

Forward and Inverse Problem of EEG

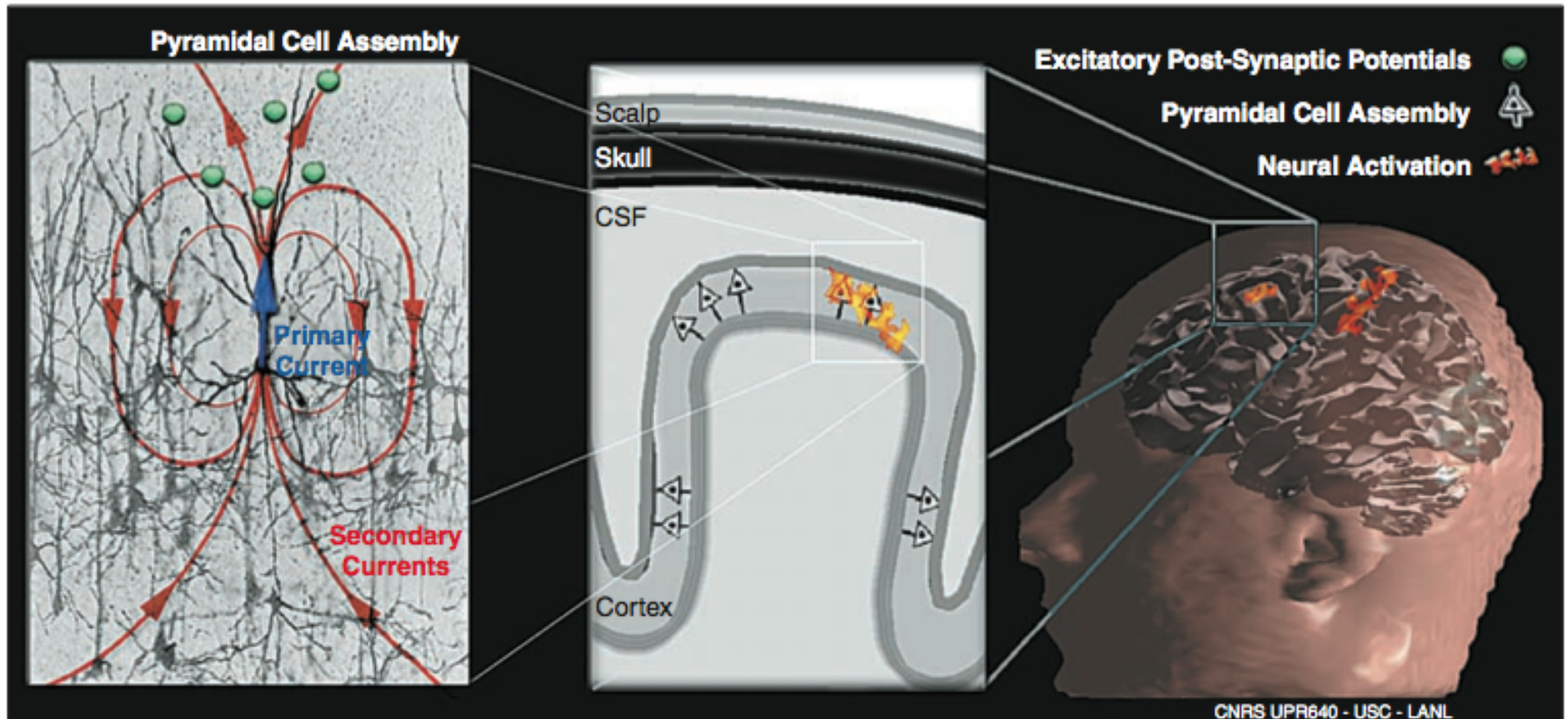
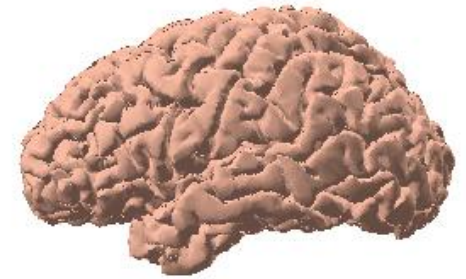
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November, 2012

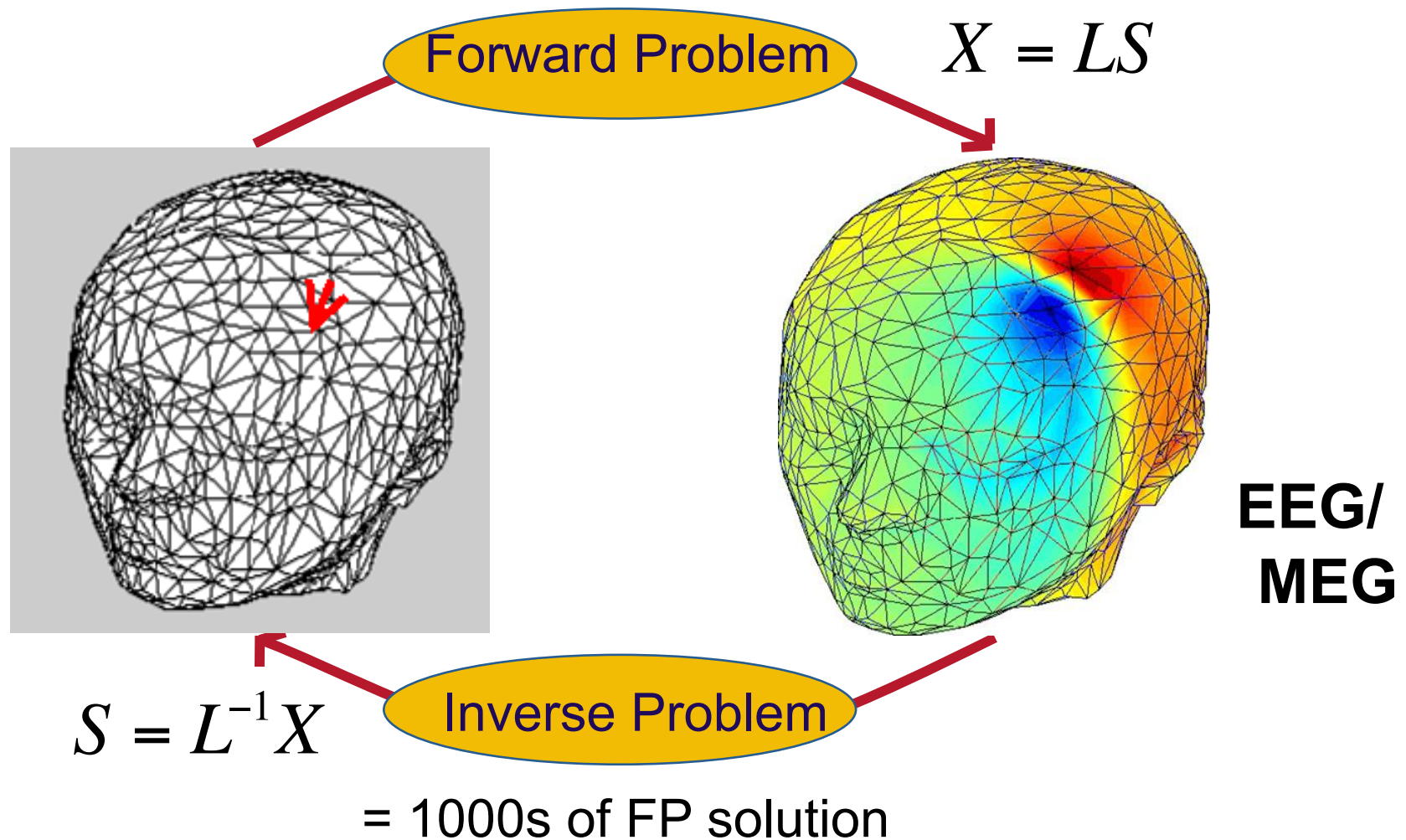
Generators of EEG

Cortical surface



Baillet et al, 2001

Forward and inverse problem



Source localization is ill-posed

$$X = LS + n$$

X: scalp recorded potentials

S: current density vector

L: transfer matrix 'the head volume conductor model'

The inverse problem refers to finding S given known X.

$$O(S) = \min \|X - LS\|^2 \quad \text{Infinite solutions!}$$

Apply electrophysiological neuroanatomical constraints

1. The electrical head model used,
2. The inverse solution itself

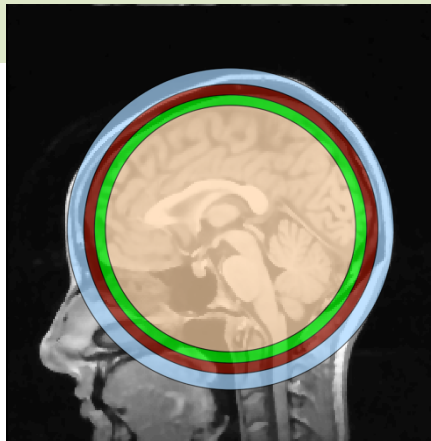
Head volume conductor model

Simple Head Models

- ◆ Single layer sphere, spheroid
- ◆ 3-4 layer sphere

ANALYTICAL SOLVER

Simple, fast, but not accurate

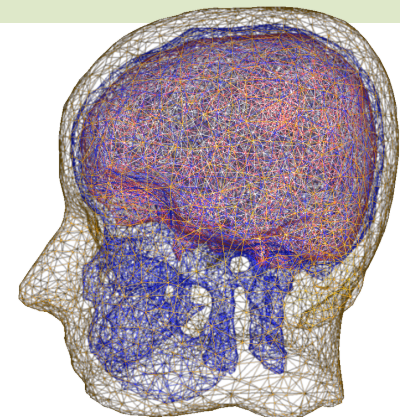


Realistic Head Models

- ◆ Boundary Element (BEM)
- ◆ Finite Element (FEM)
- ◆ Finite Difference (FDM)

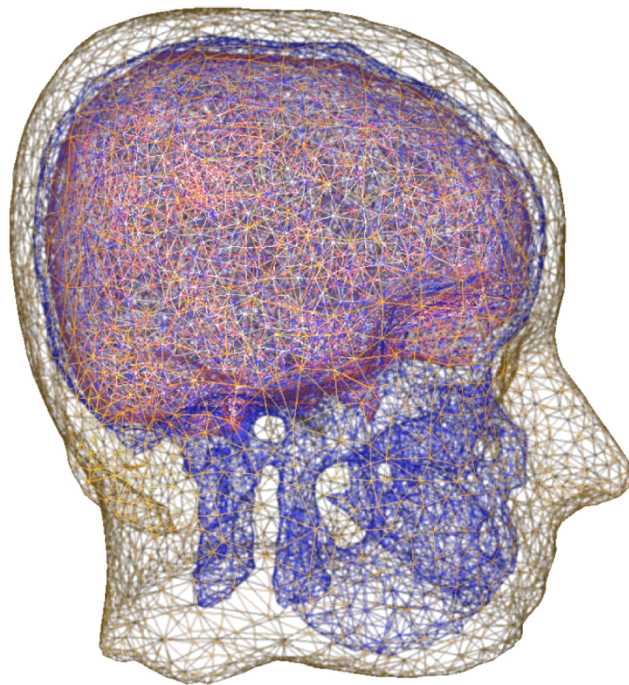
NUMERICAL SOLVER

Represents head shape better, but computationally complex



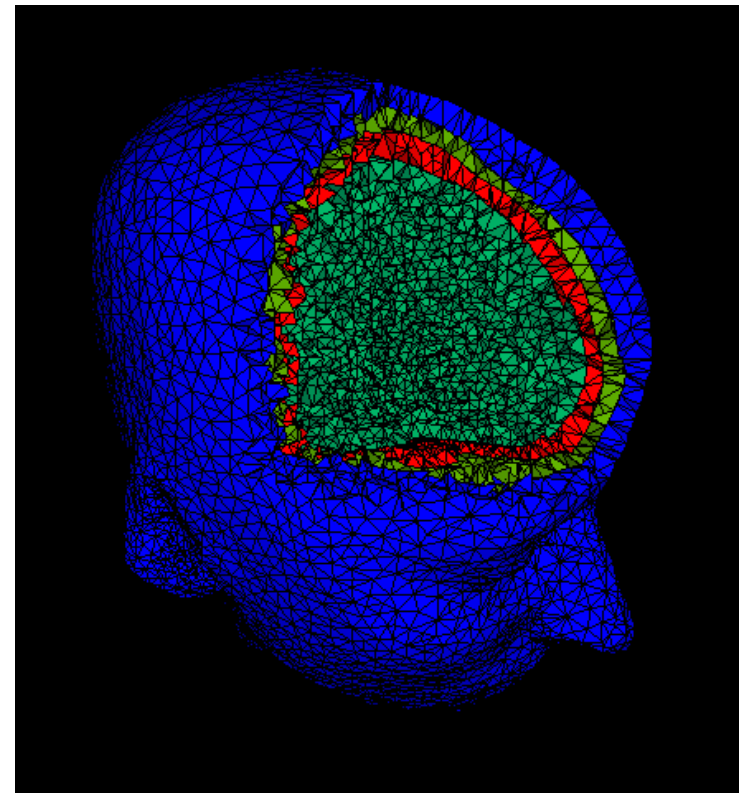
Numerical Head Models

BEM



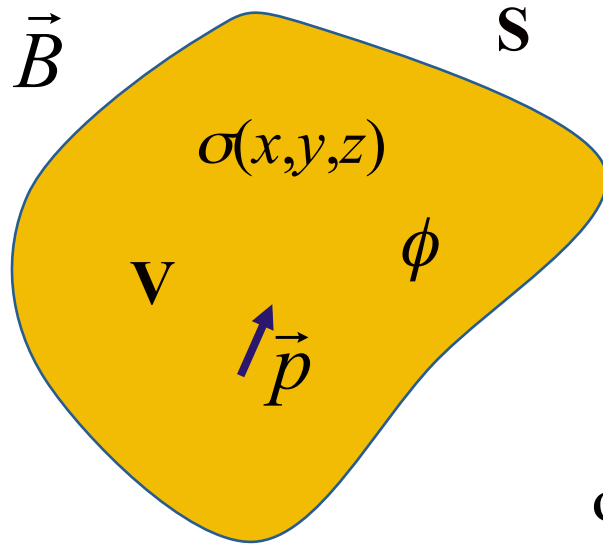
NFT BEM mesh

FEM



Generated using Tetgen
from NFT BEM mesh

Formulation of the FP



$$\nabla \cdot (\sigma \nabla \Phi) = -\nabla \cdot J^P \quad \text{inside } V$$

$$\sigma \frac{\partial \Phi}{\partial n} = 0 \quad \text{on } S$$

$\sigma(x,y,z)$: conductivity distribution

\vec{p} : current source

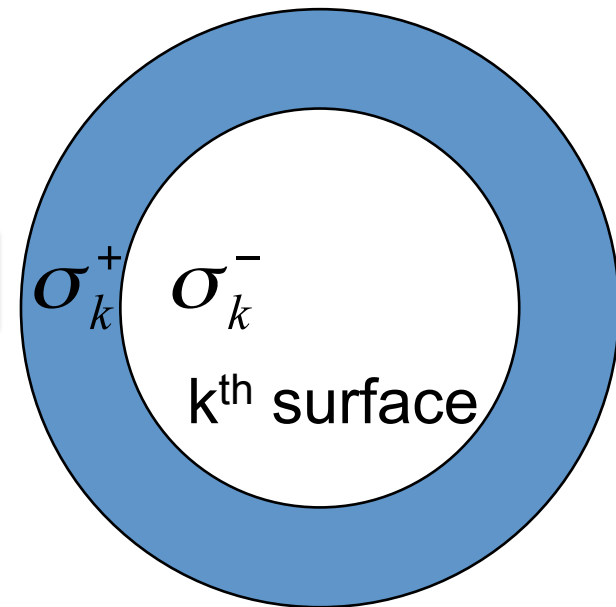
BEM Formulation

Integral equation for Potential Field:

$$\phi(\vec{r}) = 2g(\vec{r}) + \frac{1}{2\pi} \sum_{k=1}^n \left(\frac{\sigma_k^- - \sigma_k^+}{\sigma_i^- + \sigma_i^+} \right) \int_{S_k} \phi(\vec{r}') \frac{\vec{R}}{R^3} \cdot d\vec{S}_k(\vec{r}')$$

Primary sources

Secondary sources



BEM Formulation

Integrating the previous integral equation over all elements a set of equations are obtained.

