BENG280A, Principles of Biomedical Imaging 10/22/10
Fall Quarter 2010

**HOMEWORK #4**
**Due in Class on Thursday 10/28/10; Note: there are 2 pages**

## Readings:

Section 2.8 and review Chapter 6 as necessary.

## Problems:

1. Consider an X-ray imaging system with source distribution *s(x,y)* and object transmission function *t(x,y)*. The distance from the source to the detector is d; and the distance from the source to the object is z. Let $s(x,y) = rect(x,y)$ and $t(x,y) = \cos(8\pi x) rect(y)$. Find the relation between $z$ and $d$ that yields an image intensity of zero. **HINT**: Although not necessary, it may be useful to consider the Fourier transform.

2. Let $G(k,\theta)$ be the 1-D Fourier transform of the projection $g(l,\theta)$.
    a) Show that $g(l,\theta+\pi) = g(-l,\theta)$
    b) Next, show that $G(k,\theta+\pi) = G(-k,\theta)$
    c) Use your results to show that equation 6.22 follows from equation 6.21.

3. Problem 2.24
4. Consider the CT k-space filter $G(k) = |k| w(k)$ where $w(k)$ is a windowing function. For each of the following window functions, use MATLAB to plot the k-space filter. Also, derive analytical expressions for the inverse Fourier transforms of the window functions.

    a) The Ram-Lak Filter with $w(k) = rect\left(\frac{k}{2k_{\max}}\right)$ .

    b) A Hanning window defined as $w(k) = rect\left(\frac{k}{2k_{\max}}\right)\left(0.5 + 0.5\cos\left(\frac{\pi k}{k_{\max}}\right)\right)$ .

    c) Use MATLAB to plot out and compare the inverse transforms with the expressions you derived in parts (a) and (b). Comment on the relative advantages and disadvantages of the two filters for CT reconstruction.

5. A parallel beam CT imaging system is used to image an object defined as:
$$f(x,y) = rect(x,y) + \left(rect(x,y) ** \left[\left(\delta(x-2) + \delta(x+2)\right)\delta(y)\right] ** \left[\left(\delta(y-4) + \delta(y+4)\right)\delta(x)\right]\right)$$
    a) Sketch the object and draw the projections of the object at 0 degrees and 45 degrees.
    b) Derive the Fourier transform of the object
    c) Show that the Projection-slice theorem holds for the projections at 0 and 45 degrees.

6. (20 pts) Consider the object $f(x,y) = \cos\left(\frac{1}{\sqrt{3}}\pi x + 2\pi y\right)$
    a) Sketch the object by hand, labeling critical points, such as the zero-crossings and the distances between peaks. Also use MATLAB to make an image of the object and compare the MATLAB result to your sketch.
    b) Consider sampling the object in both the x and y directions with sample intervals of $\Delta_x$ and $\Delta_y$, respectively. Indicate what sample intervals should be used to avoid aliasing.
    c) Now consider imaging the object with a parallel beam CT imaging system. At what angle will the projection be non-zero?

d) We now wish to sample the non-zero projection. What sampling interval should we use to avoid aliasing?

e) Now consider the object $g(x,y) = \left(f(x,y)\right)^2$. Answer items (c) and (d) for this object

## Matlab exercise on convolution and Fourier transform

Assume you have functions A and B, A has 512 sampling points and B has 64. Now you need to do some experiments to see if the convolution in time domain is equivalent to multiplication in the frequency domain.

```
L1 = 512;
L2 = 64;
```

**Consider 3 different cases.**

```
1:
A = [zeros(1,L1/4),hamming(L1/2)',zeros(1,L1/4)];
B = [zeros(1,L2/4),ones(1,L2/2),zeros(1,L2/4)];

2:
A = [zeros(1,L1/4),cos((1:L1/2)/10),zeros(1,L1/4)];
B = [zeros(1,L2/4),ones(1,L2/2),zeros(1,L2/4)];

3:
A = [zeros(1,L1/4),cos((1:L1/2)/10),zeros(1,L1/4)];
B = [zeros(1,L2/4),cos((1:L2/2)/5),zeros(1,L2/4)];
```

**For each case, apply the following steps.**

**Step 1:**

`Let C=A*B;  where * denotes convolution.`

Do convolution in time domain, see what you have for C, you may use matlab function **conv**;

**Step 2:**

Now let's do it using Fourier transform.

Compute the Fourier transforms of A, B, and C. Denote these as Af, Bf and Cf. Remember to use **fftshift** and **abs** before you plot the Fourier transforms.

In order to do the multiplication later correctly, you need to zero-pad B on both sides instead of using B directly, e.g. B1=[zeros(1,10), B, zeros(1,10)], so after **fft**, Bf will have the same size as Af.

**Step 3:**

Now multiply Af and Bf (use the element-by-element mutiply), and inverse-Fourier transform the product back to the time domain to get your estimate C1. Compare your results with C. **ifftshift** could be used before you show the results. And since C has more than 512 points, while C1 has only 512 points, it will be easier to compare them if you truncate the edges of C.

Note: You may use **subplot** to show multiple results in a same window to compare the results. Use **help** to figure out how to use Matlab functions correctly.