

Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2010
CT/Fourier Lecture 4

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Projection Theorem

$$\begin{aligned}
 U(k_x, 0) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y) e^{-j2\pi(k_x x + k_y y)} dx dy \\
 &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} u(x, y) dy \right] e^{-j2\pi k_x x} dx \\
 &= \int_{-\infty}^{\infty} g(x, 0) e^{-j2\pi k_x x} dx \\
 &= \int_{-\infty}^{\infty} g(l, 0) e^{-j2\pi k l} dl
 \end{aligned}$$

In-Class Example:
 $\mu(x, y) = \cos 2\pi x$

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Projection Theorem

$$\begin{aligned}
 U(k_x, k_y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y) e^{-j2\pi(k_x x + k_y y)} dx dy \\
 &= F_{2D}[u(x, y)]
 \end{aligned}$$

$$\begin{aligned}
 U(k_x, k_y) &= G(k, \theta) \\
 k_x &= k \cos \theta \\
 k_y &= k \sin \theta \\
 k &= \sqrt{k_x^2 + k_y^2}
 \end{aligned}$$

$$\begin{aligned}
 G(k, \theta) &= \int_{-\infty}^{\infty} g(l, \theta) e^{-j2\pi k l} dl
 \end{aligned}$$

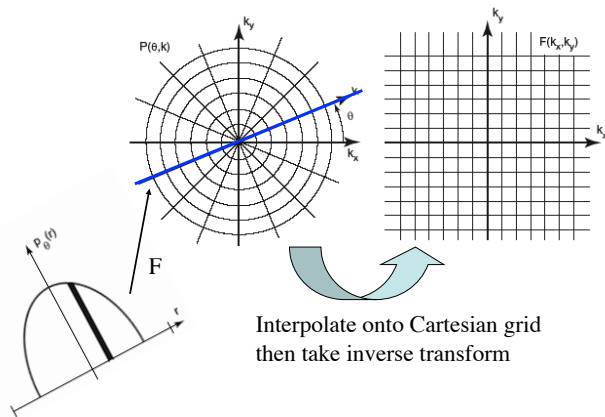
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Projection Slice Theorem

$$\begin{aligned}
 G(\rho, \theta) &= \int_{-\infty}^{\infty} g(l, \theta) e^{-j2\pi \rho l} dl \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - l) e^{-j2\pi \rho l} dx dy dl \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi \rho (x \cos \theta + y \sin \theta)} dx dy \\
 &= F_{2D}[f(x, y)] \Big|_{u=\rho \cos \theta, v=\rho \sin \theta}
 \end{aligned}$$

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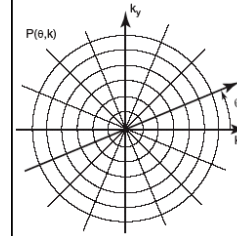
Fourier Reconstruction



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Polar Version of Inverse FT



$$\begin{aligned} \mu(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(k_x, k_y) e^{j2\pi(k_x x + k_y y)} dk_x dk_y \\ &= \int_0^{2\pi} \int_0^{\infty} G(k, \theta) e^{j2\pi(xk \cos\theta + yk \sin\theta)} k dk d\theta \\ &= \int_0^{\pi} \int_{-\infty}^{\infty} G(k, \theta) e^{j2\pi k(x \cos\theta + y \sin\theta)} |k| dk d\theta \end{aligned}$$

Note :

$$g(l, \theta + \pi) = g(-l, \theta)$$

So

$$G(k, \theta + \pi) = G(-k, \theta)$$

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Filtered Backprojection

$$\begin{aligned} \mu(x, y) &= \int_0^{\pi} \int_{-\infty}^{\infty} G(k, \theta) e^{j2\pi(xk \cos\theta + yk \sin\theta)} |k| dk d\theta \\ &= \int_0^{\pi} \int_{-\infty}^{\infty} |k| G(k, \theta) e^{j2\pi kl} dk d\theta \\ &= \int_0^{\pi} g^*(l, \theta) d\theta \quad \leftarrow \text{Backproject a filtered projection} \end{aligned}$$

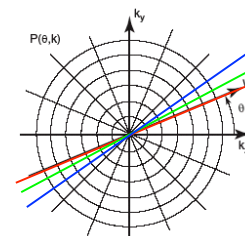
where $l = x \cos\theta + y \sin\theta$

$$\begin{aligned} g^*(l, \theta) &= \int_{-\infty}^{\infty} |k| G(k, \theta) e^{j2\pi kl} dk \\ &= g(l, \theta) * F^{-1}[|k|] \\ &= g(l, \theta) * q(l) \end{aligned}$$

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Fourier Interpretation



$$\text{Density} \approx \frac{N}{\text{circumference}} \approx \frac{N}{2\pi|k|}$$