In matrix notation **for the potential field** we obtain

$$\Phi_{M \times 1} = C_{M \times M} \Phi + g_{M \times 1} \quad \Phi = [I - C]^{-1} g \quad \Phi = \mathbf{A}^{-1} g$$

M : number of nodes

The expression **for the magnetic field**:

$$B_{n \times 1} = B_0 + \mathbf{H}_{n \times M} \Phi$$

n : number of magnetic sensors

Transfer matrix

Electrode potentials

$$\Phi_e = D\mathbf{A}^{-1}g$$

Φ_e $m \times 1$ vector of electrode potentials

D is an $m \times M$ sparse matrix to select m rows of \mathbf{A}^{-1}

Let the transfer matrix E be defined as:

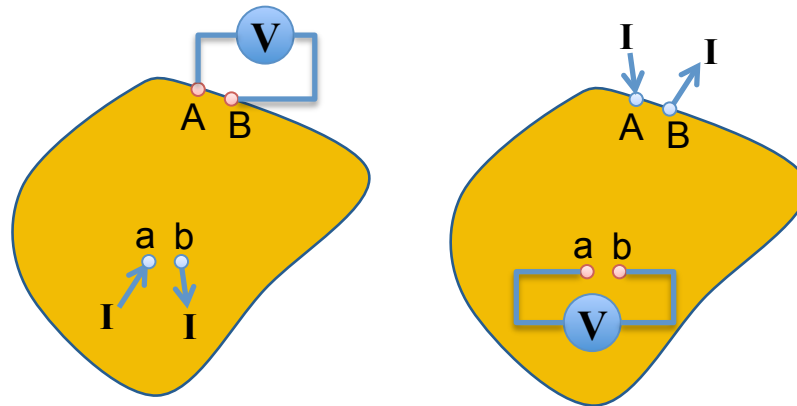
$$E = D\mathbf{A}^{-1}$$

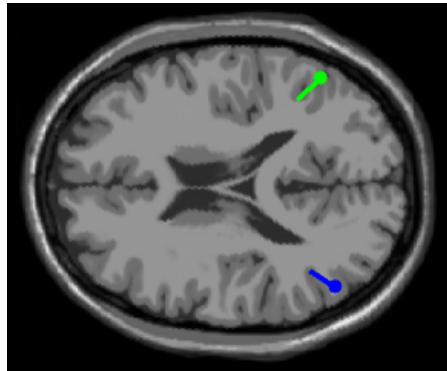
Taking the transpose of both sides, and multiplying by \mathbf{A}^T

$$\mathbf{A}^T e_i = d_i$$

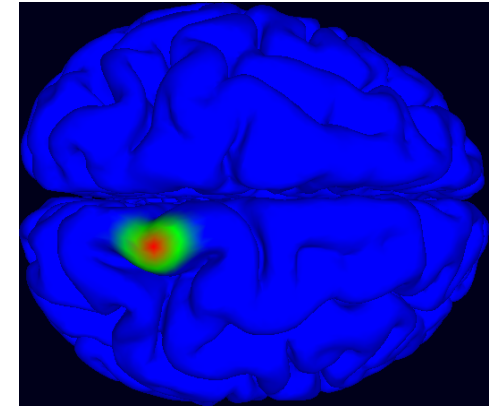
FEM transfer matrix

- ◆ FEM computes volume potentials
 - Solving the matrix for every source is slow
 - We only need potentials at electrode locations
- ◆ Use the reciprocal formulation:
 - Inject current at electrodes, solve volume potentials





Inverse Problem



Equivalent dipole Methods

- ◆ Overdetermined
- ◆ Searches for parameters of a number of dipoles
- ◆ Nonlinear optimization techniques
- ◆ May converge to local minima
- ◆ Non-linear least squares, beamforming, MUSIC, simulated annealing, genetic algorithms, etc.

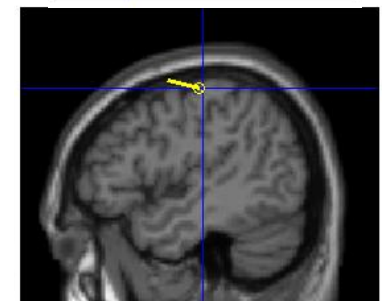
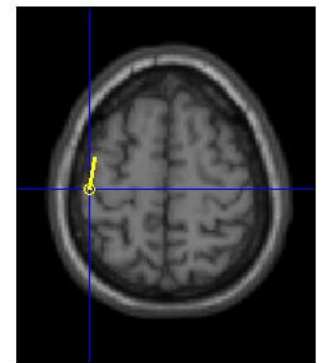
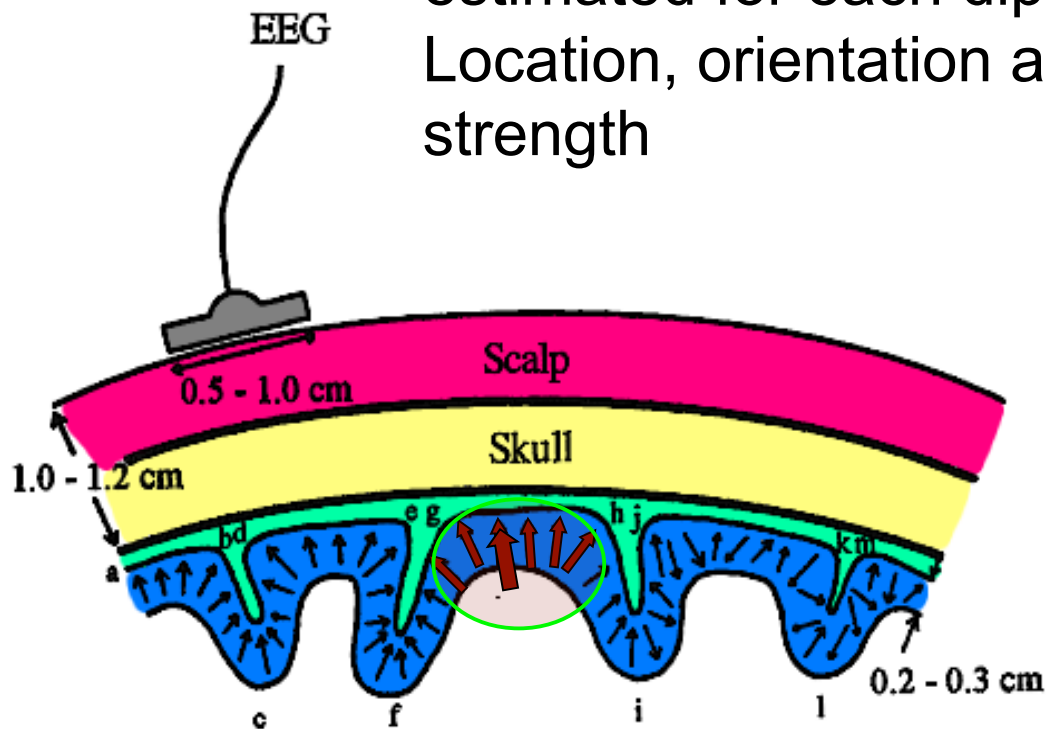
Linear distributed Methods

- ◆ Underdetermined
- ◆ Searches for activation in given locations.
- ◆ Linear optimization techniques
- ◆ Needs additional constraints
- ◆ Bayesian methods, MNE, LORETA, LAURA, etc.

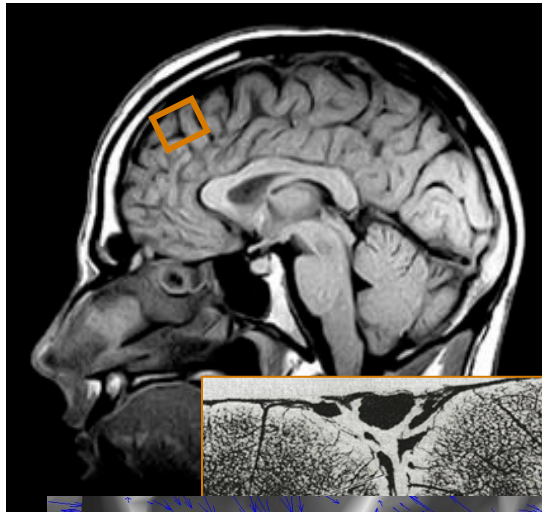
Equivalent current dipole (ECD)

$$O(S) = \min \|X - LS\|^2$$

6 parameters are estimated for each dipole:
Location, orientation and strength



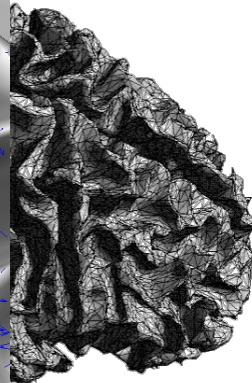
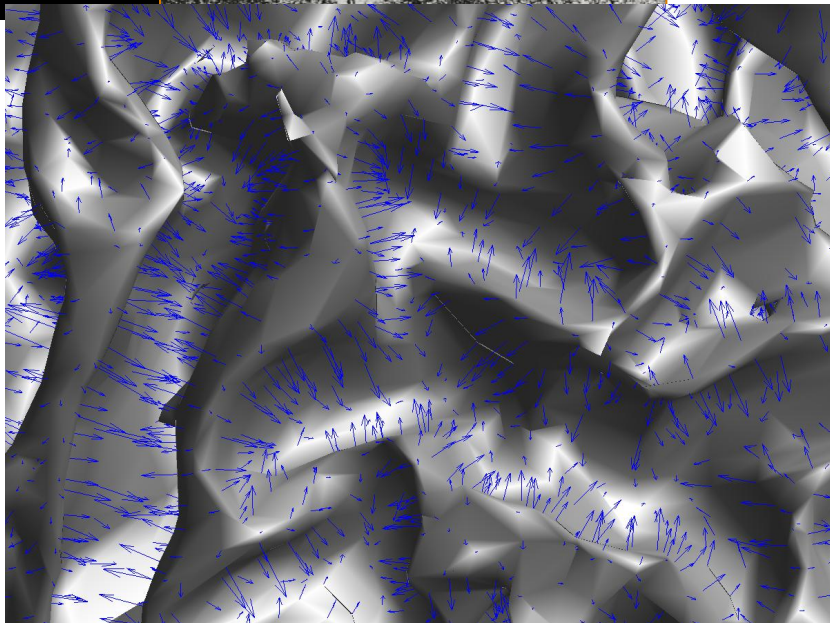
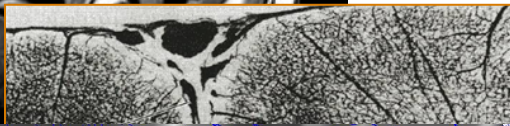
Linear distributed methods



$$X = LS$$

L is the lead field matrix:

Potential vectors of all possible solutions



Anatomical constraint:

Sources are on the cortex
perpendicular to the cortex

Multi-scale patch-basis source localization with Sparse Bayesian Learning

$D_{ij} = \text{geodesic_distance}(i, j)$

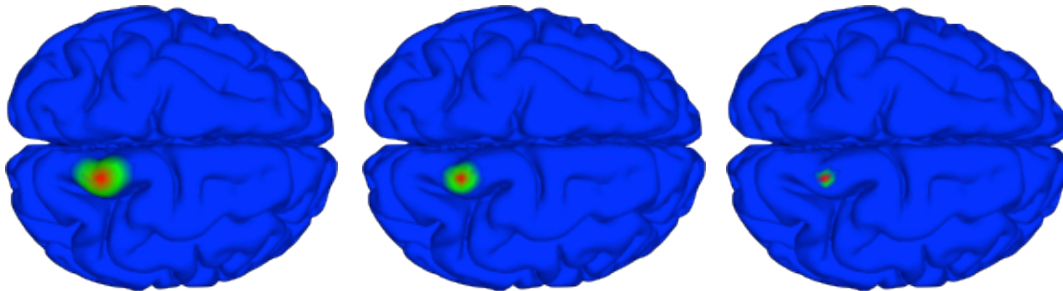
$D_{ij} = \text{Inf}$ if $D_{ij} > \text{scale}$

$$W_{ij}^{(k)} = \text{gauss}(D_{ij}, \sigma_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{D_{ij}^2}{2\sigma_k^2}\right)$$

$\sigma_k = \text{scale} / 3$

Three truncated Gaussian patches of different scales (radii)

radius	10 mm	6 mm	3 mm
σ_k	3.33 mm	2 mm	1 mm



Akalin Acar, et al (2008a, 2009) IEEE EMBC
Ramirez, et al, HBM, 2007

Forward Model

$$X = LS$$

$$L := [m \times v] \quad \text{Lead field matrix}$$

$$\tilde{L} = [LW^{(1)} \dots LW^{(3)}]_{m \times 3v}$$

ICA Model

$$X = A\hat{S}$$

$$\hat{S}_q := [1 \times T] \quad q^{\text{th}} \text{ IC activation}$$

ICA+SBL Inverse Model

$$A_q = \tilde{L}\tilde{M}_q + \tilde{U}_q$$

$$\tilde{L}^{-1} = \text{SBL}(A_q, \tilde{L})$$

$$\tilde{M}_q = [\tilde{L}^{-1}A_q]_{3v \times 1}$$

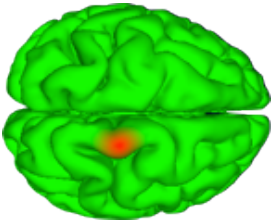
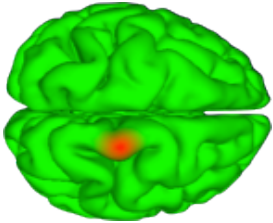
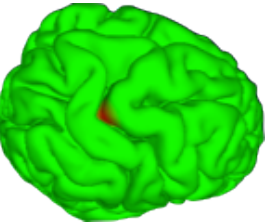
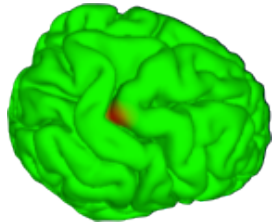
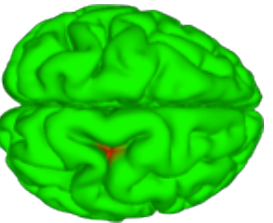
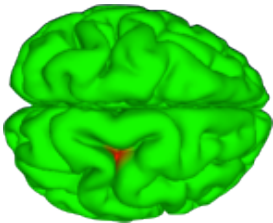
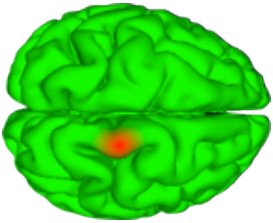
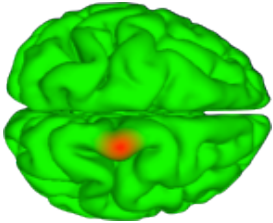
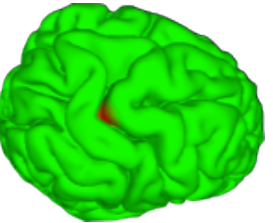
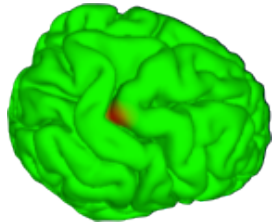
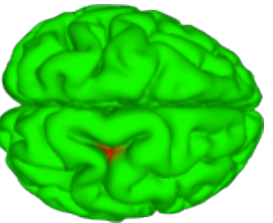
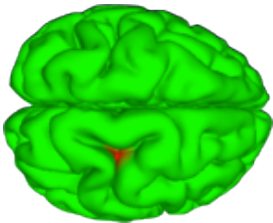
$$M_q = \text{reshape}(\tilde{M}_q, v \times 3)$$

$$M_q = \sum_{i=1}^3 \tilde{M}_q(:, i)$$

$$P_q = M_q \hat{S}_q$$

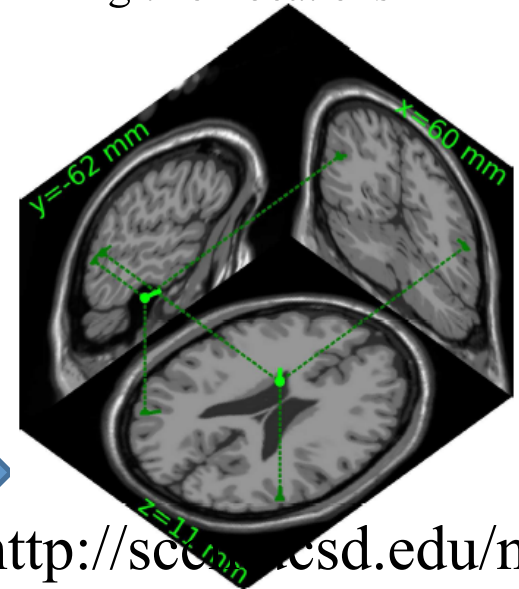
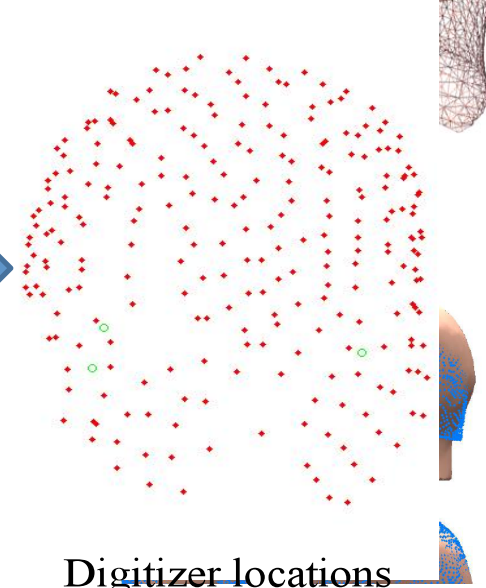
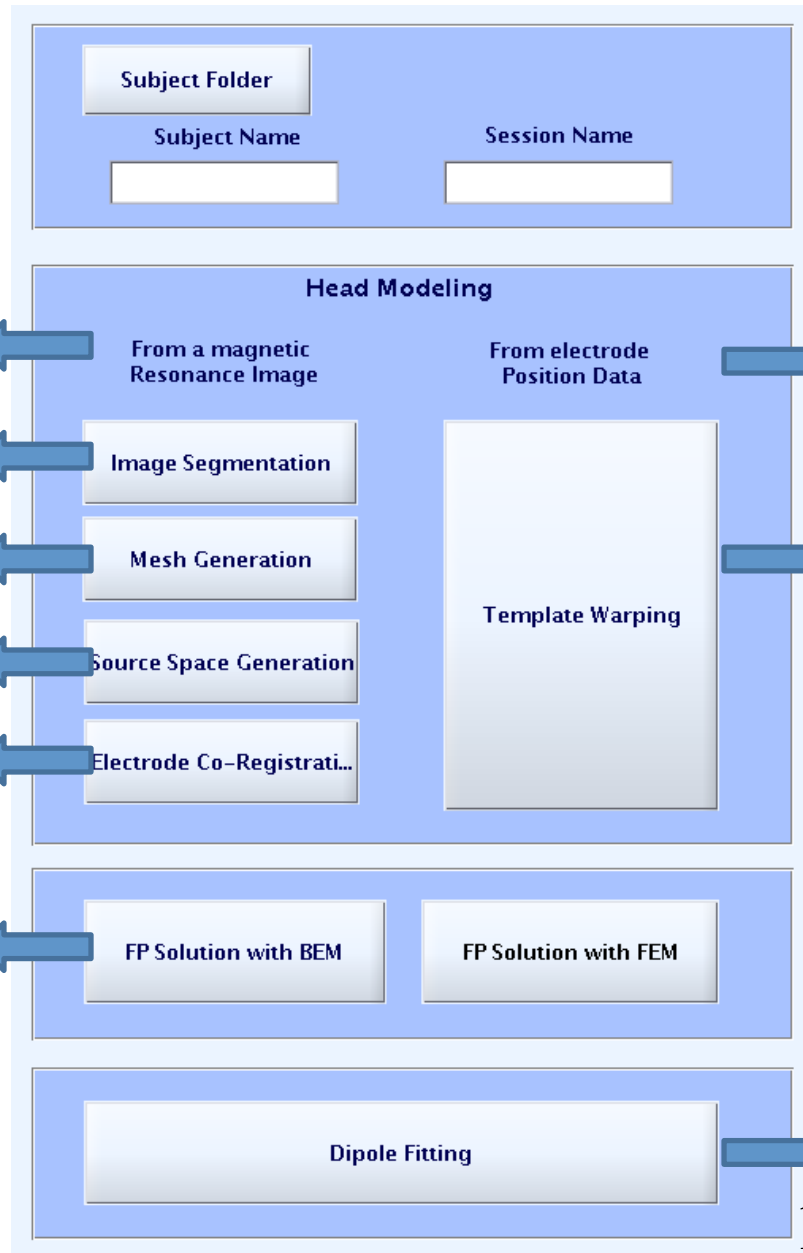
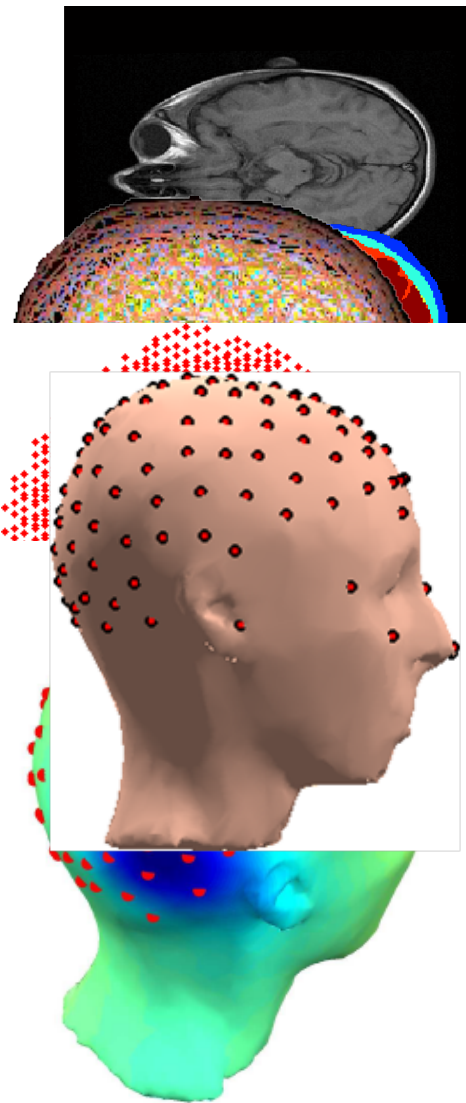
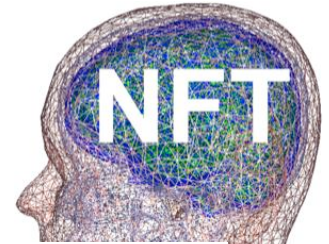
$[v \times T]$ cortical surface potentials for q^{th} IC

