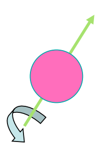
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2010
MRI Lecture 1

Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

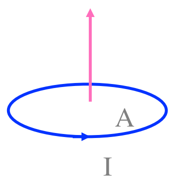
This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where γ is the gyromagnetic ratio and **S** is the spin angular momentum.

Nuclear Spin Rules

Number of Protons	Number of Neutrons	Spin	Examples
Even	Even	0	^{12}C, ^{16}O
Even	Odd	j/2	^{17}O
Odd	Even	j/2	^{1}H, ^{23}Na, ^{31}P
Odd	Odd	j	^{2}H

Classical Magnetic Moment



$$\vec{\mu} = IA\hat{n}$$

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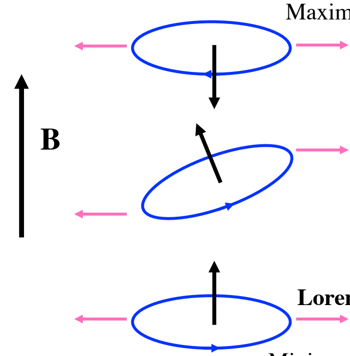
Energy in a Magnetic Field



$$E = -\vec{\mu} \bullet \mathbf{B} = -\mu_z B$$

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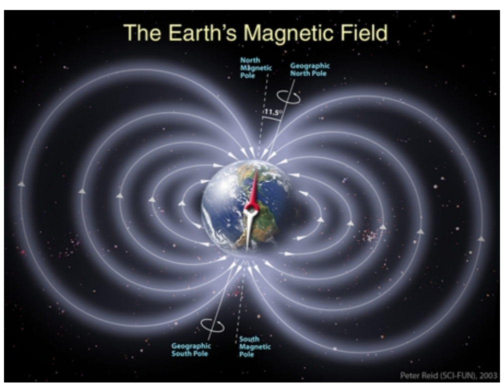
Energy in a Magnetic Field



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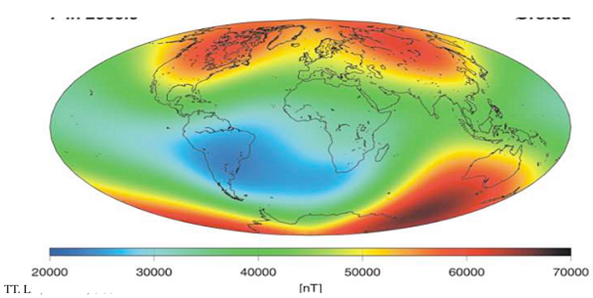
Magnetic Field Units

1 Tesla = 10,000 Gauss

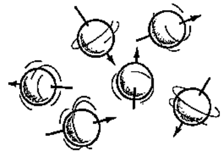
Earth's field is about 0.5 Gauss

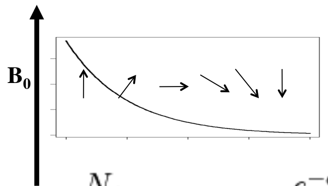
0.5 Gauss = 0.5×10^{-4} T = 50 μT



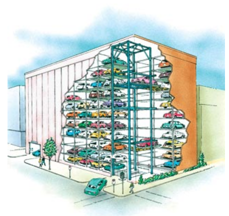
TT. L

Boltzmann Distribution

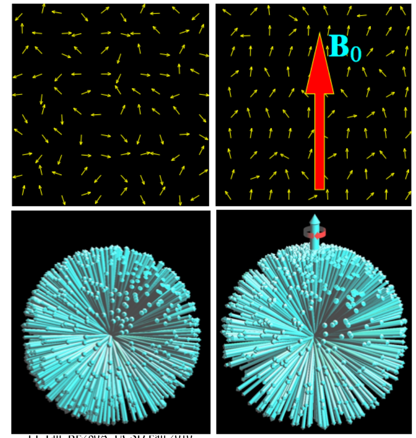




$$\frac{N_j}{N} \equiv \mathcal{P}(\epsilon_j) = \frac{e^{-\epsilon_j/(kT)}}{Q}.$$



Equilibrium Magnetization



$$\mathbf{M}_0 = N\langle \mu_z \rangle$$
$$\approx N\mu_z^2 B/(kT)$$
$$= N\gamma^2\hbar^2 B/(4kT)$$

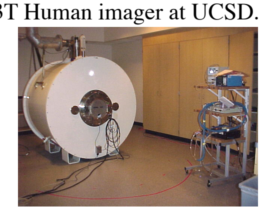
N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

Hansen 2009

Bigger is better

3T Human imager at UCSD.

