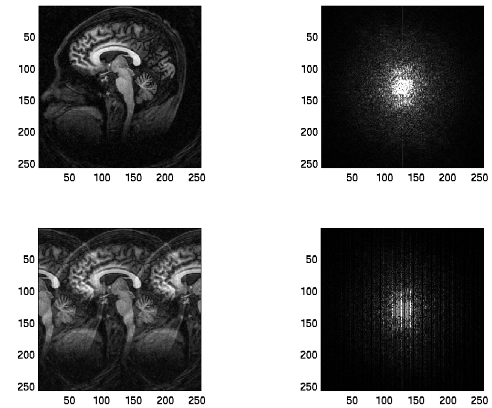
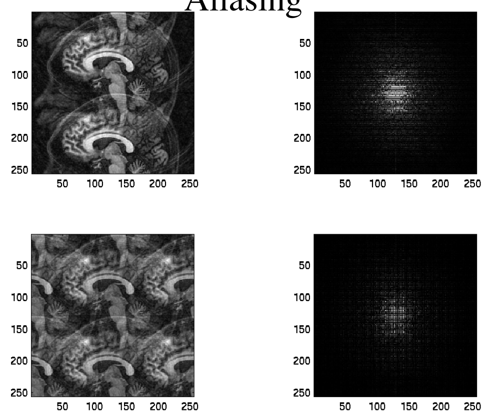
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2010
MRI Lecture 3