SBL Simulation Study with MNI model (SNR=50)

Three examples:		Source (x 15)		Max. dis. (mm)	Energy dif.	DF (%)	
original	reconstructed	Type	Scale (mm)				
Gyral			Gyral	10	0	1.5	103.8
			Sulcul	10	1.01	29.8	101.4
			Sulcul	5	2.12	4.1	37.6
Sulcal			Dual	10	11.6	29.3	89.2
			Gyral	5	1.01	4.7	41.3
Sulcal			Sulcul	12	1.8	10.6	125.5

Term	Definition
max displacement	geodesic distance between original and reconstructed patch centers
energy difference	original energy - reconstructed energy
degree of focalization (DF)	reconstructed energy / original energy

Neuroelectromagnetic Forward Head Modeling Toolbox



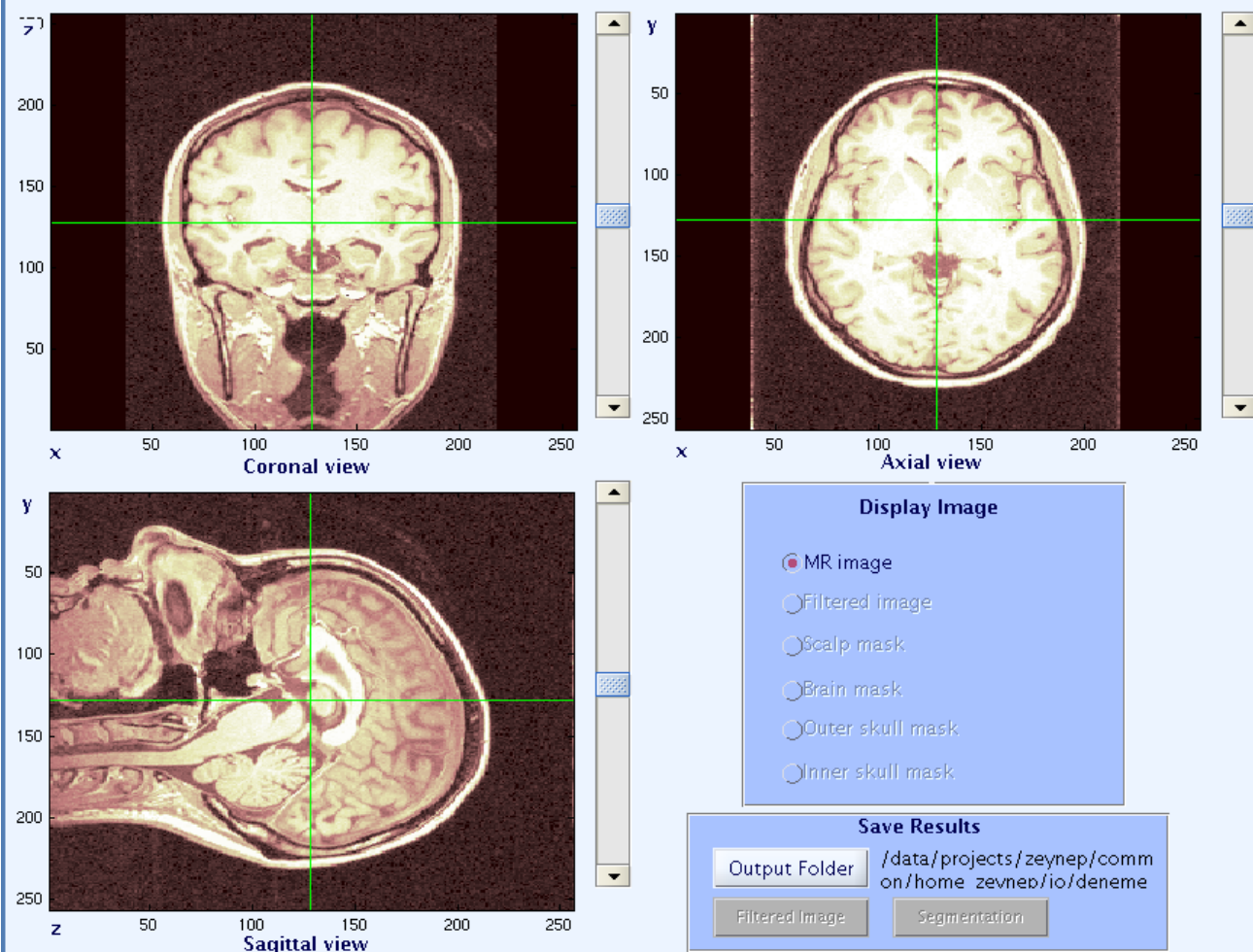
<http://scc.nyu.edu/~nft>

NFT - Segmentation

NFT: MR segmentation

File

(x,y,z)= (128, 128, 128)



Coronal view

Axial view

Sagittal view

Display Image

- MR image
- Filtered image
- Scalp mask
- Brain mask
- Outer skull mask
- Inner skull mask

Save Results

Output Folder /data/projects/zeynep/comm on/home zeynep/io/deneme

Filtered Image Segmentation

Image Segmentation

Swap L/R

1. Anisotropic Filtering

Number of iterations

Image diffusion

2. Scalp Segmentation

3. Brain Segmentation

Cerebellar low point

x

y White matter seed point

z

Fill level [0, 1] Threshold [0, 1]

4. Outer Skull Segmentation

z Center of one eye

5. Inner Skull Segmentation

NFT – Mesh generation

From a magnetic Resonance Image

Image Segmentation

Mesh Generation

Source Space Generation

Electrode Co-Registrati...

NFT: Mesh generation

Load Segmentation /data/projects/zeynep/common/home_zeynep/jo/deneme/dene_real/SubjectA_segments

Output Folder /data/projects/zeynep/common/home_zeynep/jo/deneme/dene_real

4 # of layers Mesh name: SubjectA

Linear Quadratic 7000 Number of nodes per layer

Local mesh refinement

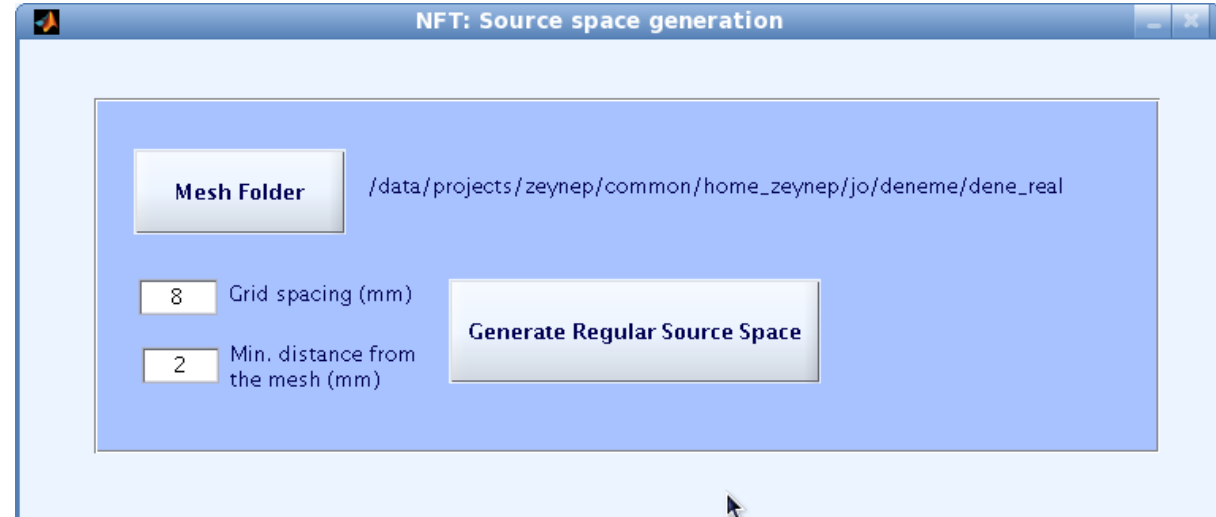
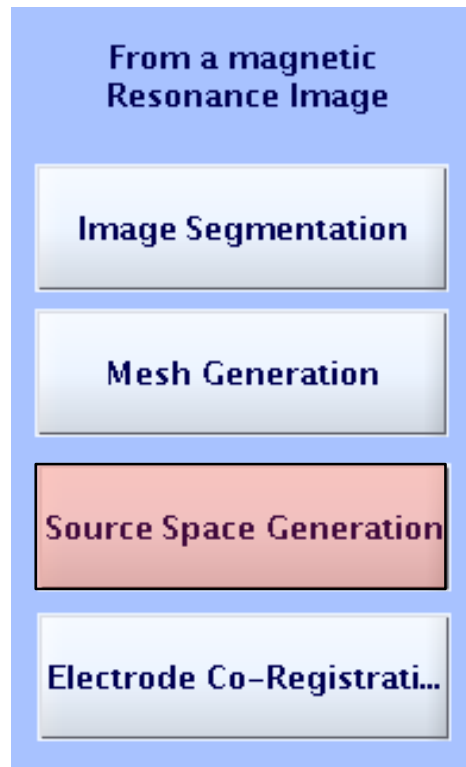
2.1 Edge length/ Distance between meshes

Start Mesh Generation

Status

Generate linear FEM mesh Generate quadratic FEM mesh

NFT – source space generation



Generates a simple source space:
Regular Grid inside the brain
With a given spacing and distance to the mesh

NFT – electrode co-registration

From a magnetic Resonance Image

Image Segmentation

Mesh Generation

Source Space Generation

Electrode Co-Registrati...

NFT: Electrode co-registration

Load sensor locations /data/projects/zeynep/common/home_zeynep/jo/deneme/jop3_raw.el

Mesh Folder /data/projects/zeynep/common/home_zeynep/jo/deneme/dene_real

Initial co-registration Translation 0 2 5
Rotation 0 0.25 -1.5708

Complete co-registration Translation
Rotation

Save initial reg. Save complete reg.

Computing translation and rotation parameters...

NFT – Template warping

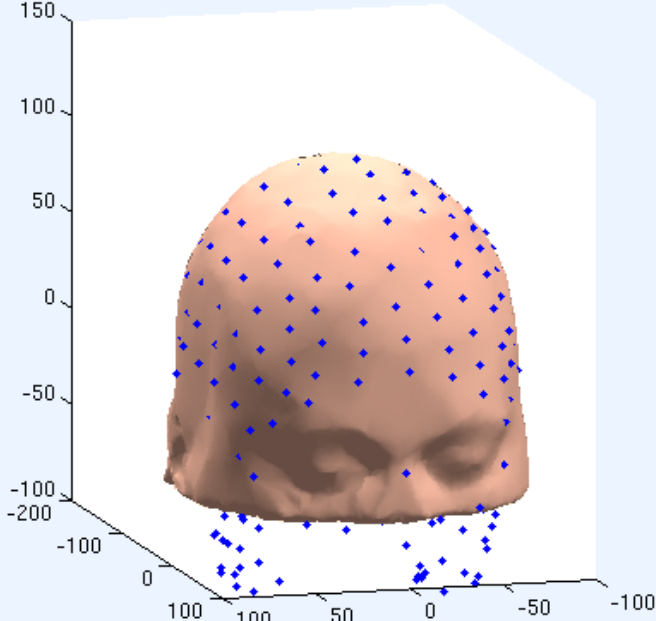
NFT: Template head model warping

of layers (3 or 4)

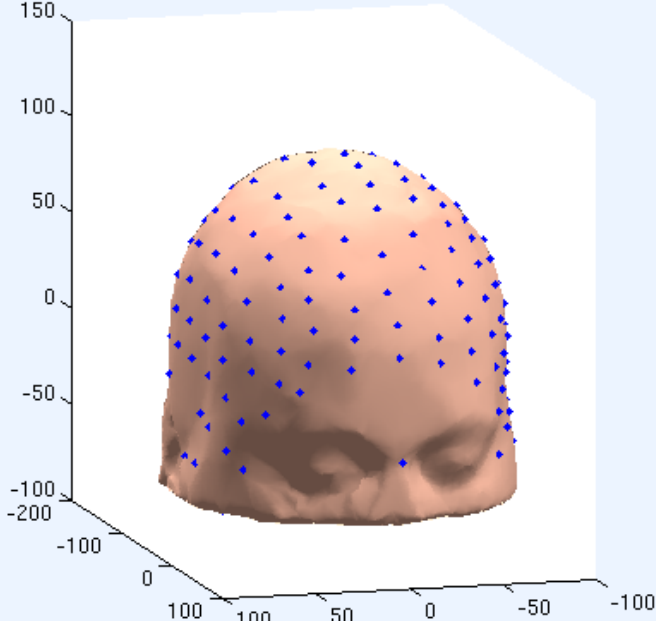
Load sensor data /data/projects/zeynep/common/home_zeynep/jo/dene
me/jop3_raw.elp

Output Folder /data/projects/zeynep/common/home_zeynep/jo/dene
me/dene_mni

MNI head model



Warped MNI head model



Mesh Warper!

