Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

Magnetic Moment and Angular Momentum

A charged sphere spinning about its axis has angular momentum and a magnetic moment. This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: \( \mu = \gamma S \) where \( \gamma \) is the gyromagnetic ratio and \( S \) is the spin angular momentum.

Nuclear Spin Rules

<table>
<thead>
<tr>
<th>Number of Protons</th>
<th>Number of Neutrons</th>
<th>Spin</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>Even</td>
<td>0</td>
<td>(^{12}\text{C}, ^{16}\text{O} )</td>
</tr>
<tr>
<td>Even</td>
<td>Odd</td>
<td>( j/2 )</td>
<td>(^{17}\text{O} )</td>
</tr>
<tr>
<td>Odd</td>
<td>Even</td>
<td>( j/2 )</td>
<td>(^{3}\text{H}, ^{23}\text{Na}, ^{31}\text{P} )</td>
</tr>
<tr>
<td>Odd</td>
<td>Odd</td>
<td>( j )</td>
<td>(^{2}\text{H} )</td>
</tr>
</tbody>
</table>
**Classical Magnetic Moment**

\[ \vec{\mu} = I A \hat{n} \]

**Energy in a Magnetic Field**

\[ E = -\dot{\mu} \cdot \vec{B} = -\mu_z B \]

**Magnetic Field Units**

- 1 Tesla = 10,000 Gauss
- Earth’s field is about 0.5 Gauss
  \[ 0.5 \text{ Gauss} = 0.5 \times 10^{-4} \text{ T} = 50 \mu\text{T} \]

Earth’s Magnetic Field Image:

![Earth’s Magnetic Field](https://www.qi-whiz.com/images/Earth-magnetic-field.jpg)
**Boltzmann Distribution**

\[ \frac{N_j}{N} = \mathcal{P}(\epsilon_j) = \frac{e^{-\epsilon_j/(kT)}}{Q} \]

**Equilibrium Magnetization**

\[ M_z = N\langle \mu_z \rangle = N\mu B/(kT) = N\gamma^2 h^2 B/(4kT) \]

\( N \) = number of nuclear spins per unit volume. Magnetization is proportional to applied field.

**Bigger is better**

- 3T Human imager at UCSD
- 7T Human imager at U. Minn.
- 7T Rodent Imager at UCSD
- 9.4T Human imager at UIC

**Torque**

\[ \text{Torque} = \mu \times B \]

For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g., compass needle).
Precession

\[ \begin{align*}
\text{Torque} & \quad N = \mu \times B \\
\text{Change in Angular momentum} & \quad \frac{dS}{dt} = N \\
\end{align*} \]

Relation between magnetic moment and angular momentum

\[ \frac{d\mu}{dt} = \mu \times \gamma B \]

Analogous to motion of a gyroscope

Precesses at an angular frequency of \( \omega = \gamma B \)

This is known as the Larmor frequency.

http://www.astrophysik.uni-kiel.de/~hhaerte/mpg_e/gyros_free.mpg

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Magnetization Vector

Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

\[ M = \frac{1}{V} \sum_{i} \mu_i \]

\[ \frac{dM}{dt} = \gamma M \times B \]

Hansen 2009

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Gyromagnetic Ratios

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Spin</th>
<th>Magnetic Moment</th>
<th>( \gamma / (2\pi) ) (MHz/Tesla)</th>
<th>Abundance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^1\text{H})</td>
<td>1/2</td>
<td>2.793</td>
<td>42.58</td>
<td>88 M</td>
</tr>
<tr>
<td>(^{23}\text{Na})</td>
<td>3/2</td>
<td>2.216</td>
<td>11.27</td>
<td>80 mM</td>
</tr>
<tr>
<td>(^{31}\text{P})</td>
<td>1/2</td>
<td>1.131</td>
<td>17.25</td>
<td>75 mM</td>
</tr>
</tbody>
</table>

Source: Haacke et al., p. 27
Larmor Frequency

\[ \omega = \gamma B \]
Angular frequency in rad/sec

\[ f = \frac{\gamma B}{2\pi} \]
Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth’s magnetic field is about 50 \( \mu T \), so that a 1.5T system is about 30,000 times stronger.

Notation and Units

1 Tesla = 10,000 Gauss
Earth's field is about 0.5 Gauss
0.5 Gauss = 0.5 \times 10^{-4} T = 50 \( \mu T \)

\[ \gamma = 26752 \text{ radians/second/Gauss} \]
\[ \gamma = \frac{\gamma}{2\pi} = 4258 \text{ Hz/Gauss} \]
\[ = 42.58 \text{ MHz/Tesla} \]

RF Excitation

Simplified Drawing of Basic Instrumentation. Body lies on table encompassed by coils for static field \( B_0 \), gradient fields (two of three shown), and radiofrequency field \( B_1 \).

Image, caption: copyright Nishimura, Fig. 3.15
B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (xy) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the z-axis.

At equilibrium, net magnetization is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B. Hanson 2009

On-Resonance Excitation

(a) (b) (c)

http://www.drcmr.dk/main/content/view/213/74/

http://www.eecs.umich.edu/~dnol/BME516/

http://www.eecs.umich.edu/~dnol/BME516/
Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

Images & caption: Nishimura, Fig. 3.3

RF Excitation

From Buxton 2002

Free Induction Decay (FID)

http://www.easymeasure.co.uk/principlesmri.aspx