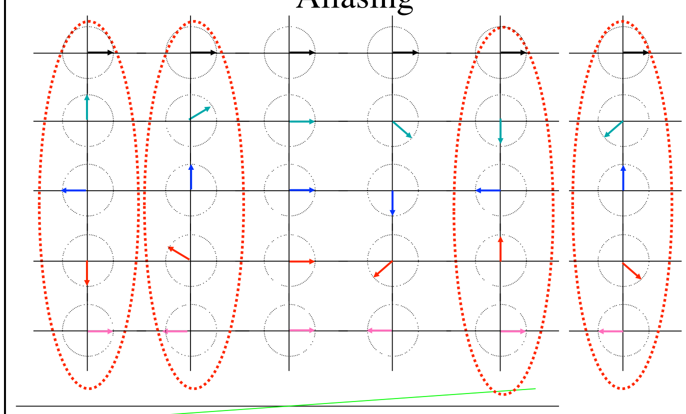
Sampling in k-space



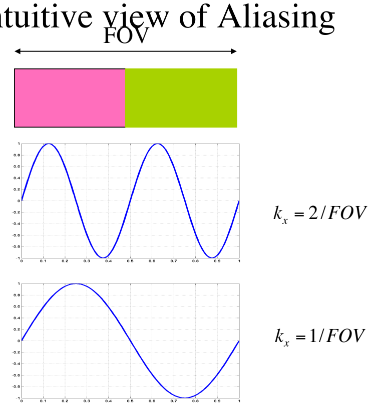
Aliasing



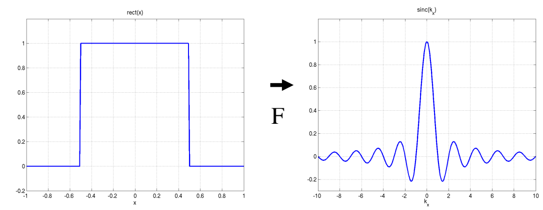
Aliasing



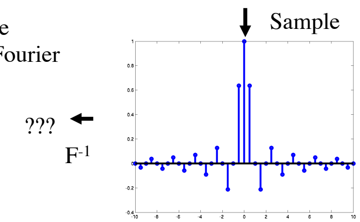
Intuitive view of Aliasing



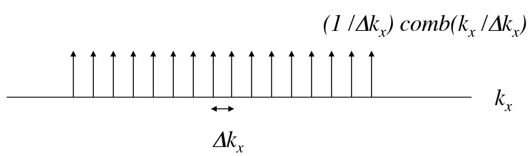
Fourier Sampling



Instead of sampling the signal, we sample its Fourier Transform



Fourier Sampling

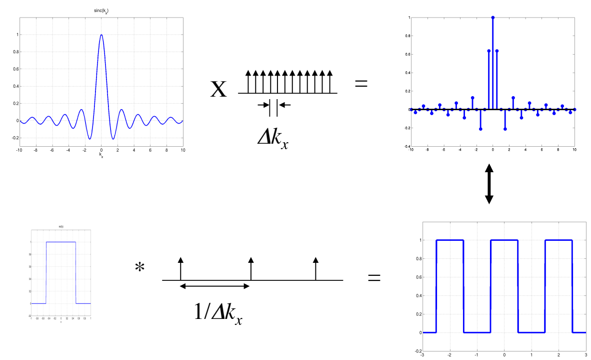


$$G_S(k_x) = G(k_x)\frac{1}{\Delta k_x} comb\left(\frac{k_x}{\Delta k_x}\right)$$

$$= G(k_x)\sum_{n=-\infty}^{\infty}\delta(k_x - n\Delta k_x)$$

$$= \sum_{n=-\infty}^{\infty} G(n\Delta k_x)\delta(k_x - n\Delta k_x)$$

Fourier Sampling -- Inverse Transform



Fourier Sampling -- Inverse Transform

$$g_S(x) = F^{-1}[G_S(k_x)]$$

$$= F^{-1}\left[G(k_x)\frac{1}{\Delta k_x} comb\left(\frac{k_x}{\Delta k_x}\right)\right]$$

$$= F^{-1}[G(k_x)] * F^{-1}\left[\frac{1}{\Delta k_x} comb\left(\frac{k_x}{\Delta k_x}\right)\right]$$

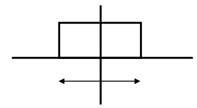
$$= g(x) * comb(x\Delta k_x)$$

$$= g(x) * \sum_{n=-\infty}^{\infty}\delta(x\Delta k_x - n)$$

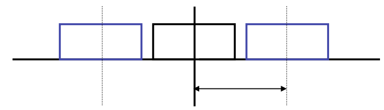
$$= g(x) * \frac{1}{\Delta k_x}\sum_{n=-\infty}^{\infty}\delta\left(x - \frac{n}{\Delta k_x}\right)$$

$$= \frac{1}{\Delta k_x}\sum_{n=-\infty}^{\infty} g\left(x - \frac{n}{\Delta k_x}\right)$$

Nyquist Condition



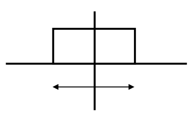
FOV (Field of View)



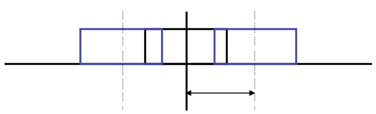
$1/\Delta k_x$

To avoid overlap, $1/\Delta k_x >$ FOV, or equivalently, $\Delta k_x < 1/\text{FOV}$

Aliasing



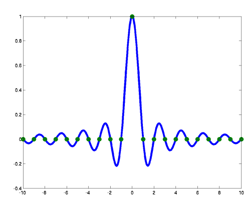
FOV (Field of View)

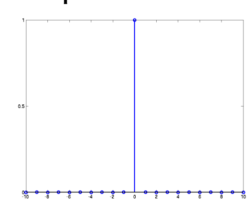


$1/\Delta k_x$

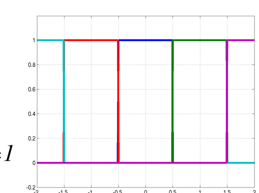
Aliasing occurs when $1/\Delta k_x <$ FOV