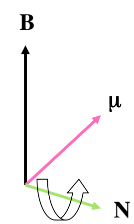
7T Human imager at U. Minn.



7T Rodent Imager at UCSD

9.4T Human imager at UIC

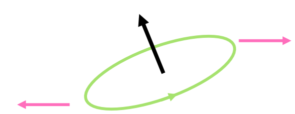
Torque



N = μ x B

Torque

For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)



Precession

Torque

$\mathbf{N} = \mu \times \mathbf{B}$

$\frac{d\mathbf{S}}{dt} = \mathbf{N}$

Change in Angular momentum

$\frac{d\mathbf{S}}{dt} = \mu \times \mathbf{B}$

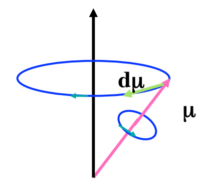
$\mu = \gamma\,\mathbf{S}$

Relation between magnetic moment and angular momentum

$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$

Precession

$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$

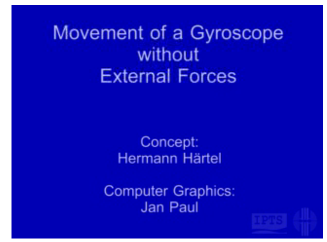


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$\omega = \gamma\,B$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertelmpg_e/gyros_free.mpg

Magnetization Vector

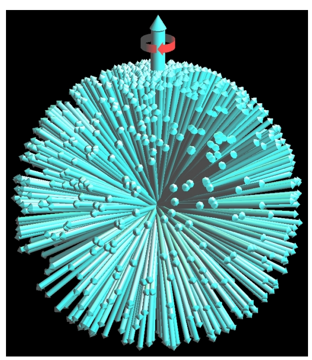
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

$$\mathbf{M} = \frac{1}{V}\sum_{\substack{\text{protons}\\ \text{in V}}} \mu_i$$

$$\frac{d\mathbf{M}}{dt} = \gamma\mathbf{M} \times \mathbf{B}$$



Hansen 2009

Gyromagnetic Ratios

Nucleus	Spin	Magnetic Moment	$\gamma/(2\pi)$ (MHz/Tesla)	Abundance
^{1}H	1/2	2.793	42.58	88 M
^{23}Na	3/2	2.216	11.27	80 mM
^{31}P	1/2	1.131	17.25	75 mM

Source: Haacke et al., p. 27

Larmor Frequency

$\omega = \gamma B$	Angular frequency in rad/sec
$f = \gamma B / (2\pi)$	Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Notation and Units

1 Tesla = 10,000 Gauss

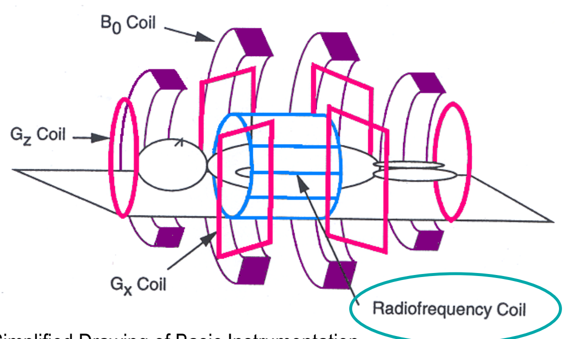
Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

$\gamma = 26752$ radians/second/Gauss

$\gamma\!\!\!{}^{-} = \gamma / 2\pi = 4258$ Hz/Gauss

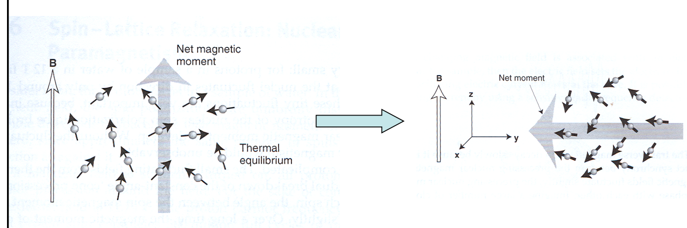
$= 42.58$ MHz/Tesla

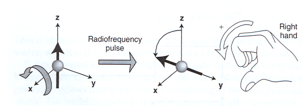


Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

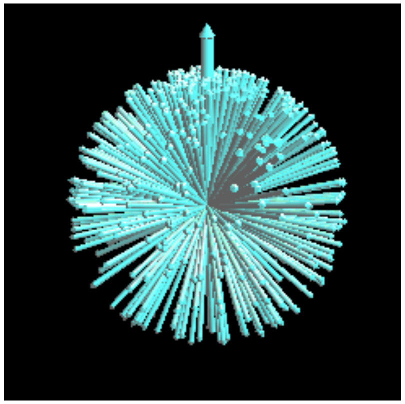
RF Excitation





From Levitt, Spin Dynamics, 2001

RF Excitation



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http://www.drcmr.dk/main/content/view/213/74/

