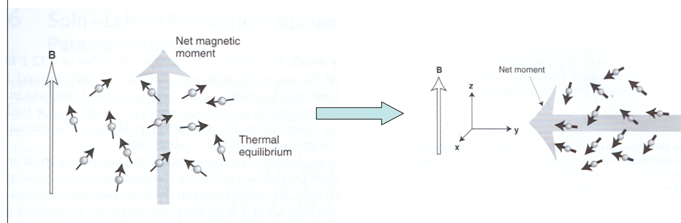
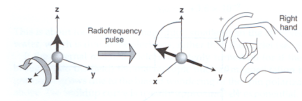
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2008
MRI Lecture 5

RF Excitation





From Levitt, Spin Dynamics, 2001

RF Excitation

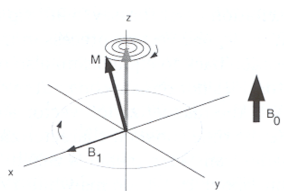
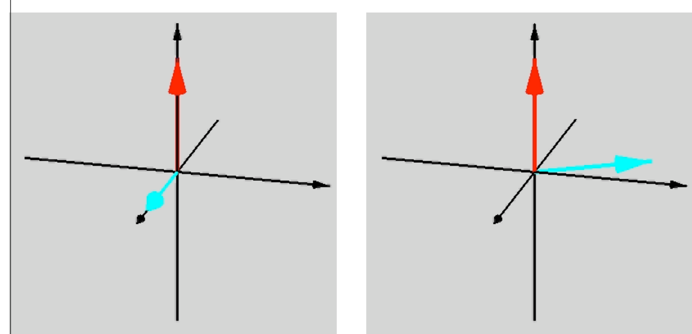


Image & caption: Nishimura, Fig. 3.2

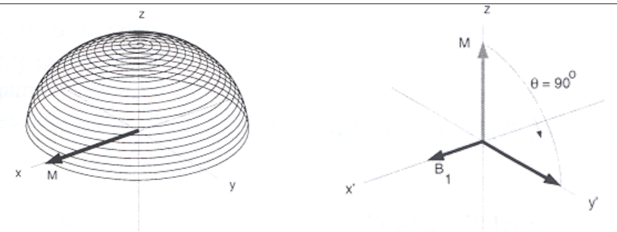
At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

RF Excitation

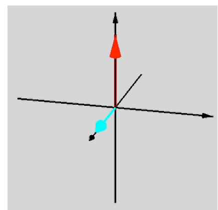


http://www.eecs.umich.edu/%7EdnollBME516/



a) Laboratory frame behavior of **M** b) Rotating frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3



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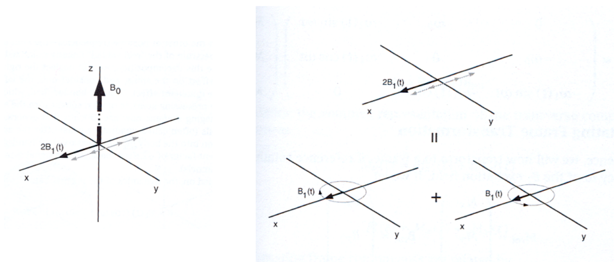


$$\mathbf{B}_1(t) = 2B_1(t)\cos(\omega t)\mathbf{i}$$
$$= B_1(t)\left(\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}\right) + B_1(t)\left(\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\right)$$

Nishimura 1996

Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

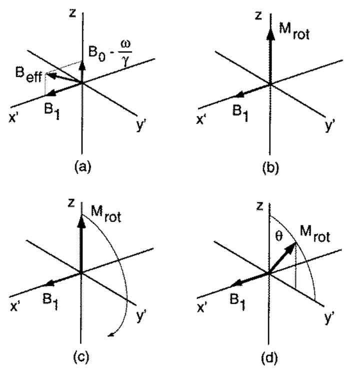
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

$$Let \;\; \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$
$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

