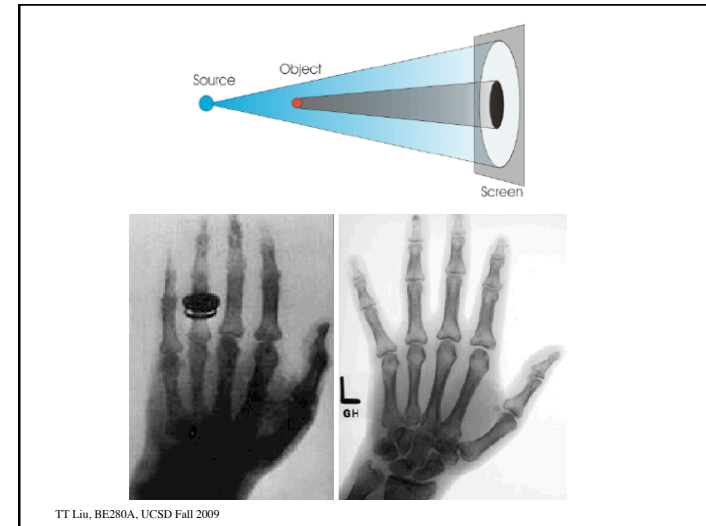


Bioengineering 280A Principles of Biomedical Imaging

Fall Quarter 2009
X-Rays Lecture 1

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EM spectrum

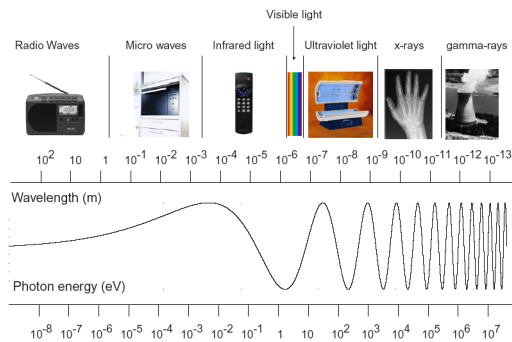
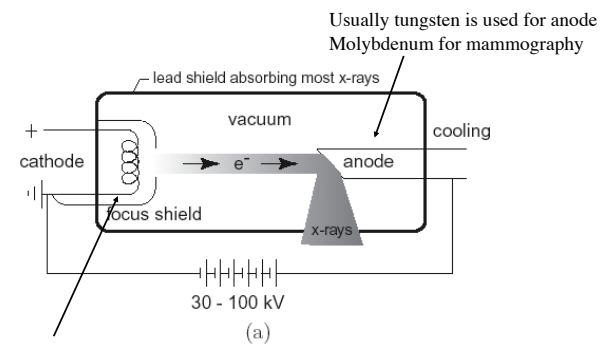


Figure 4.1: The electromagnetic spectrum.

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Suetens 2002

X-Ray Tube

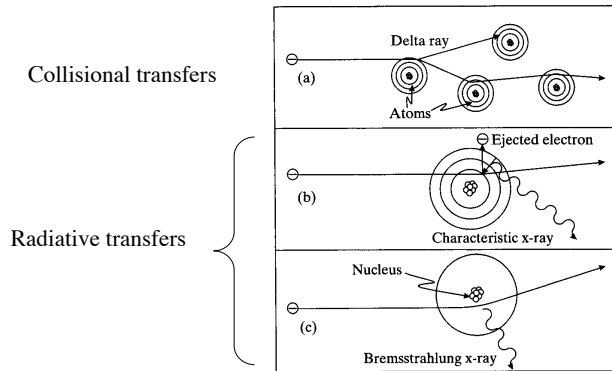


Tungsten filament heated to about 2200 C leading to thermionic emission of electrons.

