Topics

- Projection Slice Theorem
- Fourier Transforms

Signals and Images

Discrete-time/space signal/image: continuous valued function with a discrete time/space index, denoted as $s[n]$ for 1D, $s[m,n]$ for 2D, etc.

Continuous-time/space signal/image: continuous valued function with a continuous time/space index, denoted as $s(t)$ or $s(x)$ for 1D, $s(x,y)$ for 2D, etc.
1D Signal Decomposition

\[ \{2,0,2,0\} = 2 \cdot \{1,0,0,0\} + 0 \cdot \{0,1,0,0\} + 2 \cdot \{0,0,1,0\} + 0 \cdot \{0,0,0,1\} \]

\[ \{2,0,2,0\} = a \cdot \{1,1,1,1\} + b \cdot \{1,0,-1,0\} + c \cdot \{0,1,0,-1\} + d \cdot \{1,-1,-1,-1\} \]

\[ \{2,0,2,0\} = 1 \cdot \{1,1,1,1\} + 0 \cdot \{1,0,-1,0\} + 0 \cdot \{0,1,0,-1\} + 1 \cdot \{1,-1,-1,-1\} \]

Eskimo Words for Snow

tlapa: powder snow
tlacringit: snow that is crusted on the surface
tlapat: still snow
klin: remembered snow
naklin: forgotten snow
tlamo: snow that falls in large wet flakes
tlalin: snow that falls in small flakes
tlalo: snow that falls slowly
tlapiti: snow that falls quickly
kripya: snow that has melted and refrozen
tliyel: snow that has been marked by wolves
tliyelin: snow that has been marked by Eskimos
tlapaman: snow sold to German tourists
tlalan: snow sold to American tourists
tlapip: snow sold to Japanese tourists
tla-na-na: snow mixed with the sound of old rock and roll from a portable radio
depptla: a small snowball, preserved in lucite, that had been handled by Johnny Depp

Image Compression

https://www.mendosa.com/snow.html
2D Image

\[
\begin{array}{|c|c|}
\hline
a & b \\
\hline
c & d \\
\hline
\end{array}
= \begin{array}{|c|c|}
\hline
\begin{array}{|c|c|}
\hline
a & 0 \\
\hline
0 & 0 \\
\hline
\end{array} & \begin{array}{|c|c|}
\hline
0 & b \\
\hline
\end{array} \\
\hline
\end{array} + \begin{array}{|c|c|}
\hline
\begin{array}{|c|c|}
\hline
0 & 0 \\
\hline
0 & 0 \\
\hline
\end{array} & \begin{array}{|c|c|}
\hline
0 & d \\
\hline
\end{array} \\
\hline
\end{array}
\]

Image Decomposition

\[
\begin{array}{|c|c|}
\hline
a & b \\
\hline
c & d \\
\hline
\end{array}
= \begin{array}{|c|c|}
\hline
\begin{array}{|c|c|}
\hline
1 & 0 \\
\hline
0 & 0 \\
\hline
\end{array} & \begin{array}{|c|c|}
\hline
0 & 1 \\
\hline
\end{array} \\
\hline
\end{array} + \begin{array}{|c|c|}
\hline
\begin{array}{|c|c|}
\hline
0 & 0 \\
\hline
1 & 0 \\
\hline
\end{array} & \begin{array}{|c|c|}
\hline
0 & 0 \\
\hline
\end{array} \\
\hline
\end{array}
\]

1D Fourier Transform

\[
x = \begin{array}{|c|c|}
\hline
1/2 & 1/2 \\
\hline
1/2 & 1/2 \\
\hline
\end{array} + \begin{array}{|c|c|}
\hline
1/2 & -1/2 \\
\hline
1/2 & -1/2 \\
\hline
\end{array}
\]

\[
\text{Basis Functions} \quad \text{Coefficients}
\]

\[
\text{Sum}
\]

\[
\text{Object}
\]

\[
\text{KPBS} \quad \text{KIFM} \quad \text{KIOZ}
\]

\[
\text{Fourier Transform}
\]
The Fourier Transform

Fourier Transform (FT)
\[ G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt = F\{g(t)\} \]

Inverse Fourier Transform
\[ g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df = F^{-1}\{G(f)\} \]

Complex Numbers

\[ j = \sqrt{-1} \]
\[ j^2 = ? \]
\[ (3 + 2j)(3 - 2j) = ? \]
\[ j^2 = -1 \]
\[ (3 + 2j)(3 - 2j) = 9 - 4j^2 = 13 \]

Complex Numbers

\[ z = 2 + 1j \]
\[ |z| = \sqrt{2^2 + 1^2} = \sqrt{5} \]
\[ \theta = \tan^{-1}\left(\frac{1}{2}\right) = 30 \text{ degrees} \]

Euler’s Formula

\[ e^{j\theta} = \cos\theta + j\sin\theta \]
\[ z = x + jy = |z| e^{j\theta} \]
1D Fourier Transform

\[ G(k_x) = \int_{-\infty}^{\infty} g(x) \exp(-j2\pi k_x x) \, dx \]

\[ = \int_{-\infty}^{\infty} g(x) \cos(2\pi k_x x) \, dx - j \int_{-\infty}^{\infty} g(x) \sin(2\pi k_x x) \, dx \]

The part of \( g(x) \) that "looks" like \( \cos(2\pi k_x x) \)

The part of \( g(x) \) that "looks" like \( \sin(2\pi k_x x) \)

Units

Temporal Coordinates, e.g. \( t \) in seconds, \( f \) in cycles/second

\[ G(f) = \int_{-\infty}^{\infty} g(t) \exp(-j2\pi ft) \, dt \]

\[ g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft) \, df \]

Spatial Coordinates, e.g. \( x \) in cm, \( k_x \) is spatial frequency in cycles/cm

\[ G(k_x) = \int_{-\infty}^{\infty} g(x) \exp(-j2\pi k_x x) \, dx \]

\[ g(x) = \int_{-\infty}^{\infty} G(k_x) \exp(j2\pi k_x x) \, dk_x \]

Plane Waves

\[ e^{j2\pi(k_x x + k_y y)} = \cos(2\pi(k_x x + k_y y)) + j \sin(2\pi(k_x x + k_y y)) \]

\[ \frac{1}{\sqrt{k_x^2 + k_y^2}} \]
Plane Waves

\[ \Delta ABC \sim \Delta BDC \]

\[ \frac{AC}{BC} = \frac{AB}{BD} \]

\[ BD = AP \cdot \frac{BC}{AC} = \frac{1}{1/k_x + 1/k_y} = \frac{1}{\sqrt{k_x^2 + k_y^2}} \]

\[ \theta = \arctan \left( \frac{k_y}{k_x} \right) \]

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k-space

Image space

k-space

Fourier Transform

\[ \theta = \arctan \left( \frac{k_y}{k_x} \right) \]