$$\text{If } \omega = \omega_0$$
$$= \gamma B_0$$

$$\text{Then } \mathbf{B}_{eff} = B_1(t)\mathbf{i}$$



Flip angle

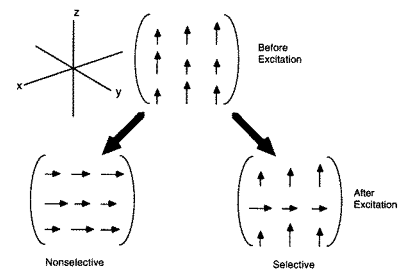
$$\theta = \int_0^\tau \omega_1(s)ds$$

where

$$\omega_1(t) = \gamma B_1(t)$$

Thomas Liu, BE280A, UCSD, Fall 2008

Nishimura 1996



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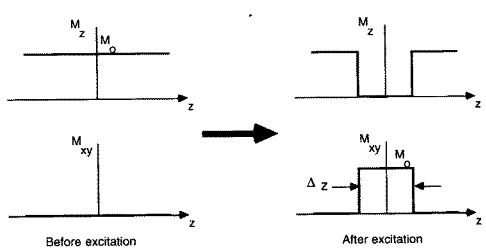

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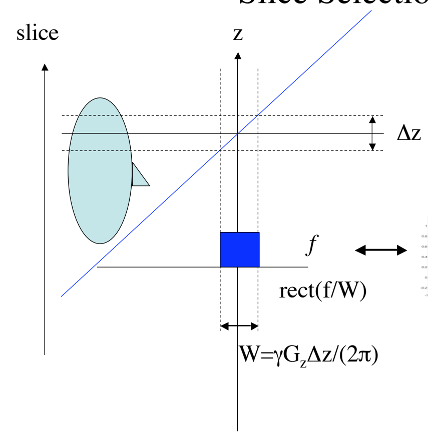
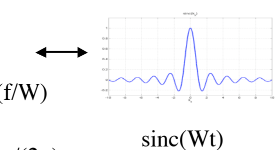
Slice Selection





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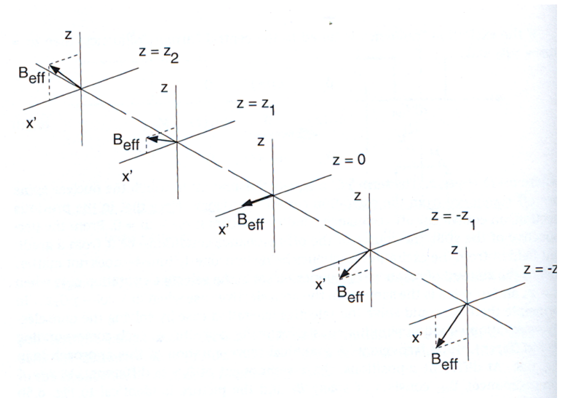
$$\textit{Let } \mathbf{B}_{rot} = B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z\right)\mathbf{k}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

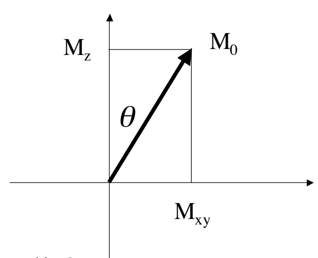
If $\omega = \omega_0$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + \left(\gamma G_z z\right)\mathbf{k}$$



Nishimura 1996

Small Tip Angle Approximation



For small θ

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$

Recall that in the rotating frame, flip angle $\theta = \gamma \int_0^\tau B_1(s)ds$

Define $\omega(z)$ as the Larmor Frequency at each location z referenced to ω_0

The effective field felt in each spin's rotating frame of reference is:

$B_1^e(t) = B_1(t)\exp(j\omega(z)t)$

Therefore the flip angle in each spin's frame of reference is

$\theta(t,z) = \gamma \int_0^t \exp(j\omega(z)s)B_1(s)ds$

With respect to the on-resonance frame of reference, there is also a relative phase shift of $\exp(-j\omega(z)t)$, so that

$$\theta(t,z) = \gamma \exp(-j\omega(z)t)\int_0^t \exp(j\omega(z)s)B_1(s)ds$$

Applying small angle approximation leads to

$$M_r(t,z) \approx jM_0\theta(t,z) = jM_0\gamma\exp(-j\omega(z)t)\int_0^t \exp(j\omega(z)s)B_1(s)ds$$