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Suetens 2002

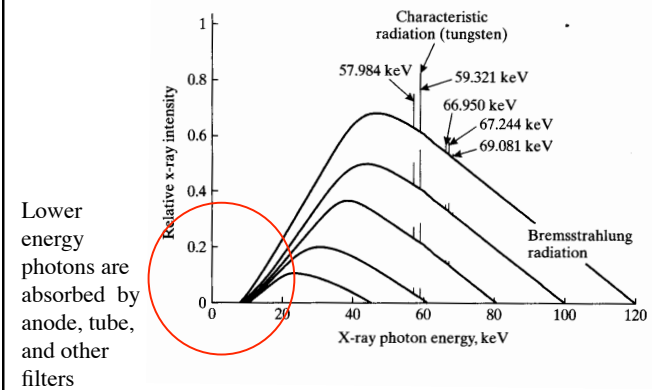
X-Ray Production



Prince and Links 2005

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X-Ray Spectrum

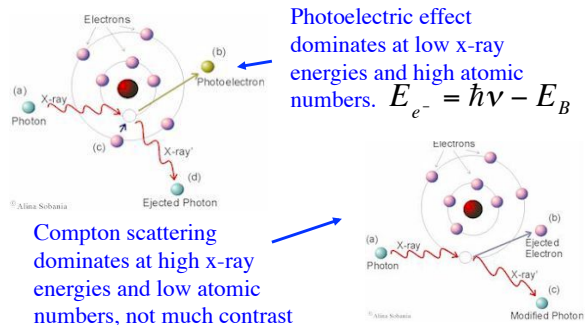


Prince and Links 2005

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Interaction with Matter

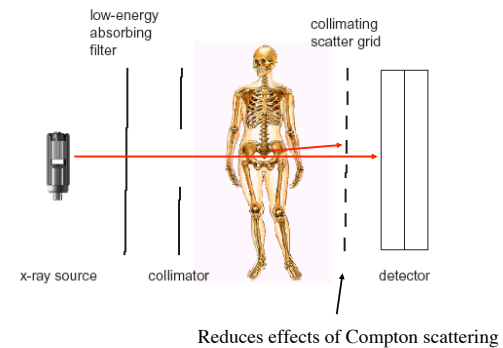
Typical energy range for diagnostic x-rays is below 200 keV. The two most important types of interaction are photoelectric absorption and Compton scattering.



<http://www.eee.ntu.ac.uk/research/vision/asobania>

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X-Ray Imaging Chain



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X-ray film

Emulsion with silver halide crystals
Each layer ~ 10 μm

Flexible base
~ 150 μm

Silver halide crystals absorb optical energy. After development, crystals that have absorbed enough energy are converted to metallic silver and look dark on the screen. Thus, parts in the object that attenuate the x-rays will look brighter.

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Intensifying Screen

X-RAY TUBE

PATIENT

PHOSPHOR SCREEN

PHOTOGRAPHIC FILM

LIGHT

FILM PROCESSOR & VIEWBOX

Phosphor crystal

Light photons emitted in all directions

Intensifying screen phosphor layer

Film emulsion

http://learntech.uwe.ac.uk/radiography/RScience/imaging_principles_d/diagimage11.htm
<http://www.sunnybrook.utoronto.ca:8080/~selenium/xray.html#Film>

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X-Ray Examples

(a)

(b)

Suetens 2002

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X-Ray w/ Contrast Agents

Angiogram using an iodine-based contrast agent.
K-edge of iodine is 33.2 keV

Barium Sulfate
K-edge of Barium is 37.4 keV

Suetens 2002

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Intensity

$$I = E\phi$$

Energy Photon flux rate

$$\phi = \frac{N}{A\Delta t}$$

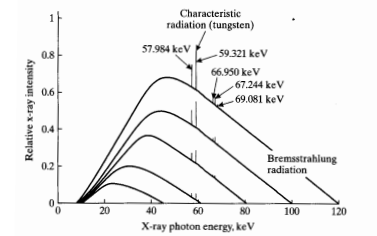
Number of photons
Unit Area Unit Time

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Intensity

$$\phi = \int_0^\infty S(E')dE'$$

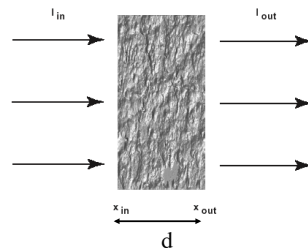
X-ray spectrum



$$I = \int_0^\infty S(E')E'dE'$$

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Attenuation



For single-energy x-rays passing through a homogenous object:

$$I_{out} = I_{in} \exp(-\mu d)$$

Linear attenuation coefficient

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Attenuation

$n = \mu N \Delta x$ photons lost per unit length

$\mu = \frac{n/N}{\Delta x}$ fraction of photons lost per unit length

$$\Delta N = -n \longrightarrow \frac{dN}{dx} = -\mu N \longrightarrow N(x) = N_0 e^{-\mu x}$$

