Neuroscience 200C **April 9, 2008**
Spring Quarter
Lecturer: Tom Liu

Magnetic Resonance Imaging Pre-lecture Notes

In these notes we will review some basic concepts that will help you follow next Tuesday's lecture on the basics of magnetic resonance imaging. Hopefully, you will have seen most of these concepts already at some point in our education (probably a trigonometry or calculus class), so this will just be a review of what you already know. Each concept will be followed by some simple exercises. Please do these exercises **before** the lecture and be prepared to hand in your solutions.

Concept 1: Imaginary and Complex Numbers.

The imaginary number is defined as the square root of -1. Mathematicians use the symbol *i,* while engineers like to use the symbol *j*. I'm an engineer, so for the lecture and these notes we'll use *j* and our definition is $j = \sqrt{-1}$.

A complex number $z = x + jy$ is simply a real number *x* plus an imaginary number multiplied by a real number *y*. For example, $3 + 2j$ is a complex number that consists of the real number 3 plus 2 times the imaginary number. In this case the number 3 is called the real part and 2*j* is the imaginary part.

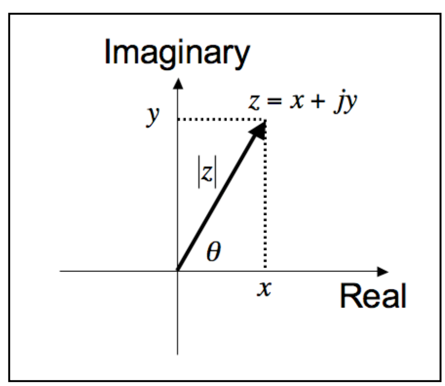
Exercises for Concept 1:

a) What is j^2 ?
b) What is $(3+2j)\cdot(3-2j)$?

Concept 2: Complex number as a vector.

Because the complex number $z = x + jy$ consists of two parts (a real part and an imaginary part), it is useful to represent it as a two-dimensional vector that starts at the origin and ends at the point (x, jy) where the "x-axis" of our graph corresponds to the real part and the "y-axis" of our graph corresponds to the imaginary part.

With the vector representation, we can also talk about the magnitude and angle of the complex number. The magnitude is simply given by the Pythagorean relation $|z| = \sqrt{x^2 + y^2}$ and the angle is given by $\theta = \tan^{-1}(y/x)$.

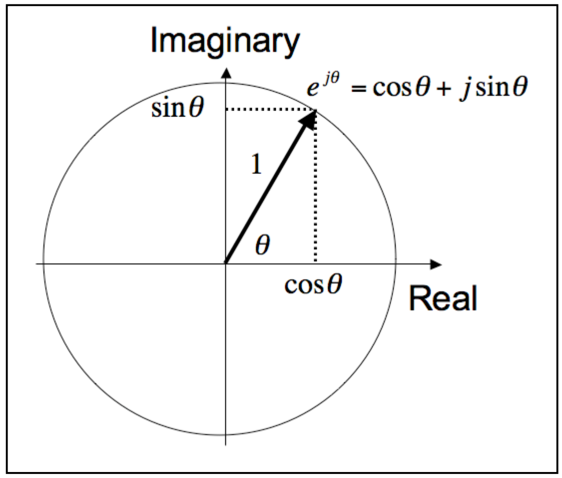


Exercises for Concept 2:

a) Plot the complex number $2 + 1j$ as a vector.
b) What are the magnitude and angle of the complex number $2 + 1j$?

Concept 3: Euler's Formula and the Complex Exponential

Euler's formula states that $e^{j\theta} = \cos\theta + j\sin\theta$. Note that this means that $e^{j\theta}$ is just a complex number with magnitude of $\sqrt{\cos^2\theta + \sin^2\theta} = 1$ and angle θ. It is commonly referred to as the complex exponential. As shown in the Figure, the complex exponential is simply the unit vector at an angle of θ. As the angle increases, the complex exponential traces out the unit circle. Using Euler's formula, any complex number can be written as $z = x + jy = |z|e^{j\theta}$.



Exercises for Concept 3:

a) Plot out the complex exponential for values of θ equal to 0, $\pi/4$, $\pi/2$, and π
b) What is $e^{-j\pi/2} + e^{j\pi/2}$?
c) Use Euler's formula to write $2 + 1j$ in the form $|z|e^{j\theta}$.

Concept 4: Functions of a complex exponential

Complex exponentials are really useful for representing any physical phenomenon that is cyclical or oscillates. For example, imagine a racecar going clockwise around a circular racetrack of radius R miles. If the car starts at an angle of zero and goes at a speed of *v* miles per hour, then its angle at time *t* is given by $\theta = \frac{vt}{R}$.

Exercise for Concept 4: Assume that the car is going 100 miles per hour and the radius of the track is 100 miles. What are the complex exponentials that correspond to the car's position at 0 hours, 0.5 hours, and 1 hour? Plot these complex exponentials. (Hint: Note that the angle is defined in units of radians and to convert from radians to degrees, divide by π and multiply by 180).

Concept 5: Signal Decomposition 101

As we'll discuss in Tuesday's lecture, a discrete-time signal (such as the audio waveform in your MP3 player), can be represented by a set of numbers such as $\{2,3,7,4\}$. This signal can be expressed as the sum of other signals. For example, we could write $\{2,3,7,4\} = 2\cdot\{1,0,0,0\} + 3\cdot\{0,1,0,0\} + 7\cdot\{0,0,1,0\} + 4\cdot\{0,0,0,1\}$.

Exercise for Concept 5

Consider the signal {2,0,2,0}. We want to write it as the sum:

$\{2,0,2,0\} = a\cdot\{1,1,1,1\} + b\cdot\{1,0,-1,0\} + c\cdot\{0,1,0,-1\} + d\cdot\{1,-1,1,-1\}$. What are the correct values of a, b, c, and d?