Small Tip Angle Approximation

$$M_r(t,z) = jM_0\exp(-j\omega(z)t)\int_0^t \exp(j\omega(z)s)\omega_1(s)ds$$

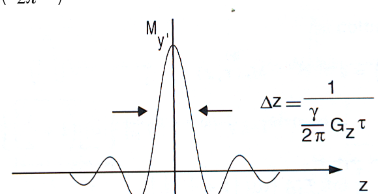
For symmetric pulse of length τ

$$\begin{aligned} M_r(\tau,z) &= jM_0\exp(-j\omega(z)\tau/2)\int_{-\tau/2}^{\tau/2}\exp(j2\pi f(z)s)\omega_1(s+\tau/2)ds \\ &= jM_0\exp(-j\omega(z)\tau/2)F\{\omega_1(t+\tau/2)\}\Big|_{f=-f(z)=-\frac{\gamma}{2\pi}G_z z} \end{aligned}$$

Small Tip Angle Example

$$B_1(t) = B_1 rect\left(\frac{t-\tau/2}{\tau}\right)$$

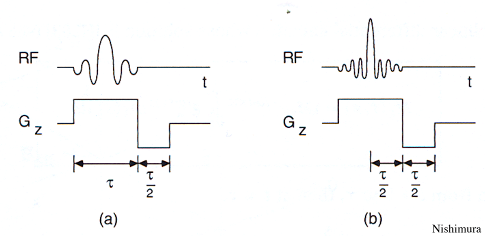
$$\begin{aligned} M_r(\tau,z) &= jM_0\exp(-j\omega(z)\tau)\int_0^\tau \exp(j\omega(z)s)\omega_1 rect\left(\frac{s-\tau/2}{\tau}\right)ds \\ &= jM_0\exp(-j\omega(z)\tau/2)F_{1D}\left(\omega_1 rect\left(\frac{t}{\tau}\right)\right)\Big|_{f=-(\gamma/2\pi)G_z z} \\ &= jM_0\exp(-j\omega(z)\tau/2)\omega_1\tau\,\mathrm{sinc}(f\tau) \\ &= jM_0\exp(-j\omega(z)\tau/2)\omega_1\tau\,\mathrm{sinc}\left(\frac{\gamma G_z\tau}{2\pi}z\right) \end{aligned}$$



Nishimura 1996

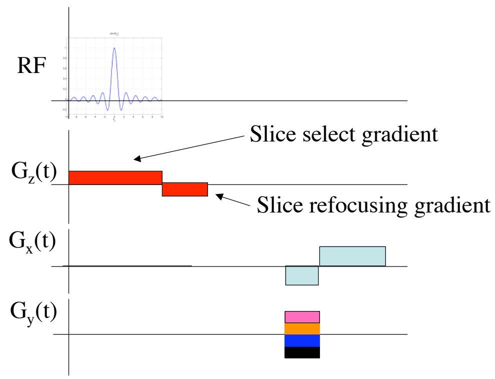
Refocusing

$$\begin{aligned} M_r(3\tau/2,z) &= \exp(j\omega(z)\tau/2)M_r(\tau,z) \\ &= jM_0\exp(j\omega(z)\tau/2)\exp(-j\omega(z)\tau/2)F\{\omega_1(t+\tau/2)\}\Big|_{f=-\frac{\gamma}{2\pi}G_z z} \\ &= jM_0F\{\omega_1(t+\tau/2)\}\Big|_{f=-\frac{\gamma}{2\pi}G_z z} \end{aligned}$$