NFT – Forward problem solver

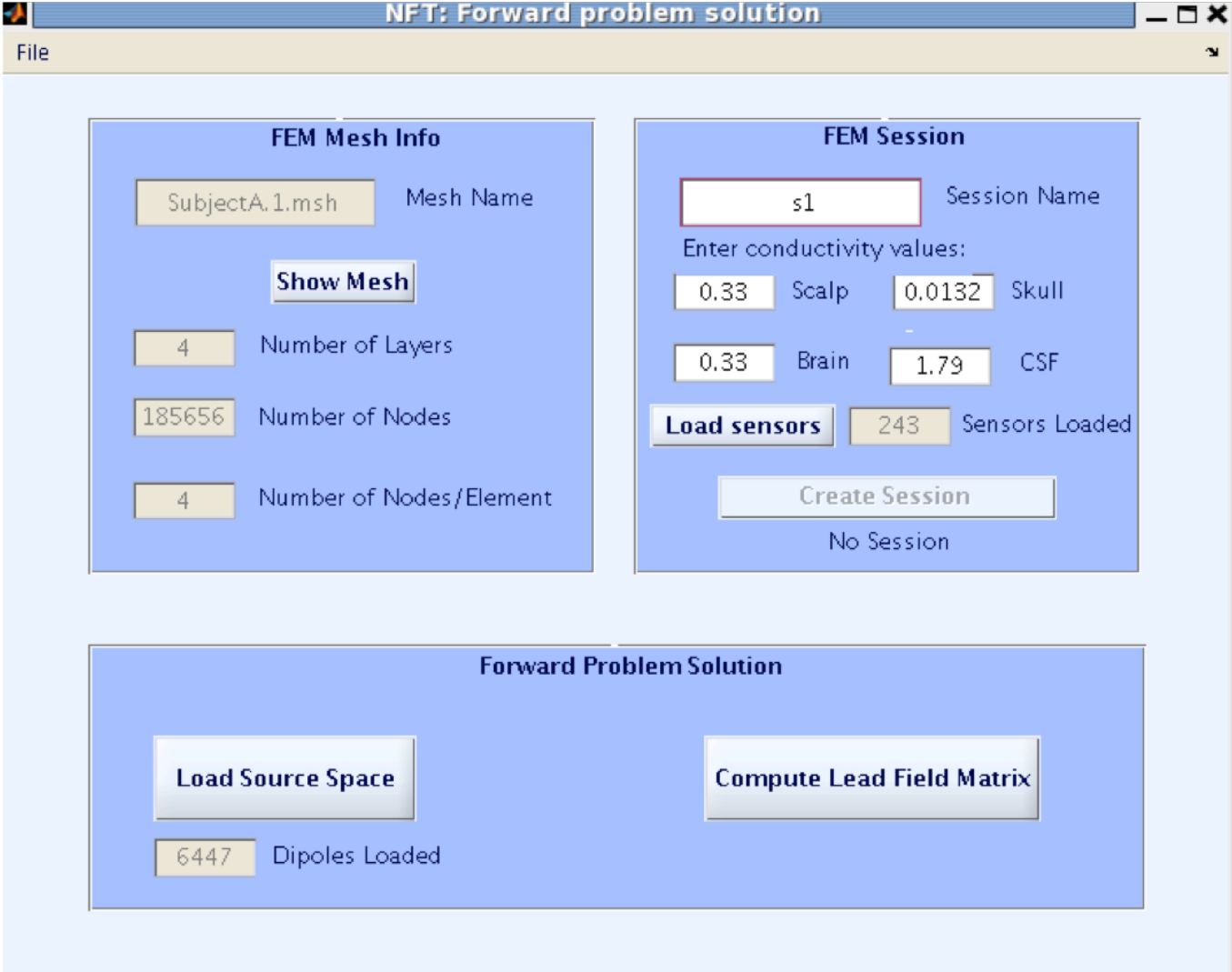
- ◆ MATLAB interface to numerical solvers
- ◆ Boundary Element Method or Finite Element Method
 - EEG Only (for now)
 - Interfaces to the Matrix generator executable written in C++
- ◆ Other computation done in MATLAB
- ◆ Generated matrices are stored on disk for future use.

NFT - Forward Problem Solver (BEM)

The screenshot shows the 'NFT: Forward problem solution' software interface. It features a menu bar with 'File' and a main workspace divided into several panels:

- BEM Mesh Info:** Contains a 'Mesh Name' field with 'SubjectA', a 'Show Mesh' button, and four statistics: 'Number of Layers' (4), 'Number of Nodes' (13724), 'Number of Elements' (27476), and 'Number of Nodes/Element' (3).
- BEM Model:** Contains a 'Model Name' field with 'SubjectA', a section for 'Enter conductivity values:' with fields for 'Scalp' (0.33), 'Skull' (0.0042), 'Brain' (0.33), and 'CSF' (1.79), a checked checkbox for 'Modified (Isolated Problem Approach)', a 'Create Model' button, and the text 'Generating matrices...'. A mouse cursor is pointing at the 'Create Model' button.
- Session:** Contains a 'Session Name' field with 's1', a 'Load Sensors' section with radio buttons for 'Mesh Coordinates' (selected) and 'Mesh Node List', a 'Load' button, a 'Show Sensors' button, a 'Generate transfer matrix' button, and the text 'Value Changed!'.
- Forward Problem Solution:** Contains three buttons: 'Load Source Space', 'Compute Lead Field Matrix', and 'Plot Potential Distribution'. Below the 'Plot Potential Distribution' button is a 'For Dipole' label and an empty text field.

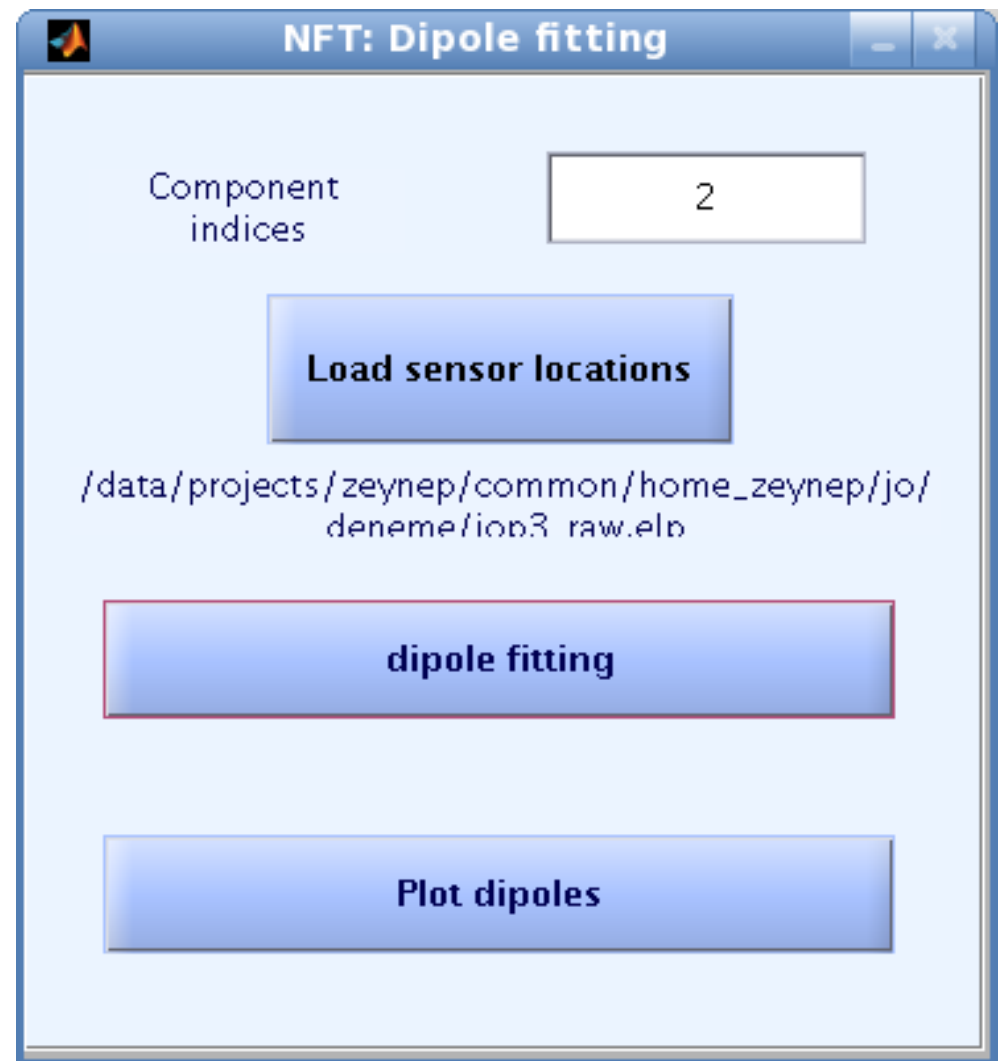
NFT – Forward Problem Solver (FEM)

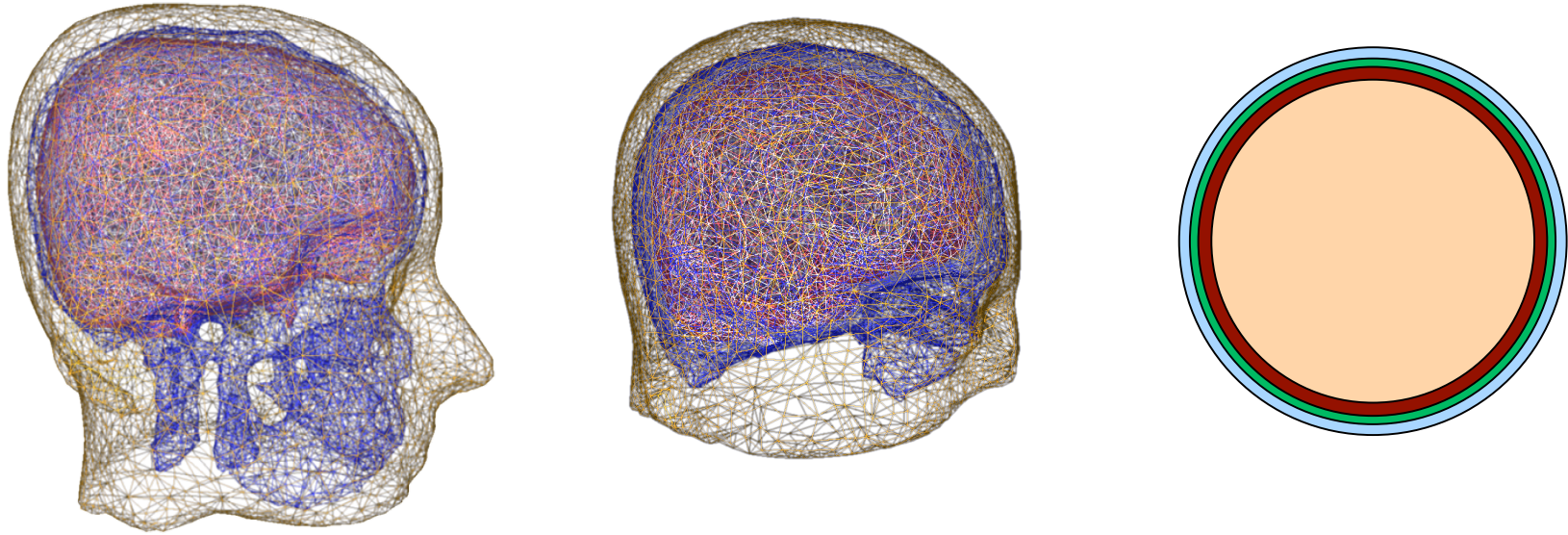


NFT – Dipole fitting

- ◆ Requires EEGLAB integration to access Component indices.
- ◆ Uses FieldTrip in EEGLAB for dipole fitting.

<http://www.sccn.ucsd.edu/nft>



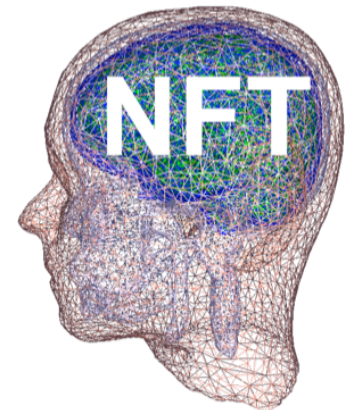


Effects of Forward Model Errors on EEG Source Localization

MODELING ERRORS

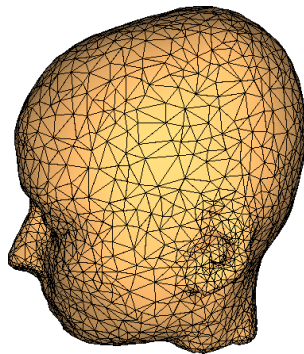
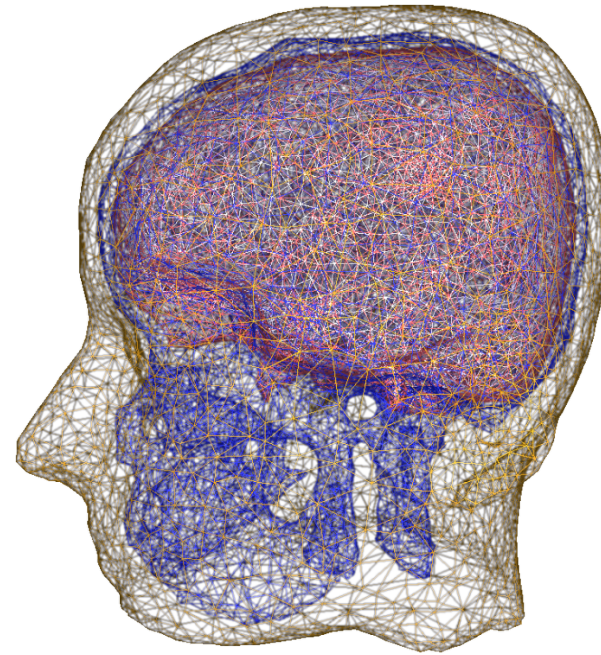
Head Model Generation

- ◆ Reference Head Model
 - From whole head T1 weighted MR of subject
 - 4-layer realistic BEM model
- ◆ MNI Head model
 - From the MNI head
 - 3-layer and 4-layer template BEM model
- ◆ Warped MNI Head Model
 - Warp MNI template to EEG sensors
- ◆ Spherical Head model
 - 3-layer concentric spheres
 - Fitted to EEG sensor locations

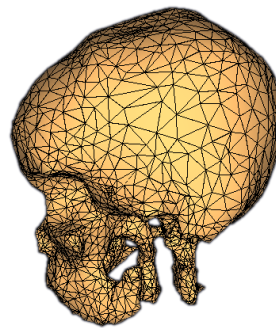


The Reference Head Model

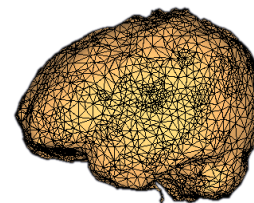
- ◆ 18541 nodes
- ◆ 37090 elements
 - 6928 Scalp
 - 6914 Skull
 - 11764 CSF
 - 11484 Brain



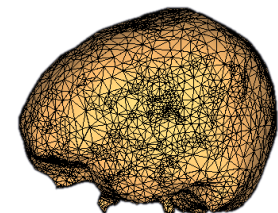
Scalp



Skull

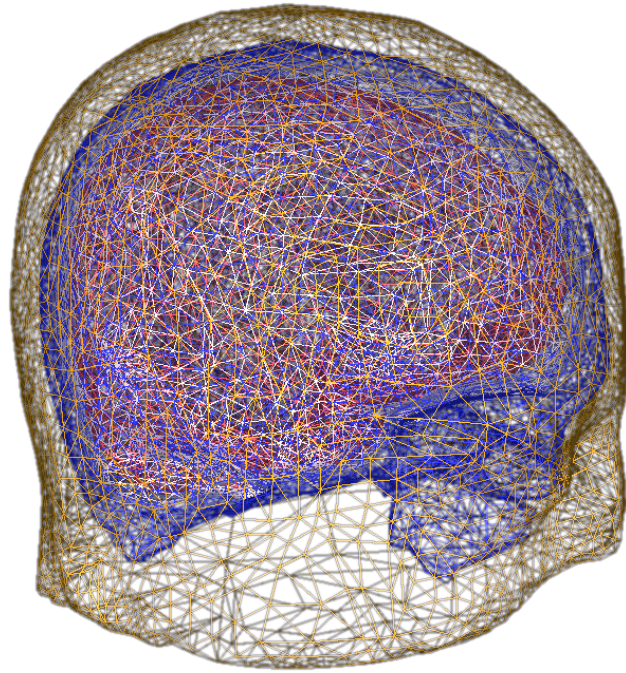


CSF

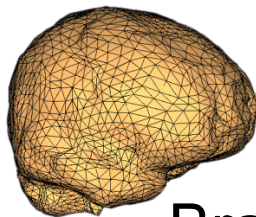


Brain

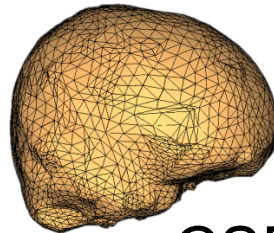
The MNI Head Model



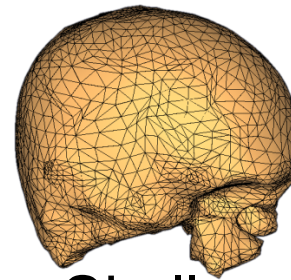
- ◆ 4-layer
 - 16856 nodes
 - 33696 elements
- ◆ 3-layer
 - 12730 nodes
 - 25448 elements



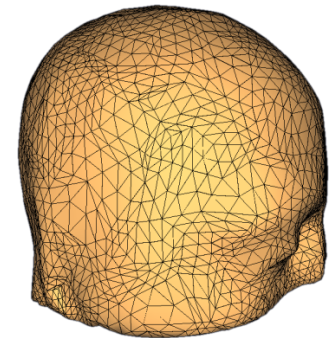
Brain



CSF

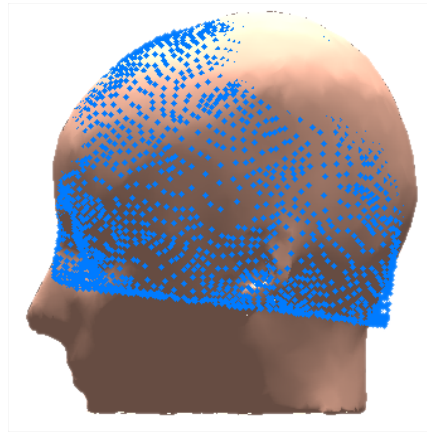
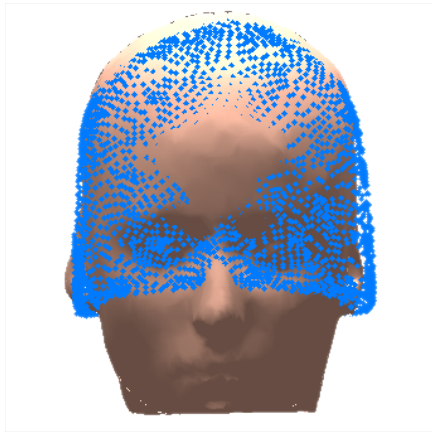


Skull

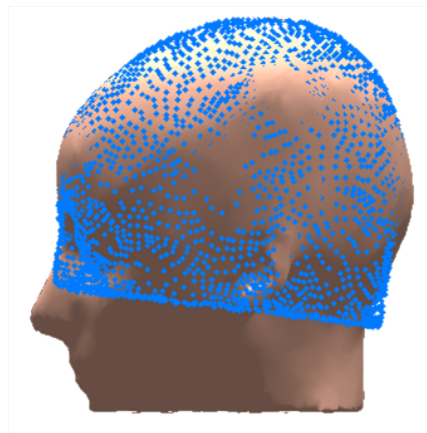
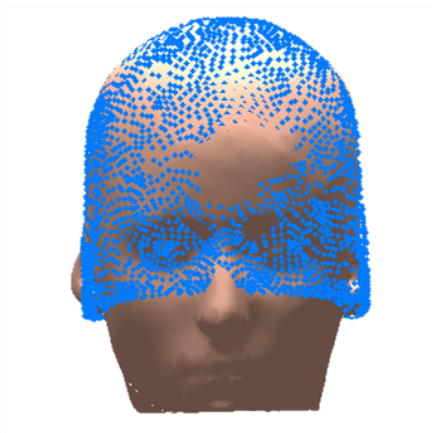


