Computed Tomography

Scanner Generations

Figure S.12: Different scanner generations: (a) first generation, (b) second generation, (c) third generation, and (d) fourth generation CT scanner.
**Scanner Generations**

**TABLE A.1**

<table>
<thead>
<tr>
<th>Generation</th>
<th>Source Collimation</th>
<th>Detector Collimation</th>
<th>Source-Detector Angle</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1G</td>
<td>Single-scan mode</td>
<td>Plane beam</td>
<td>Single</td>
<td>More sensitive to motion artifacts</td>
<td>Simpler, lower cost</td>
</tr>
<tr>
<td>2G</td>
<td>Single-scan mode</td>
<td>Plane beam</td>
<td>Multiple</td>
<td>Same as 1G</td>
<td>More complex, higher cost</td>
</tr>
<tr>
<td>3G</td>
<td>Single-scan mode</td>
<td>Line beam</td>
<td>Collimated line beam</td>
<td>Same as 1G</td>
<td>More complex, higher cost</td>
</tr>
<tr>
<td>4G (EKT)</td>
<td>Single-scan mode</td>
<td>Circular beam</td>
<td>Circular beam</td>
<td>Same as 1G</td>
<td>More complex, higher cost</td>
</tr>
<tr>
<td>5G (Spectra CT)</td>
<td>Spine beam</td>
<td>Spine beam</td>
<td>Circular beam</td>
<td>Same as 1G</td>
<td>More complex, higher cost</td>
</tr>
<tr>
<td>6G (Spiral CT)</td>
<td>Spine beam</td>
<td>Spine beam</td>
<td>Circular beam</td>
<td>Same as 1G</td>
<td>More complex, higher cost</td>
</tr>
<tr>
<td>7G (Multiplanar CT)</td>
<td>Spine beam</td>
<td>Spine beam</td>
<td>Multiple detector</td>
<td>Same as 1G</td>
<td>More complex, higher cost</td>
</tr>
</tbody>
</table>

**Prince and Links 2005**

---

**Example 6.1 from Prince and Links**

Compare 1G vs. 2G scanner whose source–detector apparatus can move linearly at speed of 1 m/sec; FOV 0.5m; 360 projections over 180 degrees; 0.5 s for apparatus to rotate one angular increment, regardless of angle.

**Question:** Scan time for 1G scanner? Scan time for 2G scanner with 9 detectors spaced 0.5 degrees apart?

**Answer:**

1G scanner:

\[
\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{0.5\text{m}}{1\text{m/s}} = 0.5\text{sec per projection.}
\]

\[
\text{Total Time} = 360 \times 0.5 = 180\text{ sec or 3 min.}
\]

2G scanner:

\[
\text{Angular resolution} = \frac{180\text{ degrees}}{360\text{ projections}} = 0.5\text{ degrees per projection.}
\]

\[
\text{40 projections} = 20\text{ degrees.}
\]

\[
\text{Total Time} = 40\times 0.5 = 20\text{ sec or 1 min.}
\]
3G, 6G, and 7G scanners

3G scanner: Typical scanner acquires 1000 projections with fanbeam angle of 30 to 60 degrees; 500 to 700 detectors; 1 to 20 seconds.

6G: Spiral/Helical CT
- 60 cm torso scan: 30s
- 24 cm lung scan: 12s
- 15 cm angio: 30s

7G: Multislice CT
- 64 or more parallel 1D projections.

CT Line Integral

$I_d = \int_0^{E_{\text{max}}} S_0(E) E \exp\left(-\int_0^E \mu(s;E)ds\right) dE$

Monoenergetic Approximation

$I_d = I_0 \exp\left(-\int_0^E \mu(s;E)ds\right)$

$g_d = -\log\left(\frac{I_d}{I_0}\right) = \int_0^E \mu(s;E)ds$

Detectors

Prince and Links 2005

Suetens 2002
CT Number

\[
CT \text{ number} = \frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}} \times 1000
\]

Measured in Hounsfield Units (HU)

Air: -1000 HU
Soft Tissue: -100 to 60 HU
Cortical Bones: 250 to 1000 HU
Metal and Contrast Agents: > 2000 HU

CT Display

Direct Inverse Approach

\[
\begin{align*}
\mu_1 & \quad \mu_2 \\
\mu_3 & \quad \mu_4 \\
p_1 & \quad p_2 \\
p_3 & \quad p_4 \\
p_1 &= \mu_1 + \mu_2 \\
p_2 &= \mu_3 + \mu_4 \\
p_3 &= \mu_1 + \mu_3 \\
p_4 &= \mu_2 + \mu_4
\end{align*}
\]

4 equations, 4 unknowns.
Are these the correct equations to use?