For mono-energetic case, intensity is

$$I(\Delta x) = I_0 e^{-\mu \Delta x}$$

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Attenuation

Inhomogeneous Slab

$$\frac{dN}{dx} = -\mu(x)N \longrightarrow N(x) = N_0 \exp\left(-\int_0^x \mu(x') dx'\right)$$

$$I(x) = I_0 \exp\left(-\int_0^x \mu(x') dx'\right)$$

Attenuation depends on energy, so also need to integrate over energies

$$I(x) = \int_0^\infty S_0(E') E' \exp\left(-\int_0^x \mu(x'; E') dx'\right) dE'$$

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Attenuation

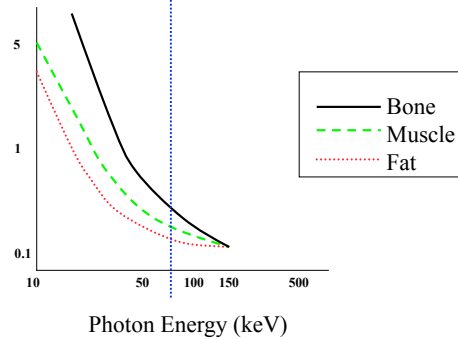
More Attenuation

Attenuation Coefficient

Less Attenuation

Photoelectric effect dominates

Compton Scattering dominates



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Half Value Layer

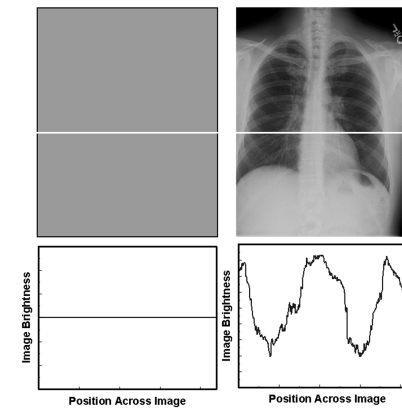
X-ray energy (keV)	HVL, muscle (cm)	HVL Bone (cm)
30	1.8	0.4
50	3.0	1.2
100	3.9	2.3
150	4.5	2.8

In chest radiography, about 90% of x-rays are absorbed by body. Average energy from a tungsten source is 68 keV. However, many lower energy beams are absorbed by tissue, so average energy is higher. This is referred to as beam-hardening, and reduces the contrast.

Values from Webb 2003

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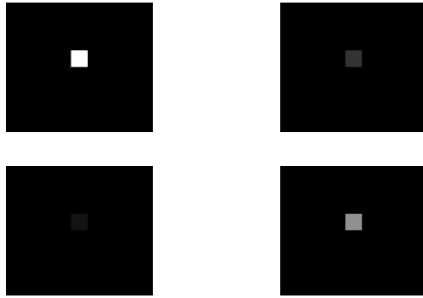
Contrast



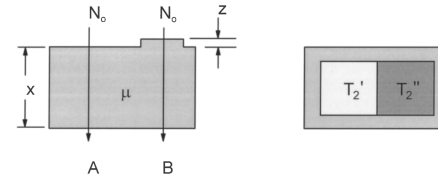
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Bushberg et al 2001