Scalp

The Warped MNI Head Model

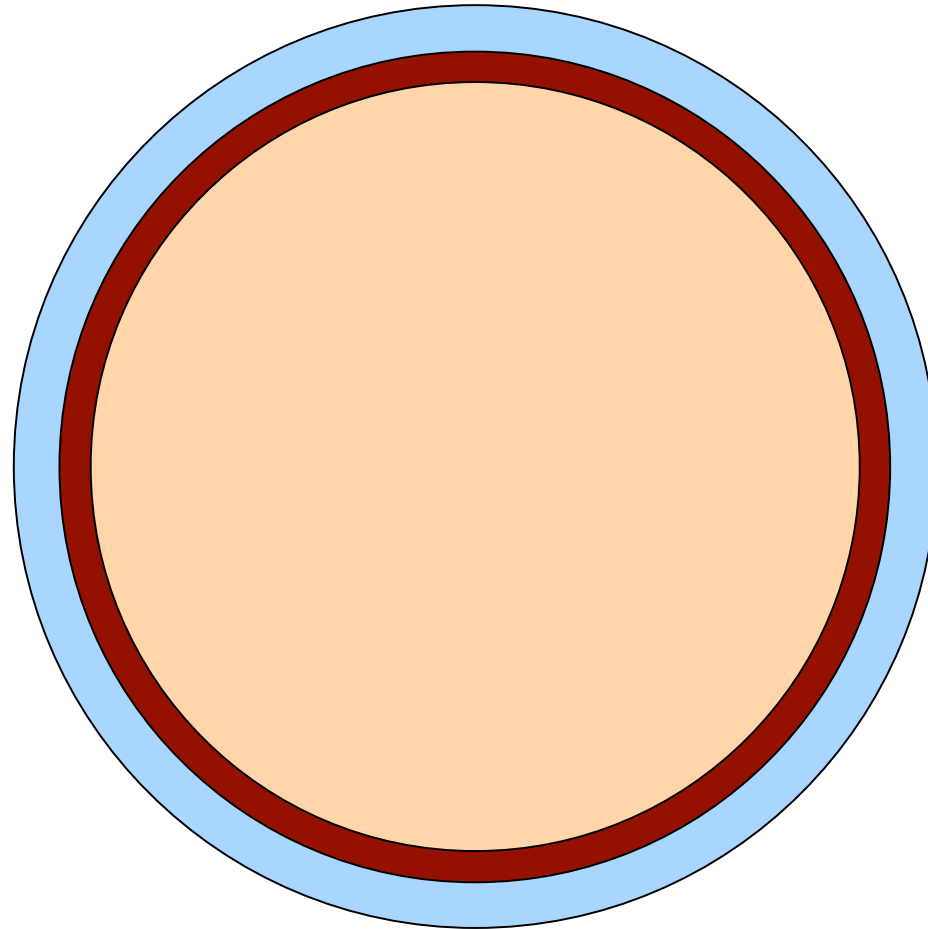


**Registered
MNI template**



**Warped
MNI mesh**

The Spherical Head Model



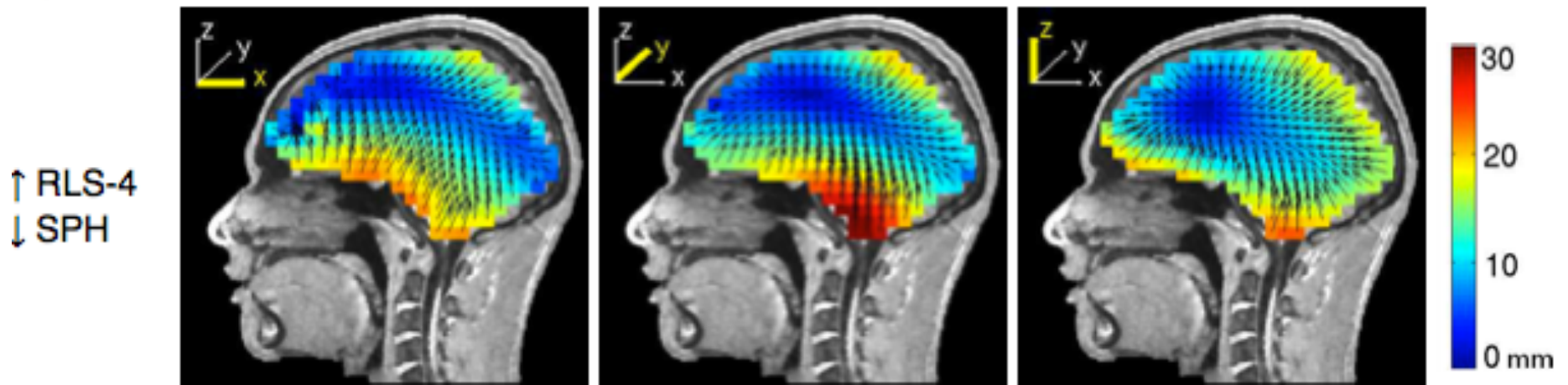
3-Layer model

Outer layer is fitted to electrode positions

Head Modeling Errors

- ◆ Solve FP with reference model
 - 3D grid inside the brain.
 - 3 Orthogonal dipoles at each point
 - ~7000 dipoles total
 - 4 subjects
- ◆ Localize using other head models
 - Single dipole search.
- ◆ Plot location and orientation errors

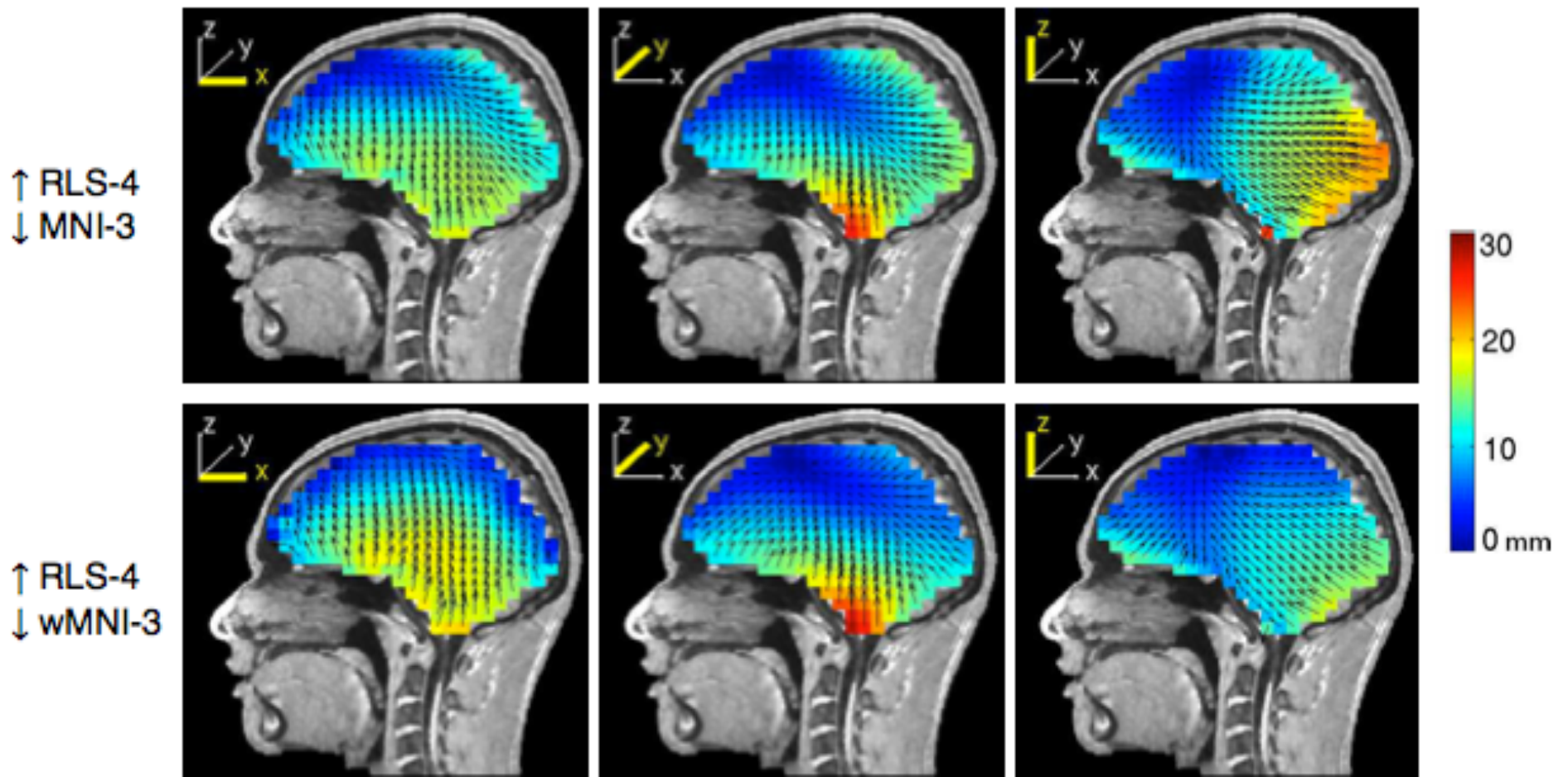
Spherical Model Location Errors



Localization errors may go up to 4 cm when spherical head models are used for source localization. The errors are largest in the inferior regions where the spherical models diverged most from the 4-layer realistic model.

3-Layer MNI Location Errors

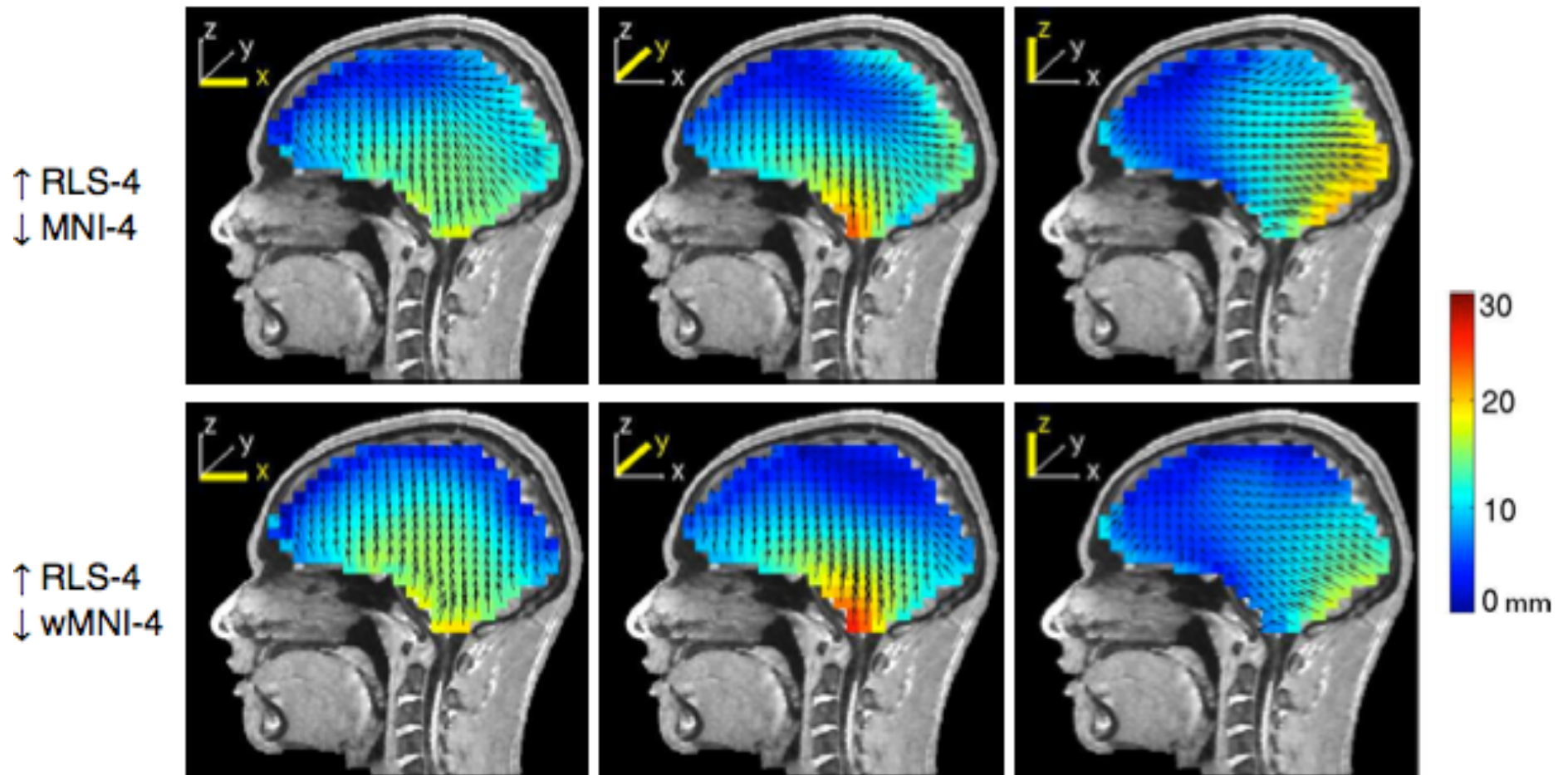
3-Layer MNI



3-Layer Warped MNI

4-Layer MNI Location Errors

4-Layer MNI



4-Layer Warped MNI

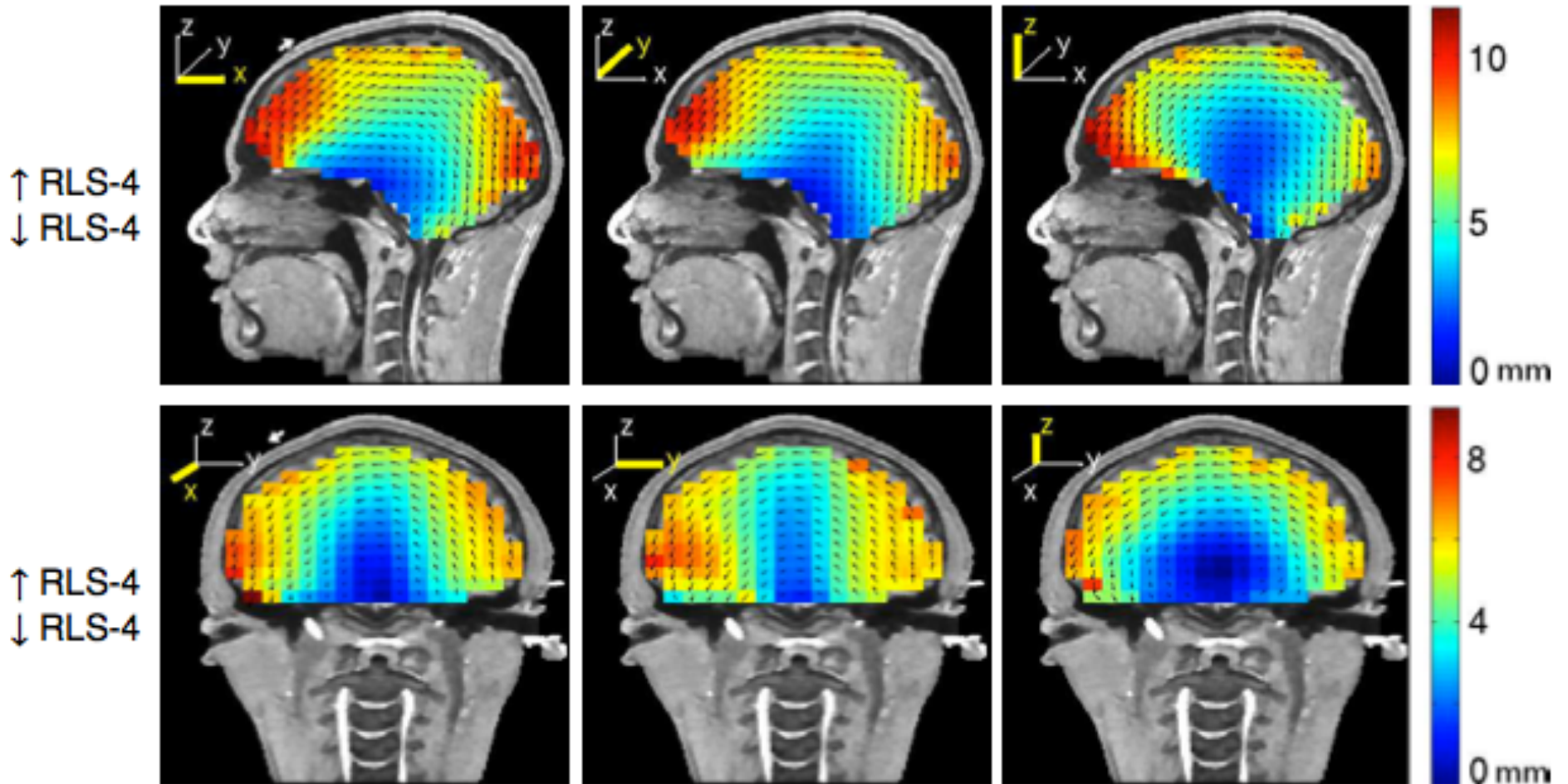
Observations

- ◆ Spherical Model
 - Location errors up to 3.5 cm. Cortical areas up to 1.5 cm.
- ◆ 3-Layer MNI
 - Large errors where models do not agree.
 - Higher around chin and the neck regions.
- ◆ 4-Layer MNI
 - Similar to 3-Layer MNI.
 - Smaller in magnitude.