Aliasing Example





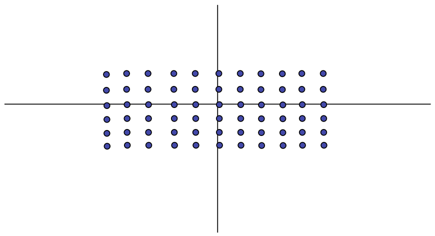
$\Delta k_x = 1$



$1/\Delta k_x = 1$

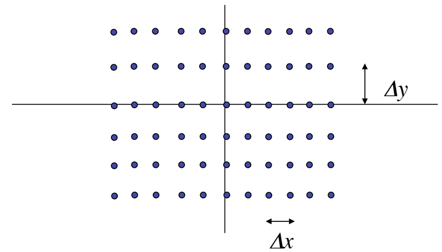
2D Comb Function

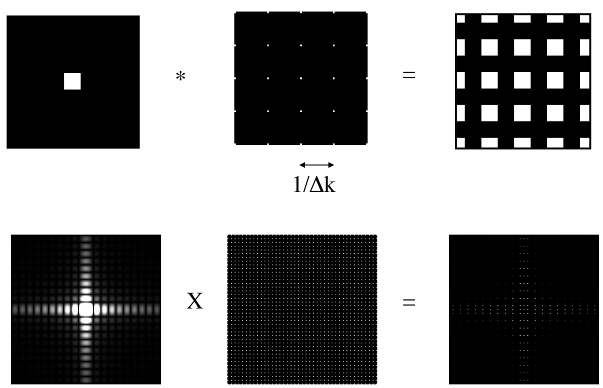
$$comb(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m, y-n)$$
$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m)\delta(y-n)$$
$$= comb(x)comb(y)$$



Scaled 2D Comb Function

$$comb(x/\Delta x, y/\Delta y) = comb(x/\Delta x)comb(y/\Delta y)$$
$$= \Delta x \Delta y \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m\Delta x)\delta(y-n\Delta y)$$





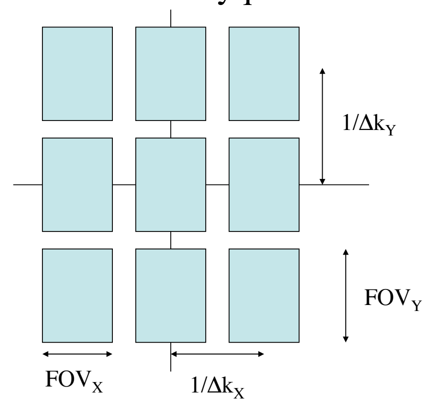
2D k-space sampling

$$G_S(k_x,k_y) = G(k_x,k_y)\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)$$
$$= G(k_x,k_y)\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(k_x - m\Delta k_x, k_y - n\Delta k_y)$$
$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}G(m\Delta k_x, n\Delta k_y)\delta(k_x - m\Delta k_x, k_y - n\Delta k_y)$$

2D k-space sampling

$$
\begin{aligned}
g_S(x,y) &= F^{-1}\left[G_S(k_x,k_y)\right] \\
&= F^{-1}\left[G(k_x,k_y)\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right)\right] \\
&= F^{-1}\left[G(k_x,k_y)\right] * F^{-1}\left[\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right)\right] \\
&= g(x,y) ** comb\left(x\Delta k_x\right) comb\left(y\Delta k_y\right) \\
&= g(x) ** \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x\Delta k_x - m)\delta(y\Delta k_y - n) \\
&= g(x) ** \frac{1}{\Delta k_x \Delta k_y}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-\frac{m}{\Delta k_x})\delta(y-\frac{n}{\Delta k_y}) \\
&= \frac{1}{\Delta k_x \Delta k_y}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g(x-\frac{m}{\Delta k_x}, y-\frac{n}{\Delta k_y})
\end{aligned}
$$

Nyquist Conditions



$1/\Delta k_Y > FOV_Y$

$1/\Delta k_X > FOV_X$

Windowing

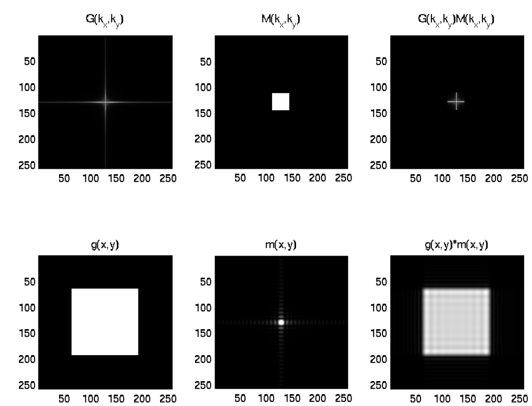
Windowing the data in Fourier space

$$G_W\left(k_x,k_y\right) = G\left(k_x,k_y\right)W\left(k_x,k_y\right)$$

Results in convolution of the object with the inverse transform of the window

$$g_W\left(x,y\right) = g\left(x,y\right) * w(x,y)$$

Resolution



Windowing Example

$$W\left(k_x,k_y\right)=rect\left(\frac{k_x}{W_{k_x}}\right)rect\left(\frac{k_y}{W_{k_y}}\right)$$