Contrast



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(A) X-ray Imaging

(B) MR Imaging

Bushberg et al 2001

$$A = N_0 \exp(-\mu x)$$

$$B = N_0 \exp(-\mu(x+z))$$

Subject Contrast

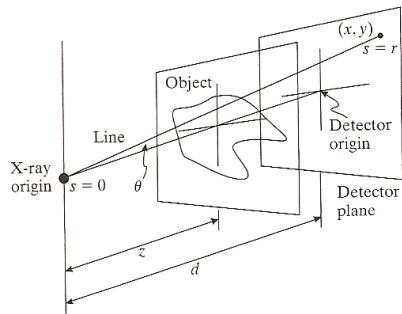
$$C_s = \frac{A-B}{A}$$

$$= \frac{N_0 \exp(-\mu x) - N_0 \exp(-\mu(x+z))}{N_0 \exp(-\mu x)}$$

$$= 1 - \exp(-\mu z)$$

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X-Ray Imaging Geometry



Prince and Links 2005

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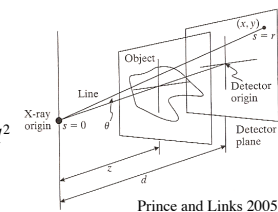
Inverse Square Law

Inverse Square Law

$$I_0 = \frac{I_s}{4\pi d^2}$$

$$I_d(x,y) = \frac{I_s}{4\pi r^2} \text{ where } r^2 = x^2 + y^2 + d^2$$

$$= \frac{I_0 d^2}{r^2} = I_0 \cos^2 \theta$$

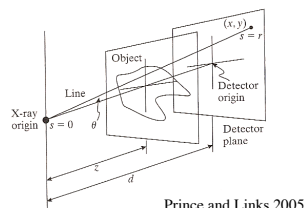
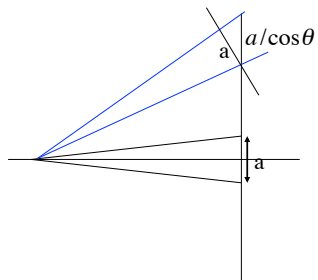


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Obliquity Factor

Obliquity Factor
 $I_d(x, y) = I_0 \cos^3 \theta$



Prince and Links 2005

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X-Ray Imaging Geometry

Beam Divergence and Flat Panel

$$I_r = I_0 \cos^3 \theta$$

Example: Chest x-ray at 2 yards with 14x17 inch film.

Question: What is the smallest ratio I_r/I_0 across the film?

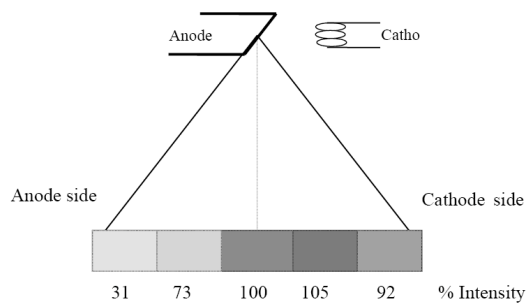
$$r_d = \sqrt{7^2 + 8.5^2} = 11$$

$$\cos \theta = \frac{d}{\sqrt{r_d^2 + d^2}} = 0.989$$

$$\frac{I_r}{I_0} = \cos^3 \theta = 0.966$$

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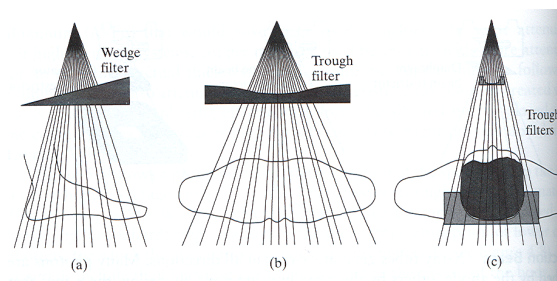
Anode Heel Effect



<http://www.animalinsides.com/radphys/chapters/Lect2.pdf>

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Compensation Filters



Prince and Links 2005

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Path Length

$L' = L / \cos \theta$

$I_d(x, y) = I_0 \cos^3 \theta \exp(-\mu L / \cos \theta)$

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Magnification of Object

$M(z) = \frac{d}{z}$

$= \frac{\text{Source to Image Distance (SID)}}{\text{Source to Object Distance (SOD)}}$