Electrode co-registration errors

- ◆ Solve FP with reference model
- ◆ Shift all electrodes and re-register
 - 5° backwards
 - 5° left
- ◆ Localize using shifted electrodes
- ◆ Plot location and orientation errors

Location Errors with 5° electrode shift

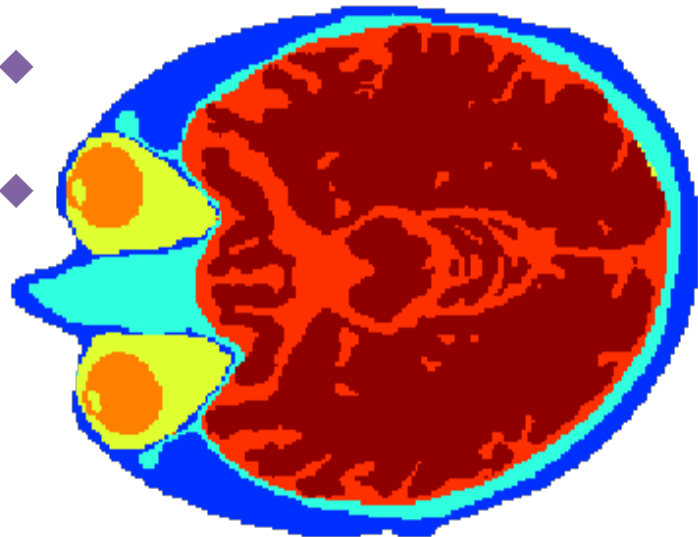


Observations

- ◆ Errors increase close to the surface near electrode locations.
- ◆ Changing or incorrectly registering electrodes may cause 5-10 mm localization error.

Head tissue conductivities

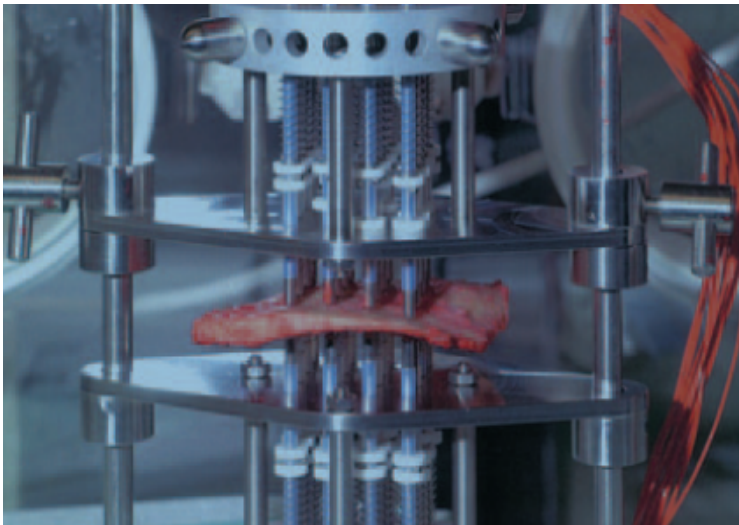
- ◆ Scalp : 0.33 S/m
- ◆ Skull: 0.0032 S/m (0.08-0.0073 S/m)
- ◆ CSF: 1.79 S/m
- ◆ Brain: 0.33 S/m



Skull conductivity measurement

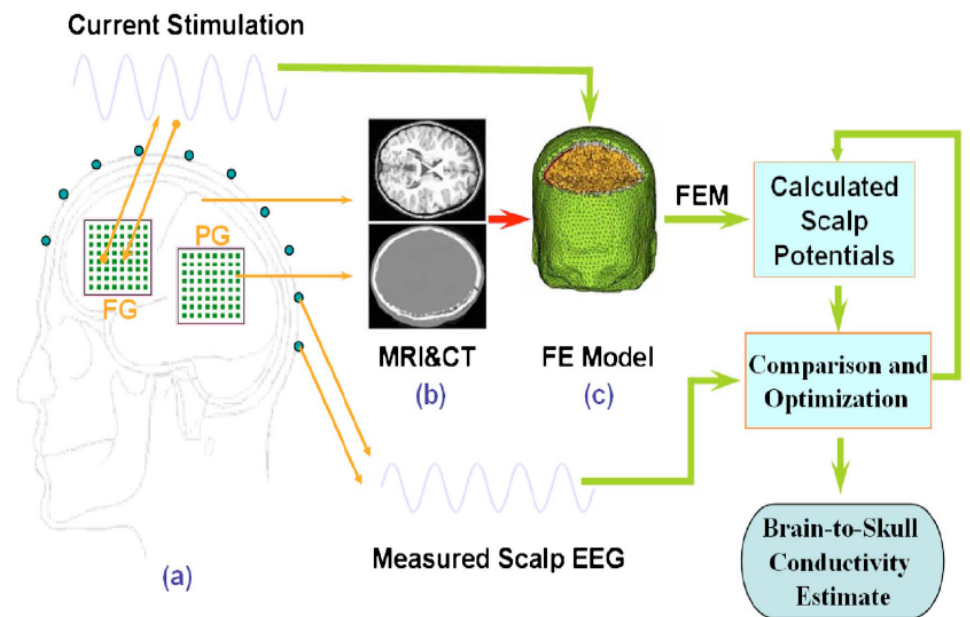
Measurement of skull conductivity

In vivo



Hoekama *et al*, 2003

In vitro



He *et al*, 2005

Skull conductivity

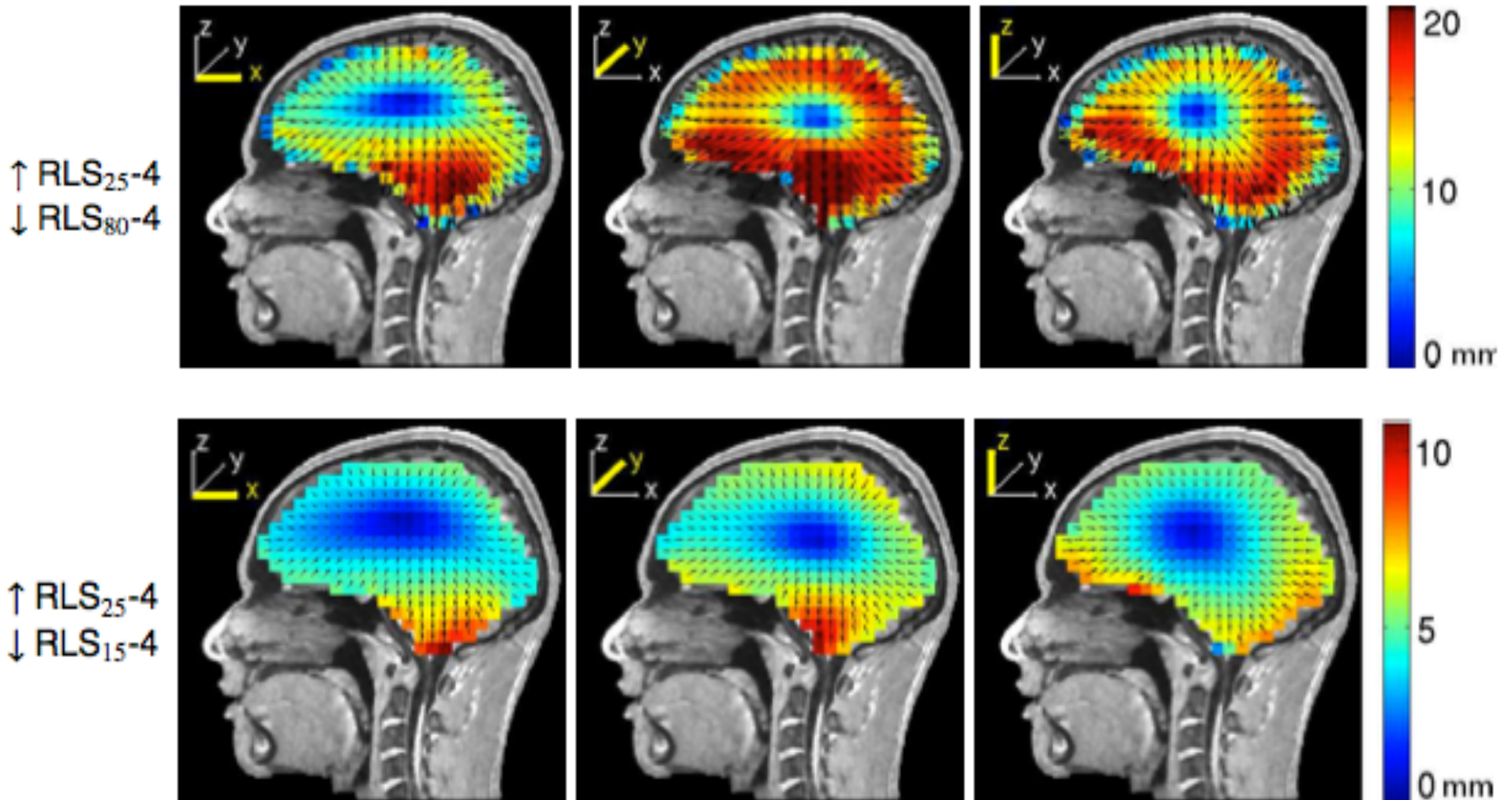
Brain to skull ratio		
Rush and Driscoll	1968	80
Cohen and Cuffin	1983	80
Oostendorp et al	2000	15
Lai et al	2005	25

Measurement	Age	σ (mS/m)	ratio
Agar-agar phantom	–	43.6	7.5
Patient 1	11	80.1	4
Patient 2	25	71.2	4.6
Patient 3	36	53.7	6.2
Patient 4	46	34.4	9.7
Patient 5	50	32.0	10.3
Post mortem skull	68	21.4	15.7

Effect of Skull Conductivity

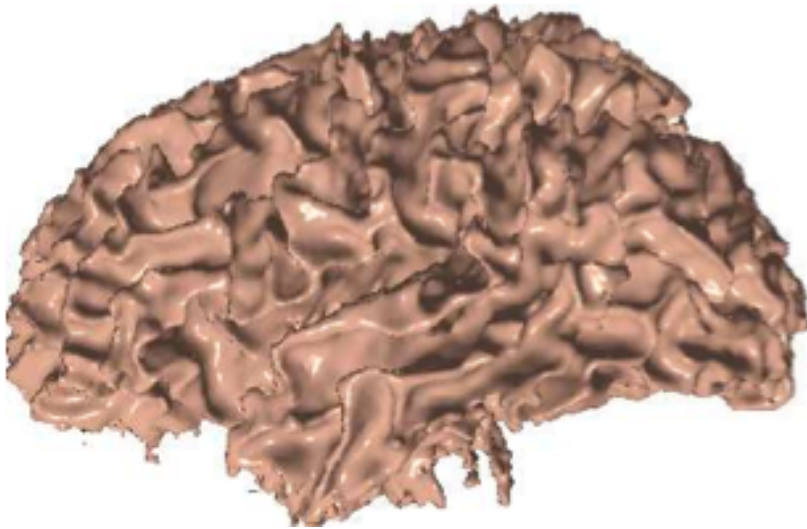
- ◆ Solve FP with reference model
 - Brain-to-Skull ratio: 25
- ◆ Generate test models
 - Same geometry
 - Brain-to-Skull ratio: 80 and 15
- ◆ Localize using test model
- ◆ Plot location and orientation errors

Skull conductivity mis-estimation

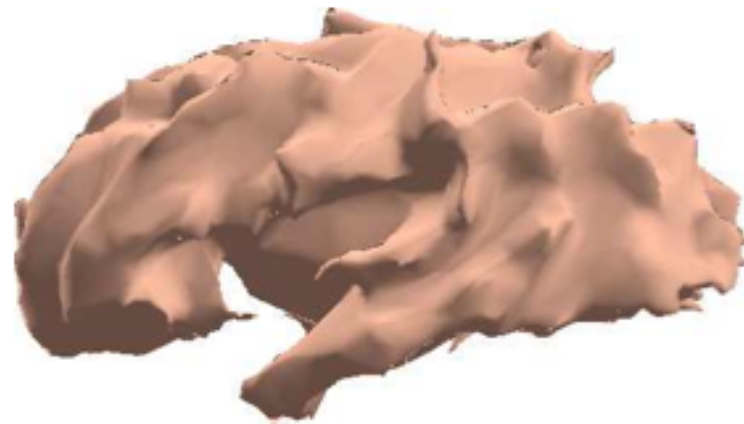


Effect of white matter

White matter conductivity: 0.14 S/m

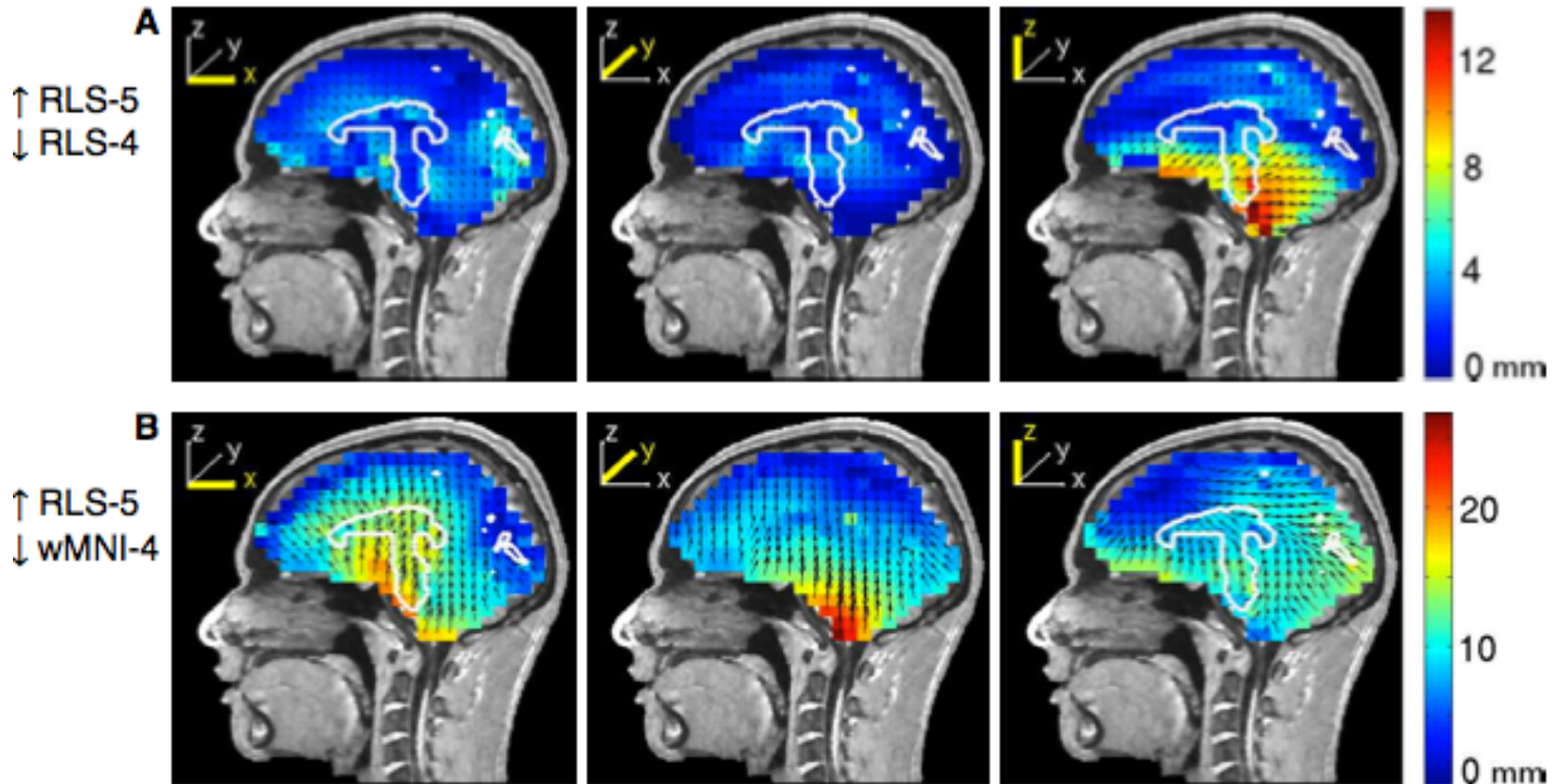


White matter surface obtained using Freesurfer

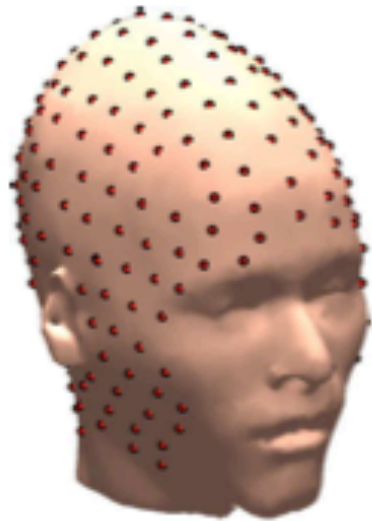


Simplified WM BEM model

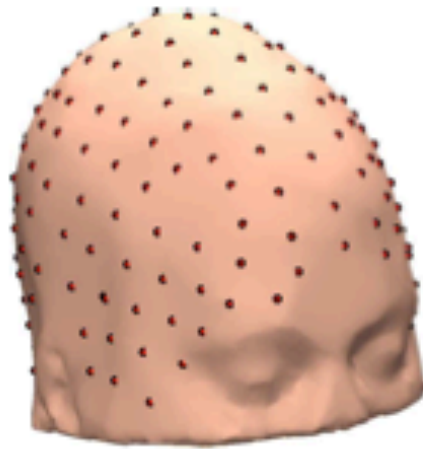
Effect of white matter



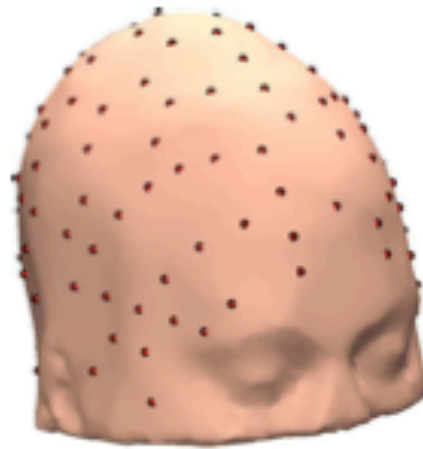
Number of electrodes and coverage



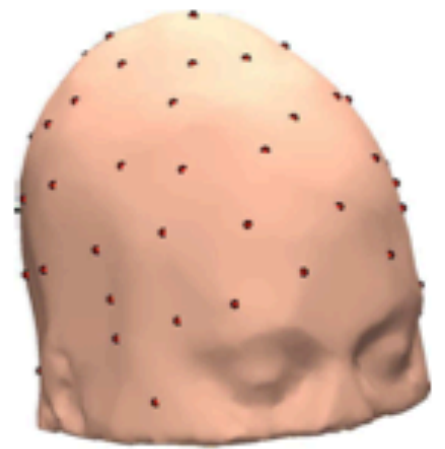
(a)