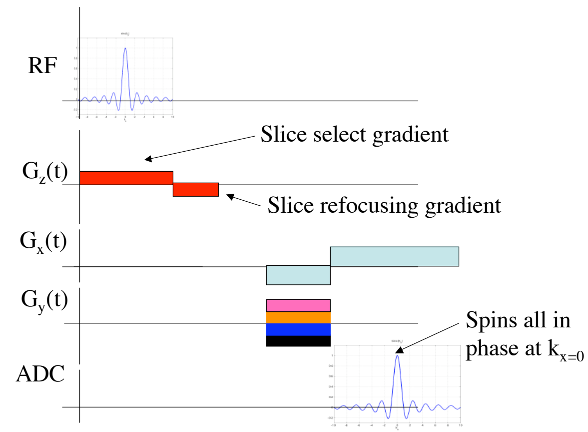


Nishimura 1996

Slice Selection



Gradient Echo

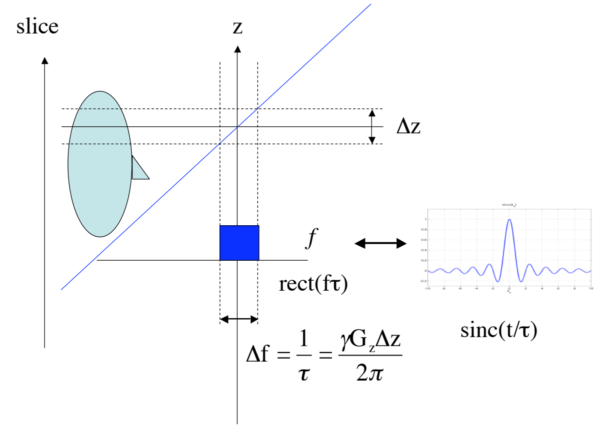


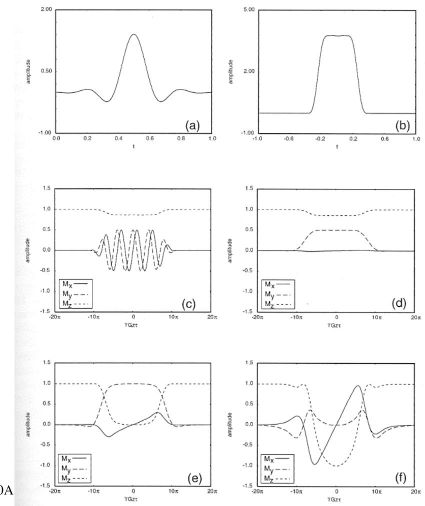
Small Tip Angle Example

$$B_1(t+\tau/2) = A\,\mathrm{sinc}(t/\tau)\left(0.5+0.46\cos\left(\frac{2\pi t}{\tau}\right)\right)$$
$$= A\,\mathrm{sinc}(t/\tau)w(t)$$
$$F(B_1(t+\tau/2)) = A\tau\ rect(f\tau) * W(f)$$
$$= A\tau\ rect\left(\frac{\gamma G_z z\tau}{2\pi}\right) * W\left(-\frac{\gamma G_z z}{2\pi}\right)$$

Width of the rect function is $\Delta z = \dfrac{2\pi}{\gamma G_z \tau}$

Slice Selection





Nishimura 1996

Example

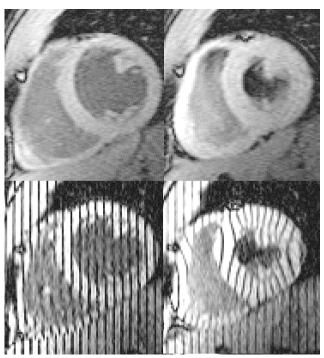
$\Delta z = 5\ mm; \tau = 400\ \mu\text{sec};\ \theta = \pi/2$

$$G_z = \frac{2\pi}{\gamma \Delta z \tau} = \frac{1}{(4257Hz/G)(0.5cm)(400e-6)} = 1.175\ G/cm$$

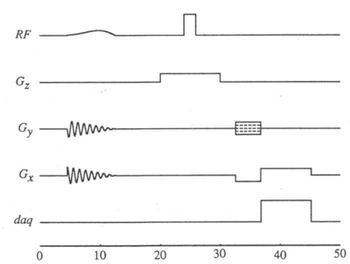
$$\theta \approx \gamma \int_0^T B_1 \operatorname{sinc}\left(\frac{s - T/2}{\tau}\right) ds \approx \gamma B_1 \cdot (\text{area of sinc}) = \gamma B_1 \tau$$

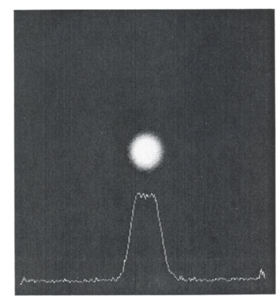
$$B_1 = \frac{\theta}{\gamma\tau} = \frac{\pi/2}{2\pi(4257Hz/G)(400e-6)} = 0.1468\ G$$

Cardiac Tagging



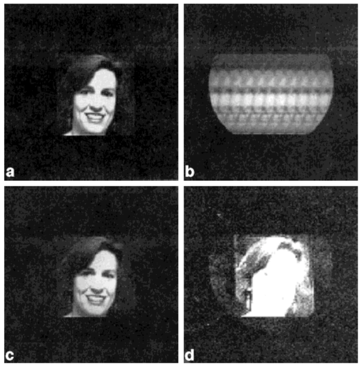
Excitation k-space





Pauly et al 1989

Excitation k-space



Panych MRM 1999