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Magnification of Object

$M = 1: I(x, y) = t(x, y)$

$M = 2: I(x, y) = t(x/2, y/2)$

In general, $I(x, y) = t(x/M(z), y/M(z))$

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X-Ray Imaging Equation

At $z = d$ there is no magnification, so

$$I_d(x, y) = I_0 \cos^3 \theta \cdot \exp\left(-\int_{L_{x,y}} \mu(s) ds / \cos \theta\right)$$

$$= I_0 \cos^3 \theta \cdot t_d(x, y)$$

where $t_z(x, y)$ is the transmittivity of the object at distance z

In general, with magnification

$$I_d(x, y) = I_0 \cos^3 \theta \cdot t_z(x/M(z), y/M(z))$$

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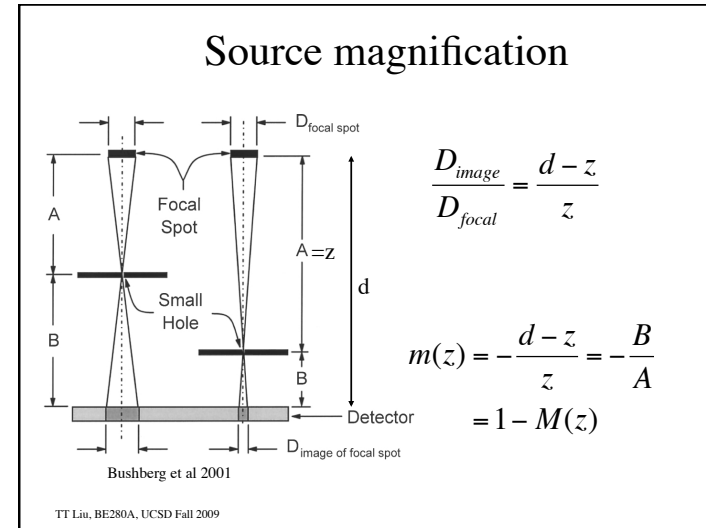
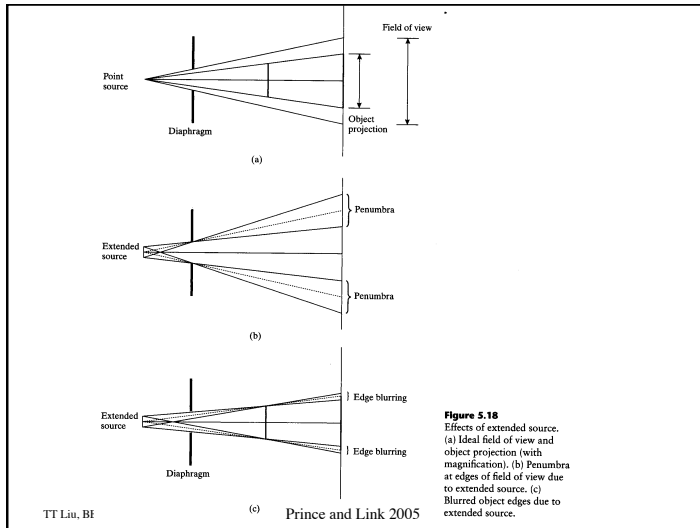


Image of a point object

$$I_d(x, y) = ks(x/m, y/m)$$

$$\iint ks(x/m(z), y/m(z)) dx dy = \text{constant}$$

$$\Rightarrow k = \frac{1}{m^2(z)}$$

$$I_d(x, y) = \lim_{m \rightarrow 0} \frac{s(x/m, y/m)}{m^2} = \delta(x, y)$$

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Image of arbitrary object

Diagram showing the source s(x,y) and the object t(x,y) on the left, and the resulting image I_d(x,y) on the right. The magnification factor m is shown as m=1.

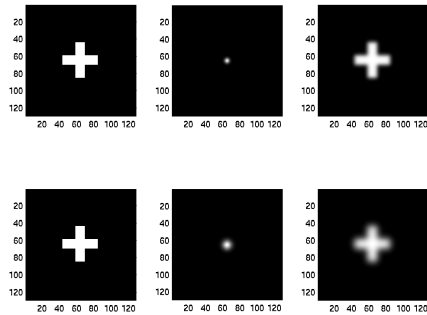
$$\lim_{m \rightarrow 0} I_d(x, y) = t(x, y)$$

$$I_d(x, y) = ???$$

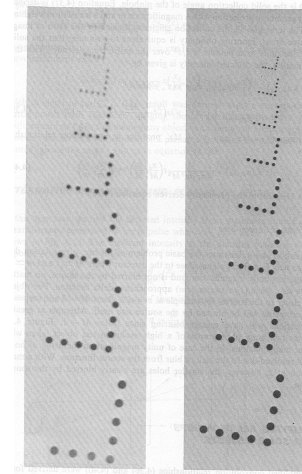
$$I_d(x, y) = \frac{\cos^3 \theta}{4\pi d^2 m^2} s(x/m, y/m) ** t(x/M, y/M)$$