Direct Inverse Approach

\[
\begin{bmatrix}
\mu_1 \\
\mu_2 \\
\mu_3 \\
\mu_4
\end{bmatrix} =
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & \mu_1 \\
0 & 0 & 1 & 1 & \mu_2 \\
1 & 1 & 0 & 0 & \mu_3 \\
0 & 1 & 0 & 1 & \mu_4
\end{bmatrix}
\]

Figure 5.4: CT image of the chest with different window/level settings: (a) for the lungs (window 1500 and level -500) and (b) for the soft tissue (window 320 and level 50).

Matrix is not full rank.
Direct Inverse Approach

\[
\begin{align*}
\mu_1 &+ \mu_2 = p_1 \\
\mu_3 &+ \mu_4 = p_2 \\
\mu_1 &+ \mu_3 = p_3 \\
\mu_1 &+ \mu_4 = p_4
\end{align*}
\]

4 equations, 4 unknowns. These are linearly independent now. In general for an N x N image, N^2 unknowns, N^2 equations. This requires the inversion of a N^2 x N^2 matrix. For a high-resolution 512 x 512 image, N^2 = 262144 equations. Requires inversion of a 262144 x 262144 matrix! Inversion process sensitive to measurement errors.

Iterative Inverse Approach

Algebraic Reconstruction Technique (ART)

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
3 & 4 & 5 & 6 \\
4 & 5 & 7 & 5 \\
5 & 5 & 5 & 5
\end{array}
\]

\[
\begin{array}{cccc}
1.5 & 1.5 & 3 & 3 \\
3.5 & 3.5 & 7 & 7 \\
5 & 5 & 5 & 5
\end{array}
\]

Backprojection

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}
\]

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1
\end{array}
\]

In-Class Exercise

\[
\begin{array}{cc}
\mu_1 & 5.7 \\
\mu_3 & 11.3 \\
\mu_2 & 8.2 \\
\mu_4 & 8.8 \\
\end{array}
\]

\[
\begin{array}{cc}
1.5 & 1.5 \\
3.5 & 3.5 \\
5 & 5 \\
10.1 & 11.3
\end{array}
\]
Projections

$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$I(\mathbf{r}, \theta) = I_0 \exp \left( - \int_{L_{r,\theta}} \mu(x, y) \, ds \right)$

$= I_0 \exp \left( - \int_{L_{r,\theta}} \mu(r \cos \theta - ss \sin \theta, r \sin \theta + ss \cos \theta) \, ds \right)$

Radon Transform

$g(r, \theta) = \int_{-\infty}^{\infty} \mu(x(s), y(s)) \, ds$

$= \int_{\mathbb{R}} \mu(r \cos \theta - ss \sin \theta, r \sin \theta + ss \cos \theta) \, ds$

$= \int_{\mathbb{R}} \int_{-\infty}^{\infty} \mu(x, y) \delta(x \cos \theta + y \sin \theta - r) \, dx \, dy$

$\mathbf{r} \cdot \mathbf{\hat{r}} = r$

$(x \hat{x} + y \hat{y}) \cdot (\cos \theta \hat{x} + \sin \theta \hat{y}) = r$

$x \cos \theta + y \sin \theta = r$
**Example**

\[ f(x, y) = \begin{cases} 1 & x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases} \]

\[ g(l, \theta = 0) = \int_{-\infty}^{\infty} f(l, y) \, dy \]
\[ = \int_{\frac{-1}{\sqrt{1-l^2}}}^{\frac{1}{\sqrt{1-l^2}}} dy \]
\[ = 2\sqrt{1-l^2} \quad \| l \| \leq 1 \]
\[ 0 \quad \text{otherwise} \]

**Sinogram**

\[ b(x_0, y) = p(l, \theta = 0) \Delta \theta \]
\[ = p(x_0, 0) \Delta \theta \]

\[ b_y(x, y) = g(x \cos \theta + y \sin \theta, \theta) \Delta \theta 
\]
\[ b(x, y) = B \{ g(l, \theta) \} = \int_0^\pi p(x \cos \theta + y \sin \theta, \theta) d\theta \]
Backprojection

\[ b(x, y) = B\{p(l, \theta)\} = \int_0^\pi p(x \cos \theta + y \sin \theta, \theta) d\theta \]

Example