(b)



(c)



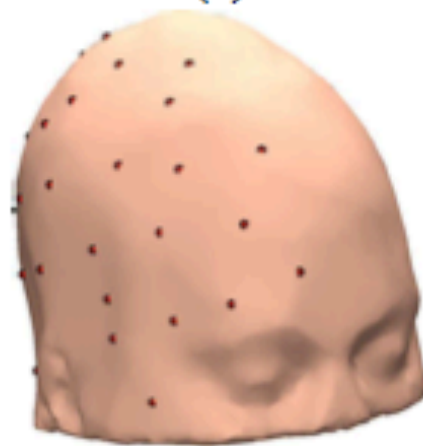
(d)



(e)



(f)

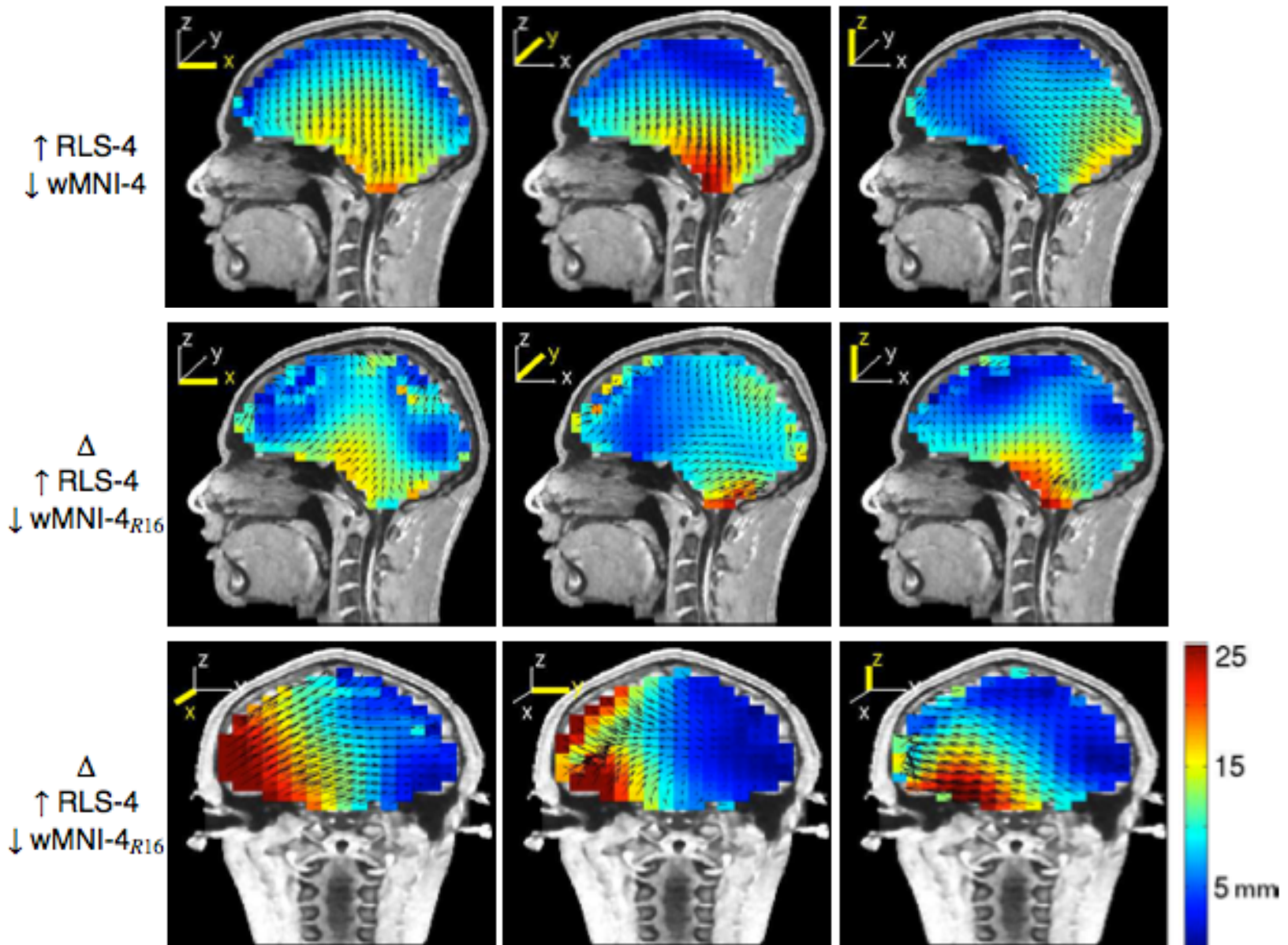


(g)



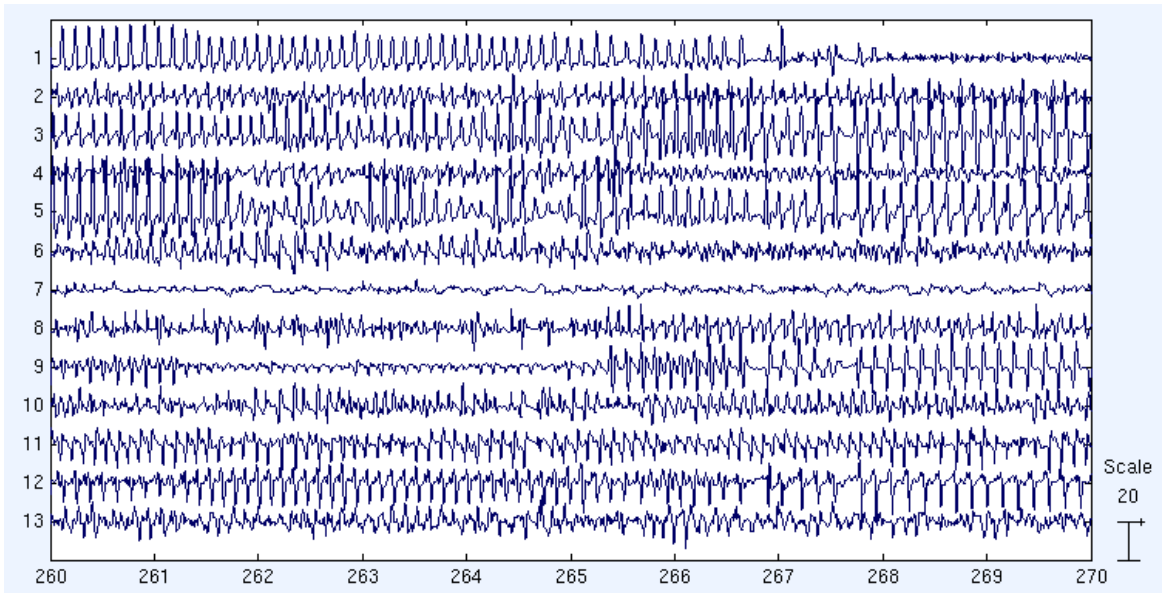
(h)

Location errors



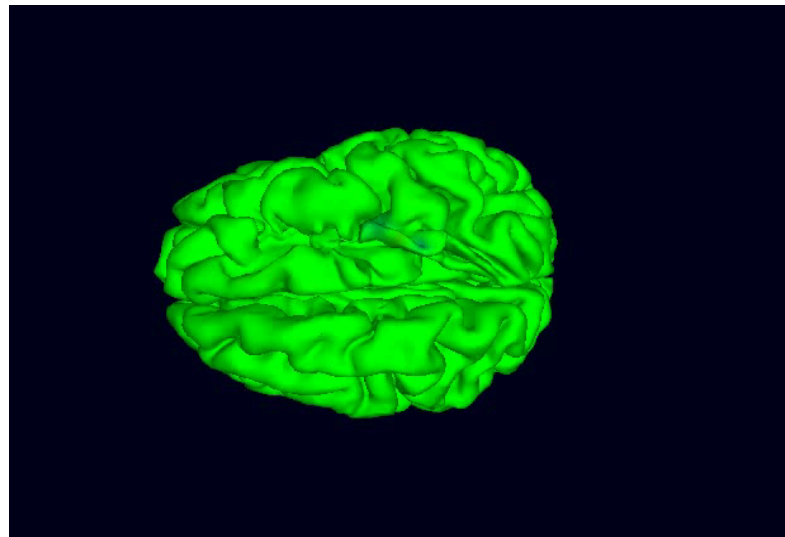
Summary

- ◆ If we have MRI of the subject:
 - Subject specific head model
 - Distributed source localization
- ◆ If we don't have MRIs
 - Warped 4-layer MNI model
 - Dipole source localization
- ◆ Skull conductivity estimation is as important as the head model used.
- ◆ WM modeling does not have much effect on source localization.

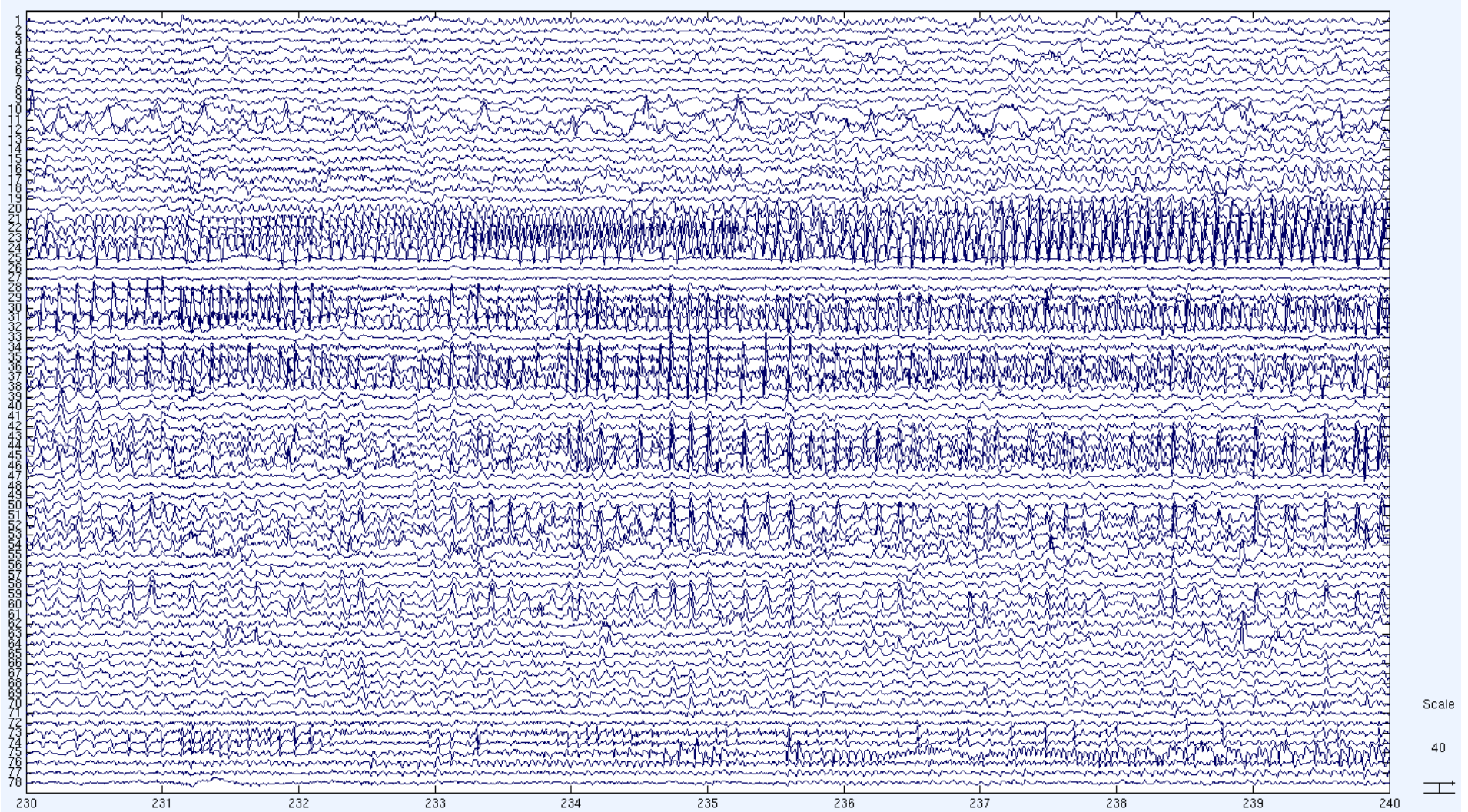


Epilepsy Head Modeling

CASE STUDY



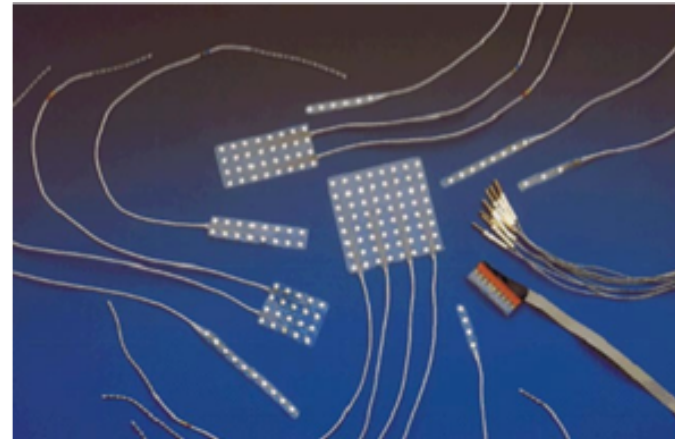
Epilepsy



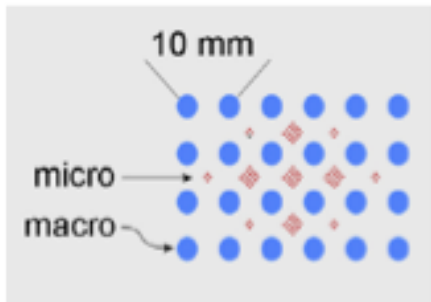
Electrocorticography



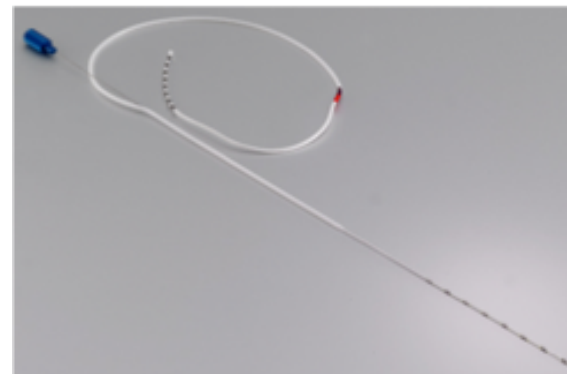
ECoG recording



ECoG grid and strips

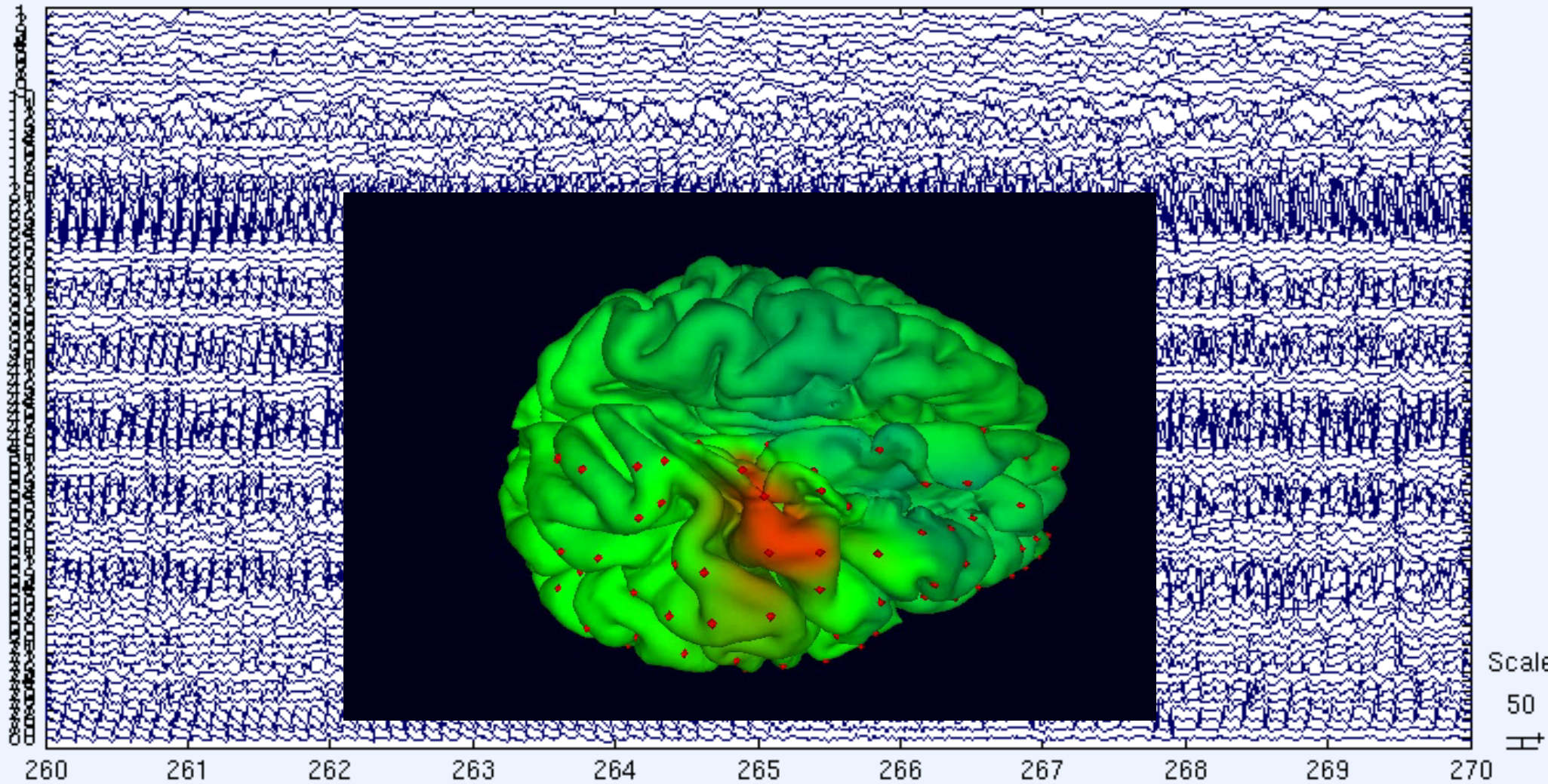


Macro and micro
Electrodes (Mayo Clinic)