$$w\left(x,y\right)=F^{-1}\left[rect\left(\frac{k_x}{W_{k_x}}\right)rect\left(\frac{k_y}{W_{k_y}}\right)\right]$$

$$=W_{k_x}W_{k_y}\,\mathrm{sinc}\left(W_{k_x}x\right)\mathrm{sinc}\left(W_{k_y}y\right)$$

$$g_W\left(x,y\right)=g(x,y)**W_{k_x}W_{k_y}\,\mathrm{sinc}\left(W_{k_x}x\right)\mathrm{sinc}\left(W_{k_y}y\right)$$

Effective Width

$$w_E=\frac{1}{w(0)}\int_{-\infty}^{\infty}w(x)dx$$

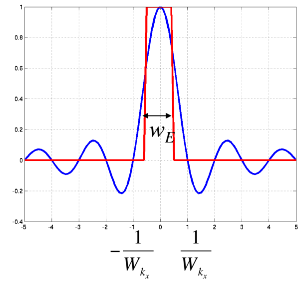
Example

$$w_E=\frac{1}{1}\int_{-\infty}^{\infty}\mathrm{sinc}(W_{k_x}x)dx$$

$$=F\left[\mathrm{sinc}(W_{k_x}x)\right]\Big|_{k_x=0}$$

$$=\frac{1}{W_{k_x}}rect\left(\frac{k_x}{W_{k_x}}\right)\Bigg|_{k_x=0}$$

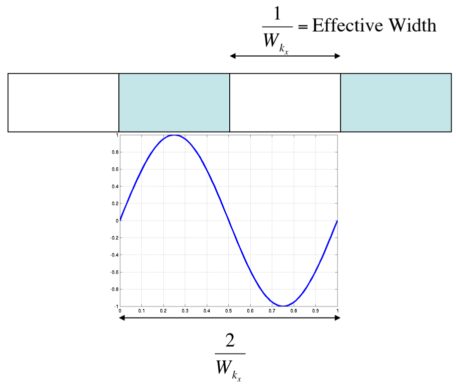
$$=\frac{1}{W_{k_x}}$$



Resolution and spatial frequency

With a window of width W_{k_x} the highest spatial frequency is $W_{k_x}/2$.

This corresponds to a spatial period of $2/W_{k_x}$.



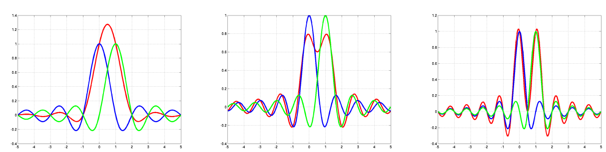
Windowing Example

$$g(x,y)=\left[\delta(x)+\delta(x-1)\right]\delta(y)$$

$$g_W\left(x,y\right)=\left[\delta(x)+\delta(x-1)\right]\delta(y)**W_{k_x}W_{k_y}\,\mathrm{sinc}\left(W_{k_x}x\right)\mathrm{sinc}\left(W_{k_y}y\right)$$

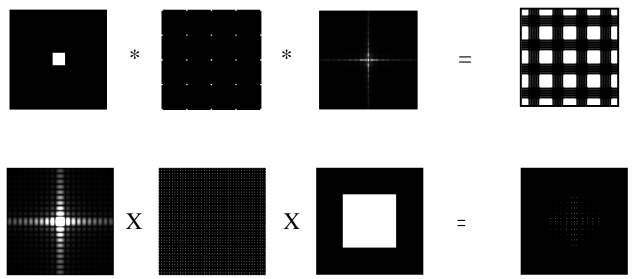
$$=W_{k_x}W_{k_y}\left(\left[\delta(x)+\delta(x-1)\right]*\mathrm{sinc}\left(W_{k_x}x\right)\right)\mathrm{sinc}\left(W_{k_y}y\right)$$

$$=W_{k_x}W_{k_y}\left(\mathrm{sinc}\left(W_{k_x}x\right)+\mathrm{sinc}\left(W_{k_x}(x-1)\right)\right)\mathrm{sinc}\left(W_{k_y}y\right)$$



