

4. (10 points)

$$G(k) = |k| w(k)$$

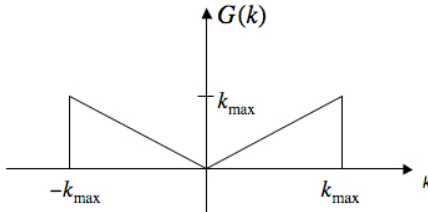
(a) (3 points) Ram-Lak Filter:

$$w(k) = \text{rect}\left(\frac{k}{2k_{\max}}\right)$$

$$G(k) = |k| \text{rect}\left(\frac{k}{2k_{\max}}\right)$$

$$= k_{\max} \left(\text{rect}\left(\frac{k}{2k_{\max}}\right) - \Lambda\left(\frac{k}{k_{\max}}\right) \right)$$

$$g(l) = 2(k_{\max})^2 \text{sinc}(2k_{\max}l) - (k_{\max})^2 \text{sinc}^2(k_{\max}l)$$



(b) (3 points) Hanning Window:

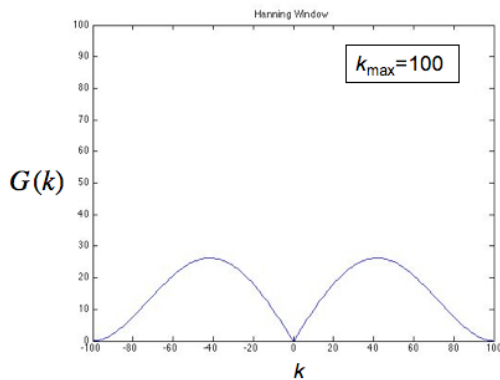
$$w(k) = \text{rect}\left(\frac{k}{2k_{\max}}\right) \left(0.5 + 0.5 \cos\left(\frac{\pi k}{k_{\max}}\right) \right)$$

$$G(k) = |k| 0.5 \text{rect}\left(\frac{k}{2k_{\max}}\right) + |k| 0.5 \left(\frac{k}{2k_{\max}}\right) \cos\left(\frac{\pi k}{k_{\max}}\right)$$

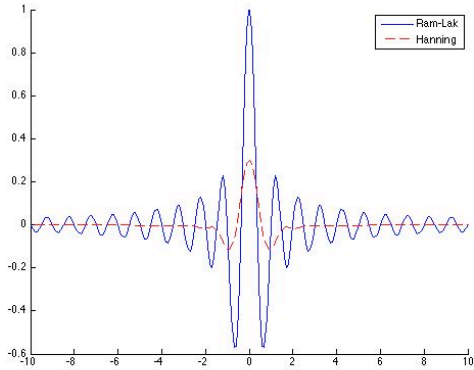
$$g(l) = 0.5 \left(2(k_{\max})^2 \text{sinc}(2k_{\max}l) - (k_{\max})^2 \text{sinc}^2(k_{\max}l) \right) * \left(1 + 0.5\delta\left(l - \frac{1}{2k_{\max}}\right) + 0.5\delta\left(l + \frac{1}{2k_{\max}}\right) \right)$$

$$= 0.5 \left[2(k_{\max})^2 \text{sinc}(2k_{\max}l) - (k_{\max})^2 \text{sinc}^2(k_{\max}l) \right] + 0.25 \left[2(k_{\max})^2 \text{sinc}\left(2k_{\max}\left(l - \frac{1}{2k_{\max}}\right)\right) - (k_{\max})^2 \text{sinc}^2\left(k_{\max}\left(l - \frac{1}{2k_{\max}}\right)\right) \right]$$

$$+ 0.25 \left[2(k_{\max})^2 \text{sinc}\left(2k_{\max}\left(l + \frac{1}{2k_{\max}}\right)\right) - (k_{\max})^2 \text{sinc}^2\left(k_{\max}\left(l + \frac{1}{2k_{\max}}\right)\right) \right]$$



(c) (4 points)



Code:

```
figure; hold on
kmax = 1;
t = -10:.1:10;
RL = 2*kmax.^2*sinc(2*kmax*t)-kmax.^2*sinc(kmax*t).^2;
plot(t,RL)
HN = 0.5*(2*kmax.^2*sinc(2*kmax*t)-kmax.^2*sinc(kmax*t).^2)...
    +0.25*(2*kmax.^2*sinc(2*kmax*(t-1/(2*kmax)))-kmax.^2*sinc(kmax*(t-1/(2*kmax))))).^2)...
    +0.25*(2*kmax.^2*sinc(2*kmax*(t+1/(2*kmax)))-kmax.^2*sinc(kmax*(t+1/(2*kmax))))).^2);
plot(t,HN,'r--')
legend('Ram-Lak','Hanning')
```

The Ram-Lak filter will preserve more high frequency information than the Hanning Window. This is good for edges but will also result in more noise. The Hanning window will create a smoother image with less noise, but also blurs the edges and reduces contrast.