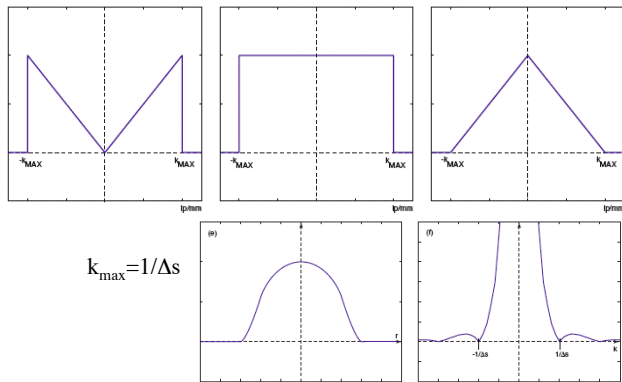
Low frequencies are oversampled. So to compensate for this, multiply the k-space data by $|k|$ before inverse transforming.



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Kak and Slaney; Suetens 2002

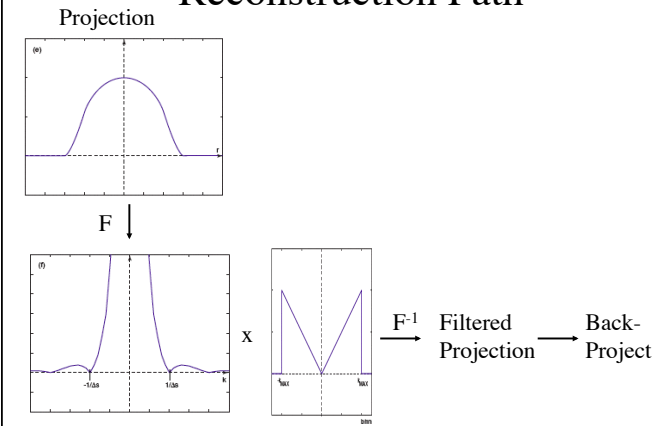
Ram-Lak Filter



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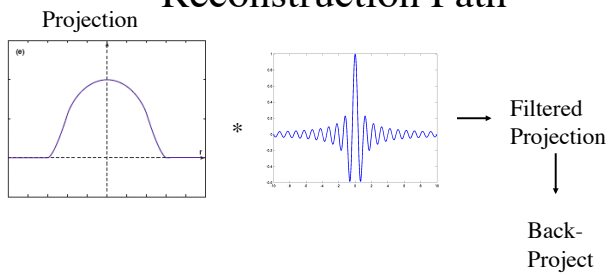
Reconstruction Path



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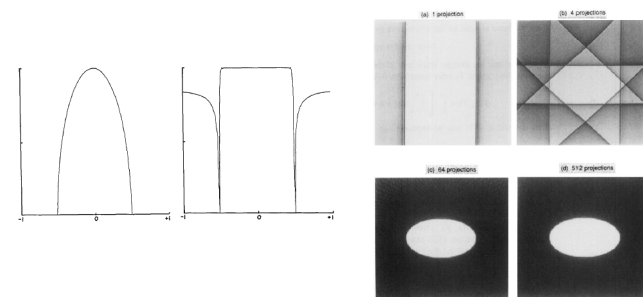
Reconstruction Path



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Example



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Example

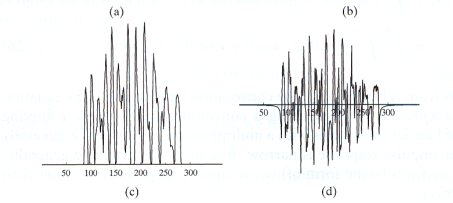
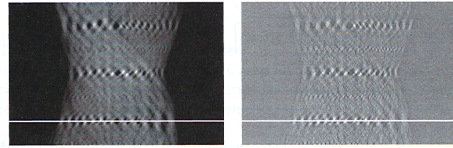


Figure 6.15
Convolution step:
(a) Original sinogram;
(b) filtered sinogram;
(c) profile of sinogram row
[white line in (a)]; and
(d) profile of filtered
sinogram row [white line in
(b)].

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Example

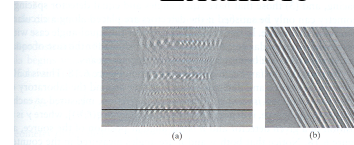


Figure 6.16
Backprojection step.

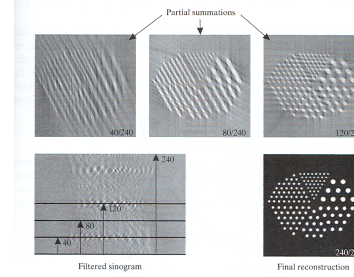


Figure 6.17
Summation step.

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