$W_{k_x}=1$ $\quad$ $W_{k_x}=1.5$ $\quad$ $W_{k_x}=2$

Sampling and Windowing



Sampling and Windowing

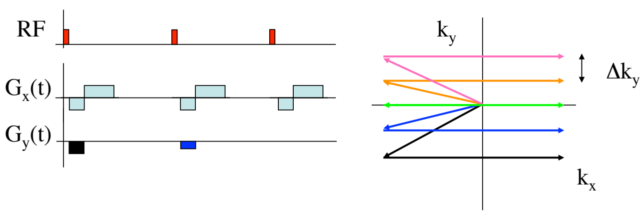
Sampling and windowing the data in Fourier space

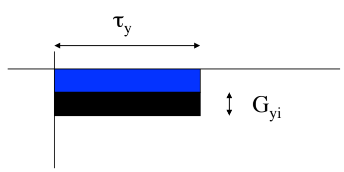
$$G_{SW}(k_x,k_y) = G(k_x,k_y)\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right) rect\left(\frac{k_x}{W_{k_x}},\frac{k_y}{W_{k_y}}\right)$$

Results in replication and convolution in object space.

$$g_{SW}(x,y) = W_{k_x} W_{ky} g(x,y) ** comb(\Delta k_x x, \Delta k_y y) ** \mathrm{sinc}(W_{k_x} x)\,\mathrm{sinc}(W_{k_y} y)$$

Sampling in k_y

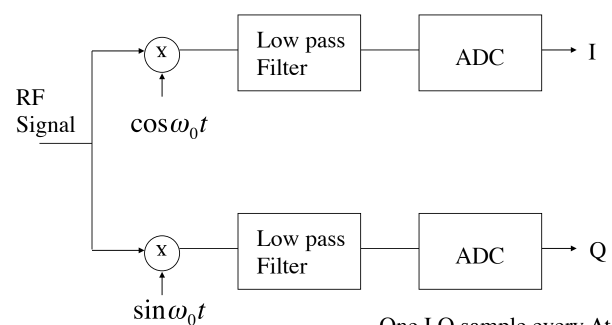




$$\Delta k_y = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

$$FOV_y = \frac{1}{\Delta k_y}$$

Sampling in k_x

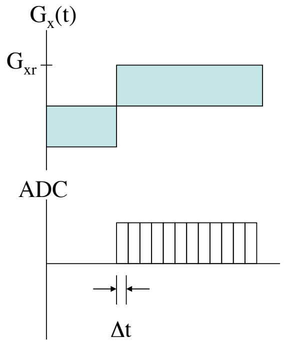


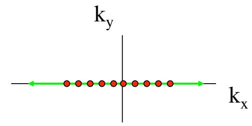
One I,Q sample every Δt

M= I+jQ

Note: In practice, there are number of ways of implementing this processing.

Sampling in k_x



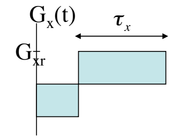


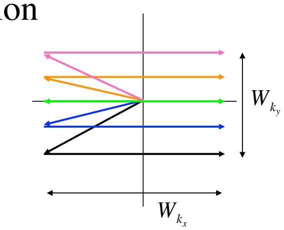
$$\Delta k_x = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

$$FOV_x = \frac{1}{\Delta k_x}$$

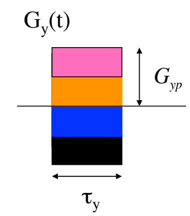
Resolution

$$\delta_x = \frac{1}{W_{k_x}} = \frac{1}{2k_{x,\max}} = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$





$$\delta_y = \frac{1}{W_{k_y}} = \frac{1}{2k_{y,\max}} = \frac{1}{\frac{\gamma}{2\pi} 2G_{yp} \tau_y}$$