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Convolution



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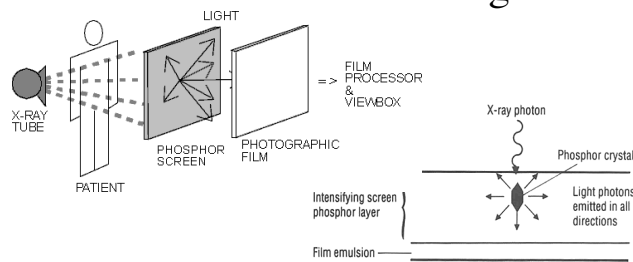
M=2
m=-1

M=1
m=0

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Macovski 1983

Film-screen blurring



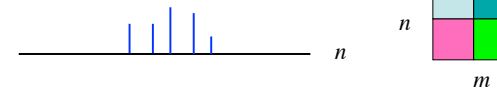
$$I_d(x, y) = \frac{\cos^3 \theta}{4\pi d^2 m^2} s(x/m, y/m) * t(x/M, y/M) * h(x, y)$$

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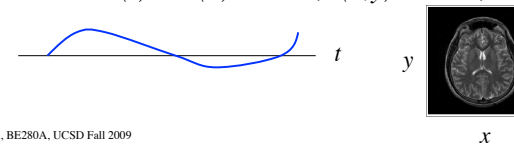
http://learntech.uwe.ac.uk/radiography/RScience/imaging_principles_d/diagram11.htm
<http://www.sunybrook.utoronto.ca:8080/~selenium/xray.html#Film>

Signals and Images

Discrete-time/space signal/image: continuous valued function with a discrete time/space index, denoted as $s[n]$ for 1D, $s[m, n]$ for 2D, etc.



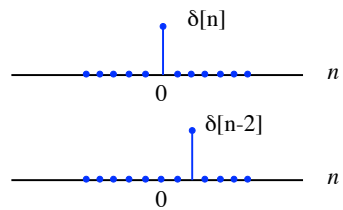
Continuous-time/space signal/image: continuous valued function with a continuous time/space index, denoted as $s(t)$ or $s(x)$ for 1D, $s(x, y)$ for 2D, etc.



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Kronecker Delta Function

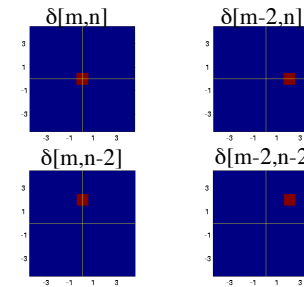
$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{otherwise} \end{cases}$$



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Kronecker Delta Function

$$\delta[m,n] = \begin{cases} 1 & \text{for } m=0, n=0 \\ 0 & \text{otherwise} \end{cases}$$

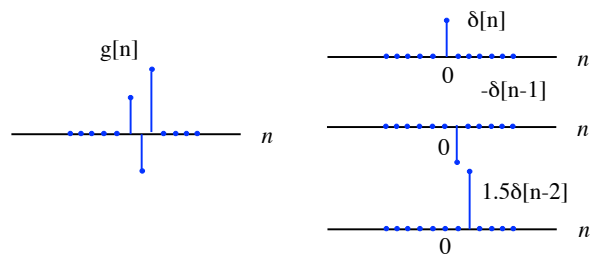


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Discrete Signal Expansion

$$g[n] = \sum_{k=-\infty}^{\infty} g[k]\delta[n-k]$$

$$g[m,n] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} g[k,l]\delta[m-k,n-l]$$



