BENG280A, Principles of Biomedical Imaging 10/21/08
Fall Quarter 2008

HOMEWORK #4
Due at the start of Class on Thursday 10/30/08

Readings:
Section 2.8 and review Chapter 6 as necessary.

Problems:
1. Generalized functions. Recall that delta functions are not ordinary functions, and are defined by what they "do". In class, we showed how to integrate a delta function with a "test" function in order to see what it does. Using this approach, show that $\delta(x) = \int_{-\infty}^{\infty} e^{j2\pi k_x x} dk_x$. HINT: Multiply the expression by a test function $g(x)$ and integrate over *x*; then consider that the resulting expression is in the form of an inverse Fourier transform evaluated at a specific *x* location. This will allow you to show that $\int_{-\infty}^{\infty} e^{j2\pi k_x x} dk_x$ "acts" like $\delta(x)$.

2. Let $G(k,\theta)$ be the 1-D Fourier transform of the projection $g(l,\theta)$.
   a) Show that $g(l,\theta+\pi) = g(-l,\theta)$
   b) Next, show that $G(k,\theta+\pi) = G(-k,\theta)$

3. Problem 2.24
4. Consider the CT k-space filter $G(k) = |k|w(k)$ where $w(k)$ is a windowing function. For each of the following window functions, use MATLAB to plot the k-space filter and then derive its inverse Fourier transform.

   a) The Ram-Lak Filter with $w(k) = rect\left(\frac{k}{2k_{\max}}\right)$.

   b) A Hanning window defined as $w(k) = rect\left(\frac{k}{2k_{\max}}\right)\left(0.5 + 0.5\cos\left(\frac{\pi k}{k_{\max}}\right)\right)$.

   c) Use MATLAB to plot out and compare the inverse transforms from parts (a) and (b). Comment on the relative advantages and disadvantages of the two filters for CT reconstruction.

5. A parallel beam CT imaging system is used to image an object defined as:
   $$f(x,y) = rect(x,y) + \left(rect(x,y) ** \left[(\delta(x-3) + \delta(x+3))\delta(y)\right] ** \left[(\delta(y-4) + \delta(y+4))\delta(x)\right]\right)$$
   a) Sketch the object and draw the projections of the object at 0 degrees and 45 degrees.
   b) Derive the Fourier transform of the object
   c) Show that the Projection-slice theorem holds for the projections at 0 and 45 degrees.

6. (20 pts) Consider the object $f(x,y) = \cos\left(\frac{2}{\sqrt{3}}\pi x + 2\pi y\right)$
   a) Sketch the object.
   b) Consider sampling the object in both the x and y directions with sample intervals of Δ_x and Δ_y, respectively. Indicate what sample intervals should be used to avoid aliasing.
   c) Now consider imaging the object with a parallel beam CT imaging system. At what angle will the projection be non-zero?
   d) We now wish to sample the non-zero projection. What sampling interval should we use to avoid aliasing?

e) Now consider the object $g(x,y) = \left(f(x,y)\right)^2$. Answer items (c) and (d) for this object.