RF Excitation

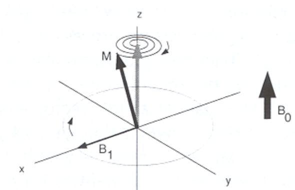

Image & caption: Nishimura, Fig. 3.2

At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

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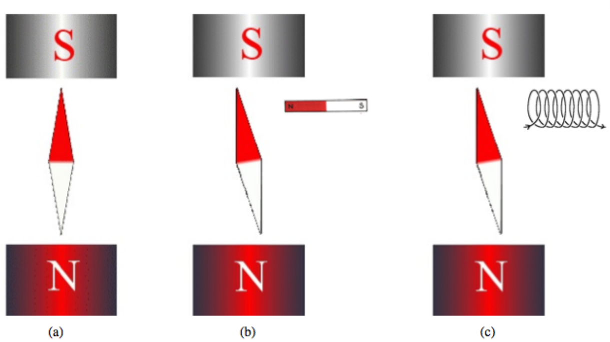
http://www.eecs.umich.edu/%7EdnollBME516/

On-Resonance Excitation



Hanson 2009

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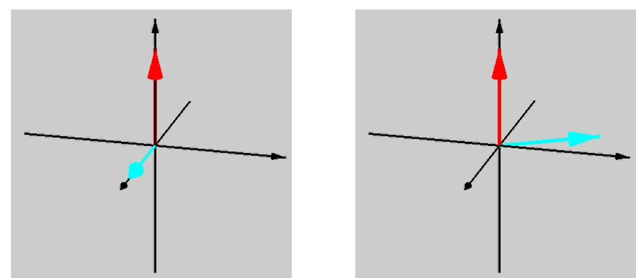
http://www.drcmr.dk/JavaCompass/

RF Excitation



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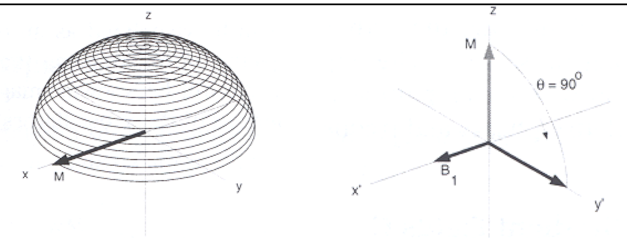
http://www.eecs.umich.edu/%7EdnollBME516/

Rotating Frame of Reference

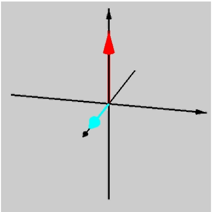
Reference everything to the magnetic field at isocenter.



a) Laboratory frame behavior of **M**

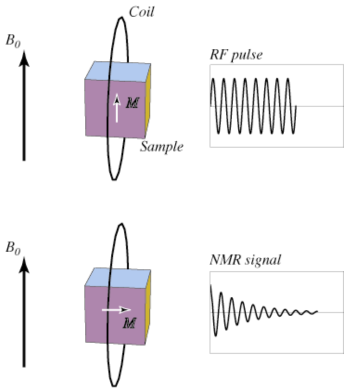
Images & caption: Nishimura, Fig. 3.3

b) Rotating frame behavior of **M**



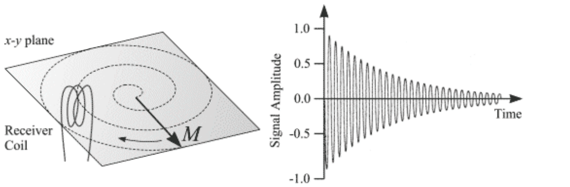
http://www.eecs.umich.edu/%7Ednoll/BME516/

RF Excitation



From Buxton 2002

Free Induction Decay (FID)



http://www.easymeasure.co.uk/principlesmri.aspx