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2D Signal

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}$$

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Image Decomposition

$$\begin{array}{|c|c|} \hline c & d \\ \hline a & b \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 0 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 0 & 1 \\ \hline 0 & 0 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 1 & 0 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{aligned}
 g[m,n] &= a\delta[m,n] + b\delta[m,n-1] + c\delta[m-1,n] + d\delta[m-1,n-1] \\
 &= \sum_{k=0}^1 \sum_{l=0}^1 g[k,l]\delta[m-k,n-l]
 \end{aligned}$$

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Dirac Delta Function

Notation :

$\delta(x)$ - 1D Dirac Delta Function

$\delta(x,y)$ or ${}^2\delta(x,y)$ - 2D Dirac Delta Function

$\delta(x,y,z)$ or ${}^3\delta(x,y,z)$ - 3D Dirac Delta Function

$\delta(\vec{r})$ - N Dimensional Dirac Delta Function

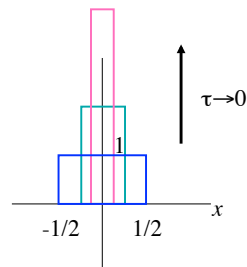
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1D Dirac Delta Function

$$\delta(x) = 0 \text{ when } x \neq 0 \text{ and } \int_{-\infty}^{\infty} \delta(x) dx = 1$$

Can interpret the integral as a limit of the integral of an ordinary function that is shrinking in width and growing in height, while maintaining a constant area. For example, we can use a shrinking rectangle function

$$\text{such that } \int_{-\infty}^{\infty} \delta(x) dx = \lim_{\tau \rightarrow 0} \int_{-\infty}^{\infty} \tau^{-1} \Pi(x/\tau) dx.$$



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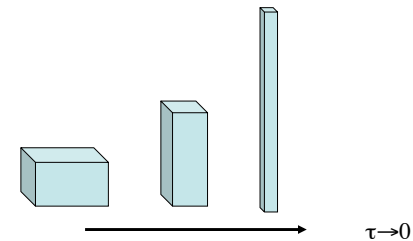
2D Dirac Delta Function

$$\delta(x,y) = 0 \text{ when } x^2 + y^2 \neq 0 \text{ and } \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x,y) dx dy = 1$$

where we can consider the limit of the integral of an ordinary 2D function that is shrinking in width but increasing in height while maintaining constant area.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x,y) dx dy = \lim_{\tau \rightarrow 0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tau^{-2} \Pi(x/\tau, y/\tau) dx dy.$$

Useful fact : $\delta(x,y) = \delta(x)\delta(y)$



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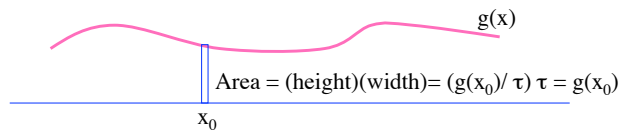
Generalized Functions

Dirac delta functions are not ordinary functions that are defined by their value at each point. Instead, they are generalized functions that are defined by what they do underneath an integral.

The most important property of the Dirac delta is the sifting property

$\int_{-\infty}^{\infty} \delta(x - x_0)g(x)dx = g(x_0)$ where $g(x)$ is a smooth function. This sifting property can be understood by considering the limiting case

$$\lim_{\tau \rightarrow 0} \int_{-\infty}^{\infty} \tau^{-1} \Pi(x/\tau)g(x)dx = g(x_0)$$



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Representation of 1D Function

From the sifting property, we can write a 1D function as

$$g(x) = \int_{-\infty}^{\infty} g(\xi)\delta(x - \xi)d\xi. \text{ To gain intuition, consider the approximation}$$

$$g(x) \approx \sum_{n=-\infty}^{\infty} g(n\Delta x) \frac{1}{\Delta x} \Pi\left(\frac{x - n\Delta x}{\Delta x}\right)\Delta x.$$



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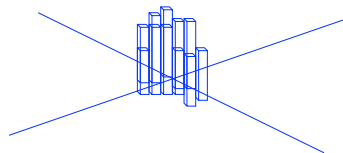
Representation of 2D Function

Similarly, we can write a 2D function as

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\xi, \eta)\delta(x - \xi, y - \eta)d\xi d\eta.$$