Example

Goal :

$$FOV_x = FOV_y = 25.6 \text{ cm}$$

$$\delta_x = \delta_y = 0.1 \text{ cm}$$

Readout Gradient :

$$FOV_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t}$$

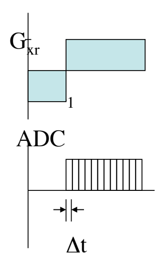
Pick $\Delta t = 32\ \mu\text{sec}$

$$G_{xr} = \frac{1}{FOV_x \frac{\gamma}{2\pi} \Delta t} = \frac{1}{(25.6cm)(42.57 \times 10^6 T^{-1} s^{-1})(32 \times 10^{-6} s)}$$

$$= 2.8675 \times 10^{-5} \text{T/cm}$$

$$= .28675 \text{ G/cm}$$

1 Gauss = 1×10^{-4} Tesla



Example

Readout Gradient :

$$\delta_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$

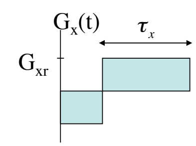
$$\tau_x = \frac{1}{\delta_x \frac{\gamma}{2\pi} G_{xr}} = \frac{1}{(0.1cm)(4257\ G^{-1} s^{-1})(0.28675 \text{ G/cm})}$$

$$= 8.192 \text{ ms}$$

$$= N_{read} \Delta t$$

where

$$N_{read} = \frac{FOV_x}{\delta_x} = 256$$



Example

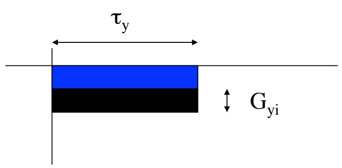
Phase - Encode Gradient :

$$FOV_y = \frac{1}{\frac{\gamma}{2\pi} G_{yi} \tau_y}$$

Pick $\tau_y = 4.096\ msec$

$$G_{yi} = \frac{1}{FOV_y \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(25.6cm)(42.57 \times 10^6 T^{-1} s^{-1})(4.096 \times 10^{-3} s)}$$

$$= 2.2402 \times 10^{-7} \text{ T/cm}$$
$$= .00224 \text{ G/cm}$$



Example

Phase - Encode Gradient :

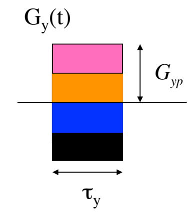
$$\delta_y = \frac{1}{\frac{\gamma}{2\pi} 2 G_{yp} \tau_y}$$

$$G_{yp} = \frac{1}{\delta_y 2 \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(0.1cm)(4257\ G^{-1} s^{-1})(4.096 \times 10^{-3} s)}$$

$$= 0.2868 \text{ G/cm}$$
$$= \frac{N_p}{2} G_{yi}$$

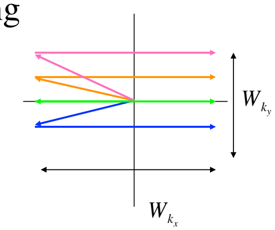
where

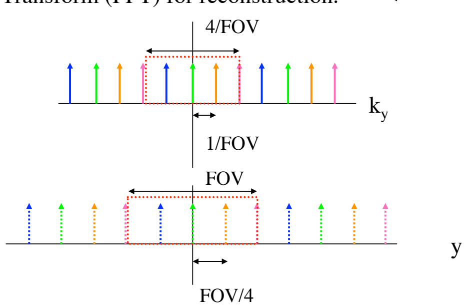
$$N_p = \frac{FOV_y}{\delta_y} = 256$$



Sampling

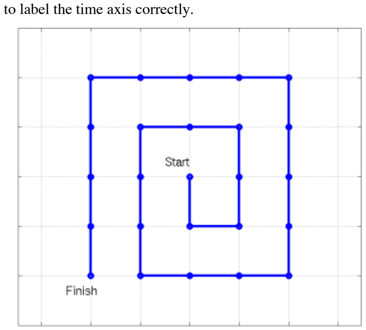
In practice, an even number (typically power of 2) sample is usually taken in each direction to take advantage of the Fast Fourier Transform (FFT) for reconstruction.





