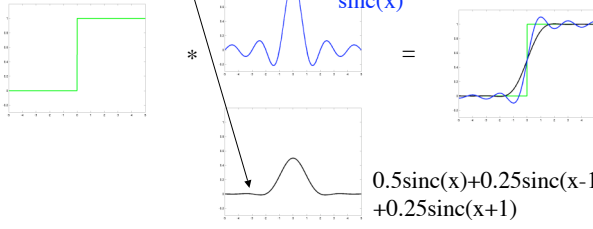


Hanning Window

$h(k_x) = 1/2(1 + \cos(2\pi k_x))$



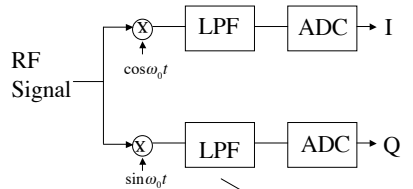
sinc(x)



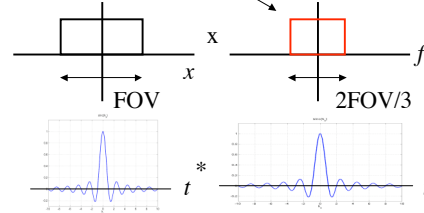
$0.5\text{sinc}(x) + 0.25\text{sinc}(x-1) + 0.25\text{sinc}(x+1)$

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Aliasing and Bandwidth



Temporal filtering in the readout direction limits the readout FOV. So there should never be aliasing in the readout direction.



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