To gain intuition, consider the approximation

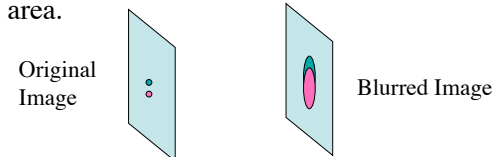
$$g(x, y) \approx \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g(n\Delta x, m\Delta y) \frac{1}{\Delta x} \Pi\left(\frac{x - n\Delta x}{\Delta x}\right) \frac{1}{\Delta y} \Pi\left(\frac{y - m\Delta y}{\Delta y}\right) \Delta x \Delta y.$$



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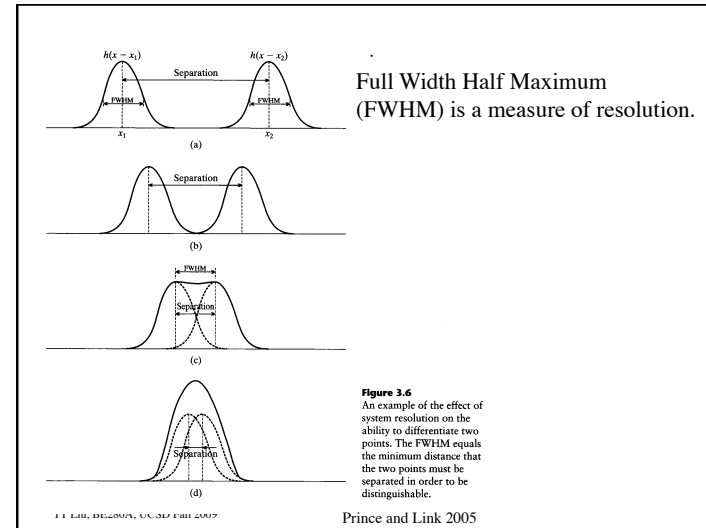
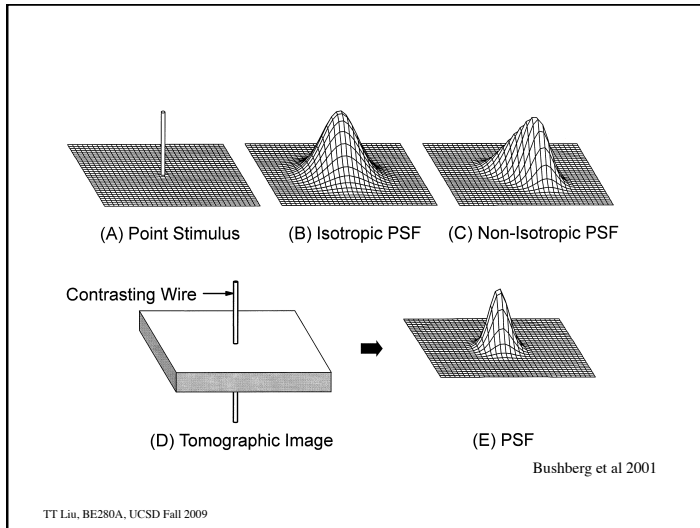
Impulse Response

Intuition: the impulse response is the response of a system to an input of infinitesimal width and unit area.



Since any input can be thought of as the weighted sum of impulses, a linear system is characterized by its impulse response(s).

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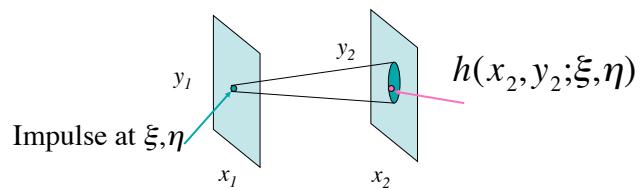


Impulse Response

The impulse response characterizes the response of a system over all space to a Dirac delta impulse function at a certain location.

$$h(x_2; \xi) = L[\delta(x_1 - \xi)] \quad \text{1D Impulse Response}$$

$$h(x_2, y_2; \xi, \eta) = L[\delta(x_1 - \xi, y_1 - \eta)] \quad \text{2D Impulse Response}$$



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