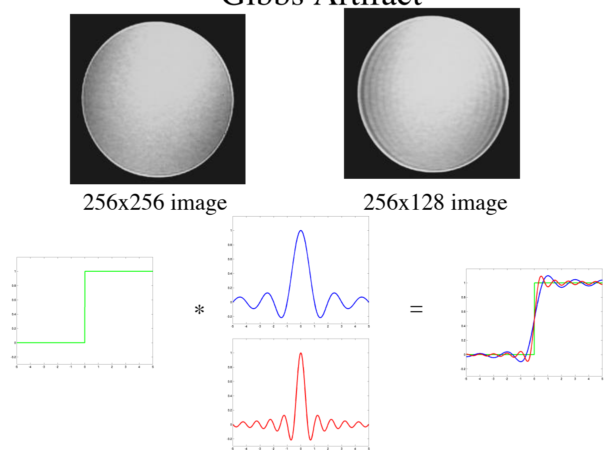
Example

Consider the k-space trajectory shown below. ADC samples are acquired at the points shown with $\Delta t = 10\ \mu sec$. The desired FOV (both x and y) is 10 cm and the desired resolution (both x and y) is 2.5 cm. Draw the gradient waveforms required to achieve the k-space trajectory. Label the waveform with the gradient amplitudes required to achieve the desired FOV and resolution. Also, make sure to label the time axis correctly.



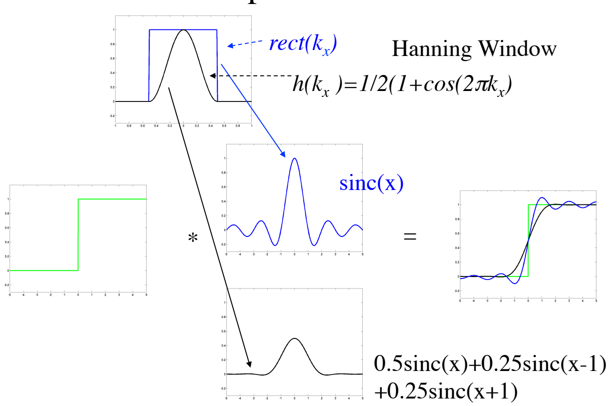
GE Medical Systems 2003

Gibbs Artifact



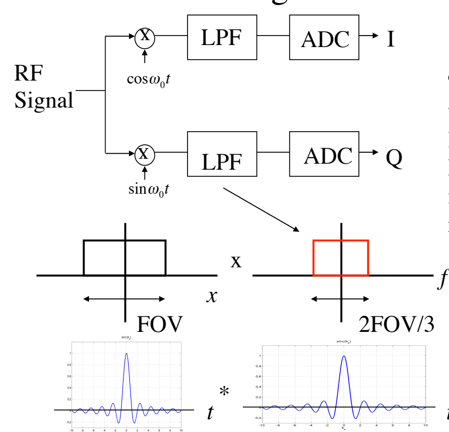
Images from http://www.mritutor.org/mritutor/gibbs.htm

Apodization



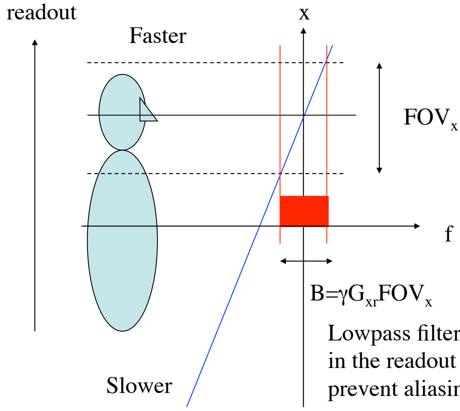
Images from http://www.mritutor.org/mritutor/gibbs.htm

Aliasing and Bandwidth



Temporal filtering in the readout direction limits the readout FOV. So there should never be aliasing in the readout direction.

Aliasing and Bandwidth



Lowpass filter
in the readout direction to
prevent aliasing.

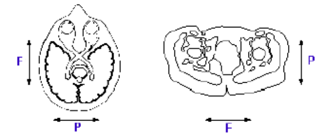


Figure 7-31 Default Axial Directions

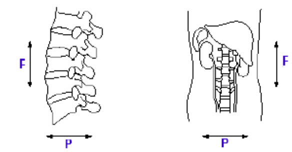


Figure 7-32 Default Sagittal and Coronal Directions

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