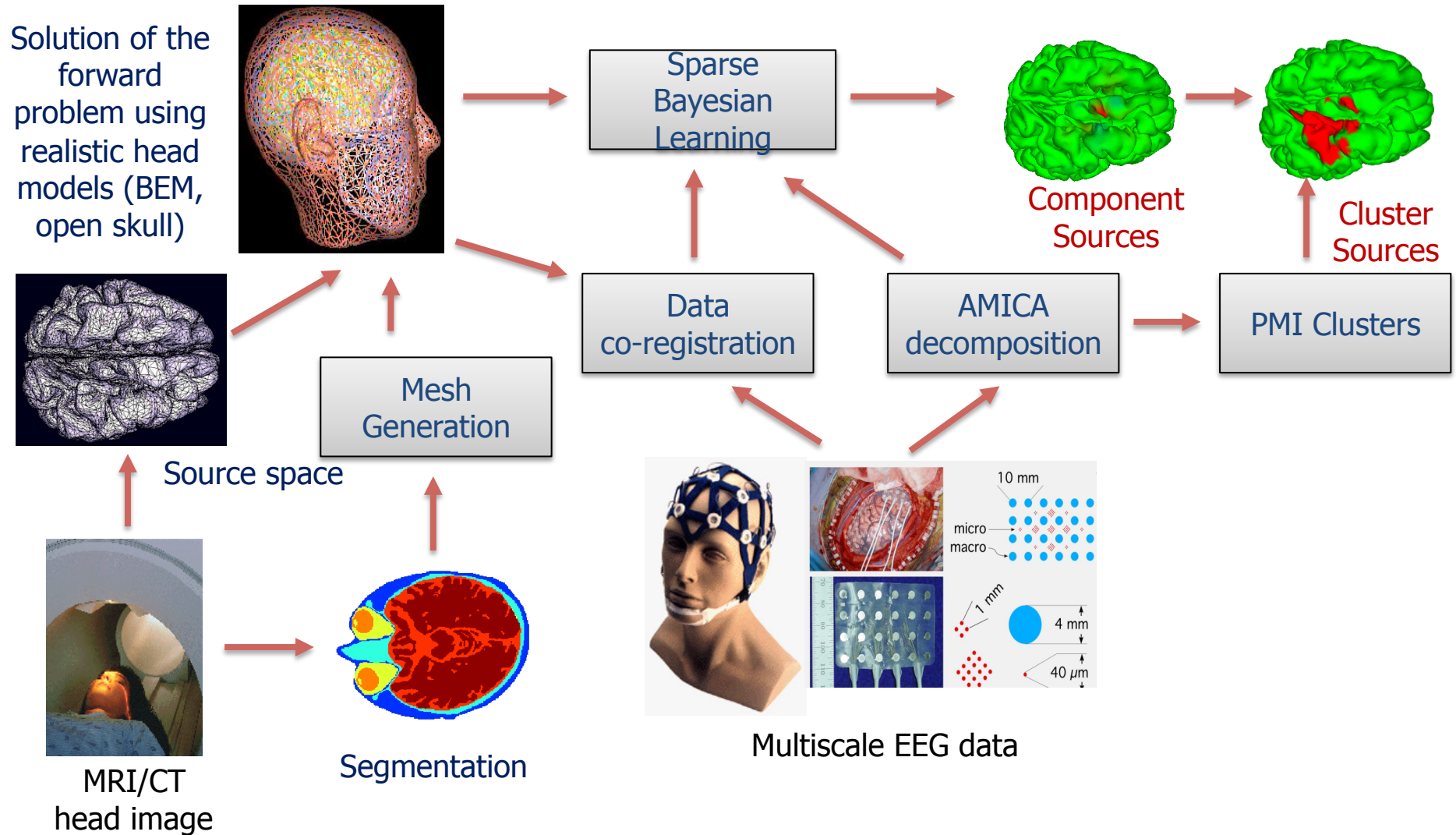


Depth electrodes

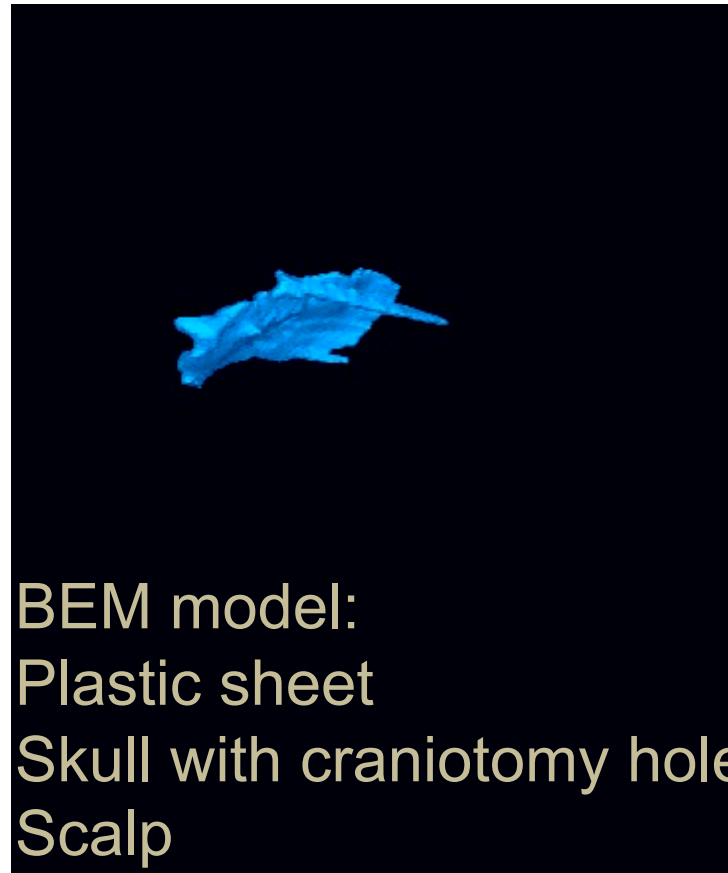
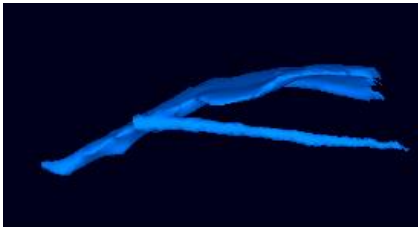
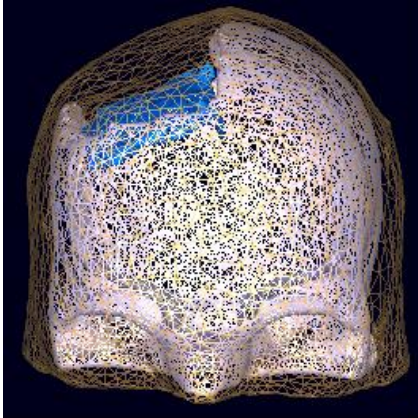
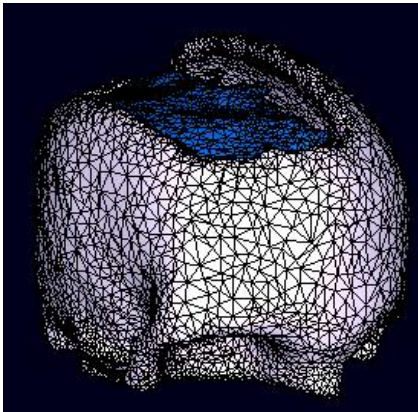
iEEG data



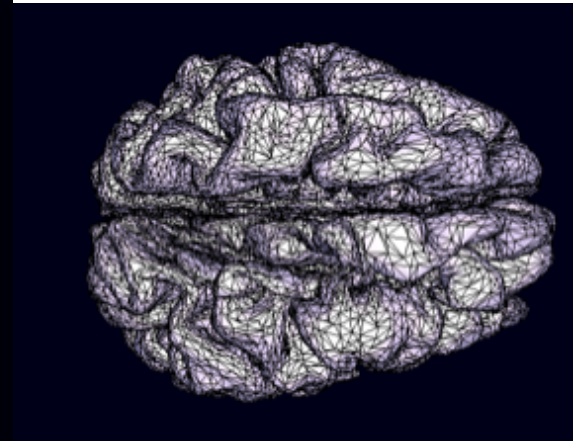
Project Summary



Forward modeling

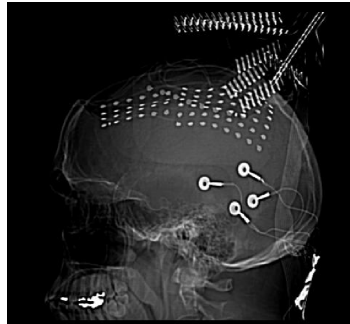
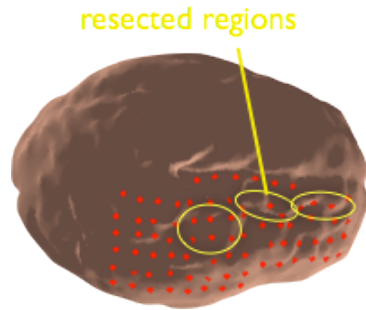


Cortex (Freesurfer)



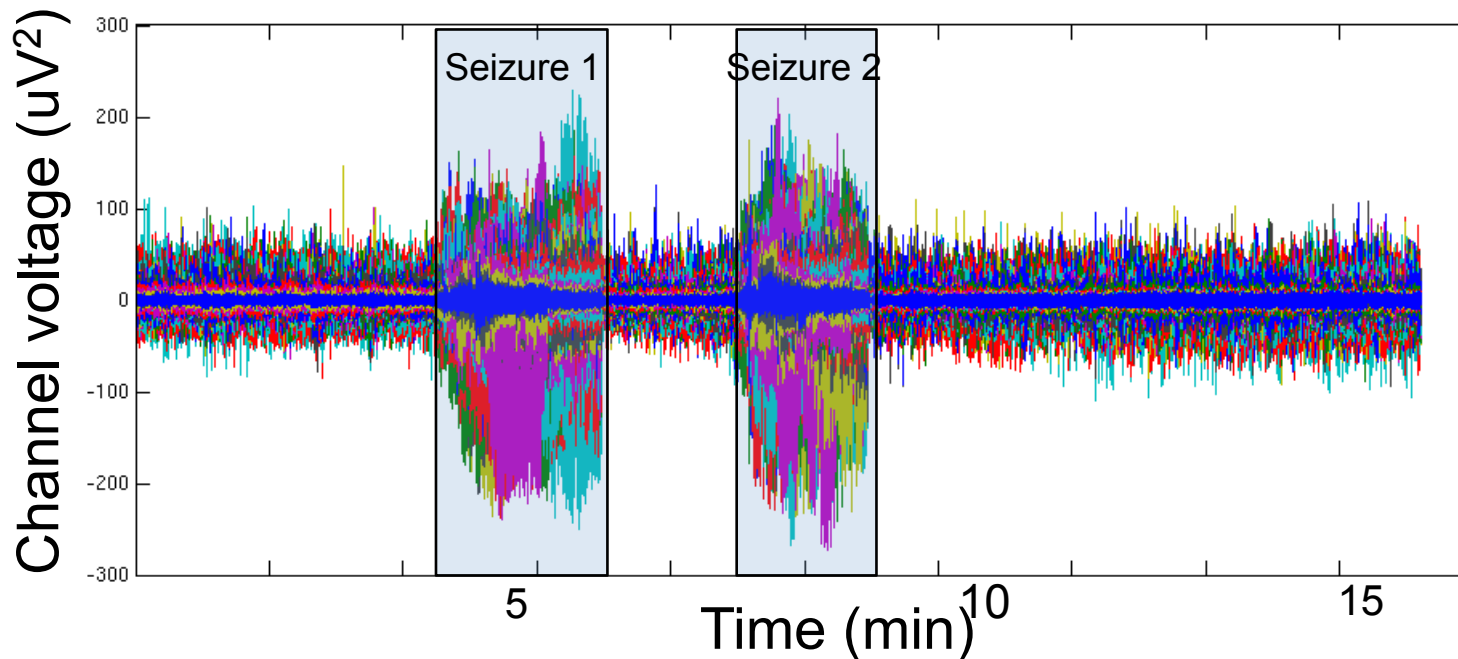
80 000 source vertices

Analyzing Epilepsy Recordings

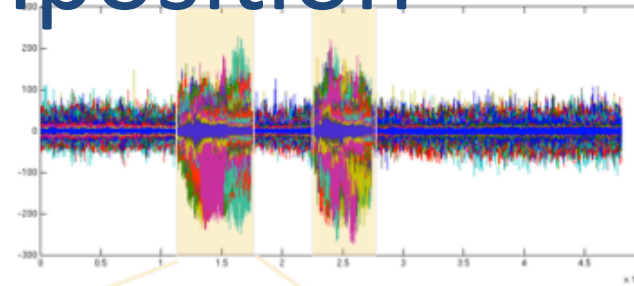
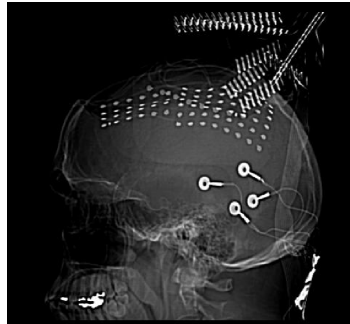
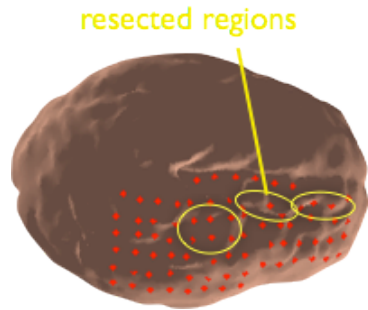


- ◆ Pre-Surgical Evaluation
- ◆ Rest Data
- ◆ 78 ECoG (subdural EEG) electrodes
- ◆ 29 scalp electrodes
- ◆ Surgical Outcome: Positive (seizure free)
- ◆ Provided by Dr. Greg Worrell, Mayo Clinic

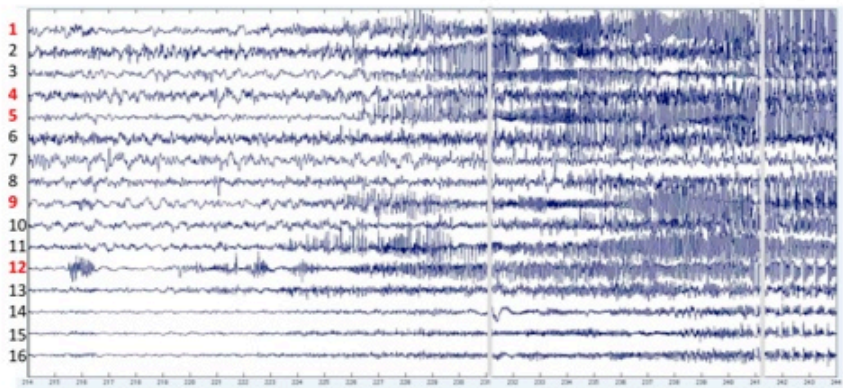
16 min of data, 2 seizures



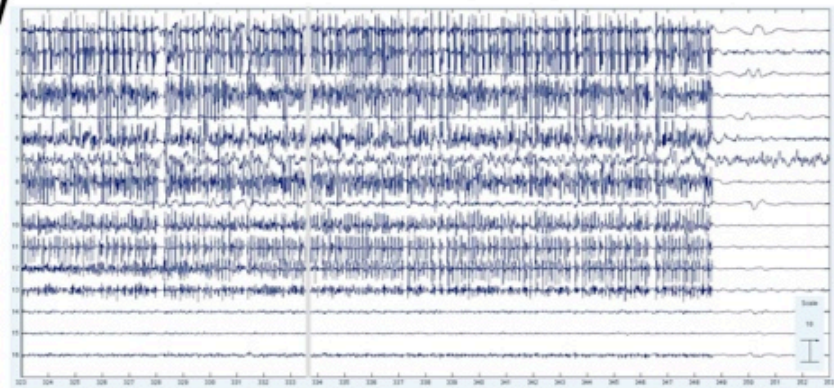
ICA decomposition



Extended Infomax ICA Decomposition
16 seizure components (ICs) selected

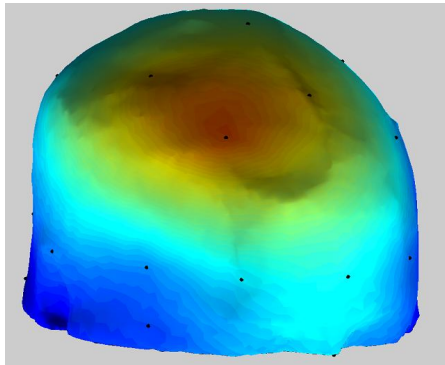


//

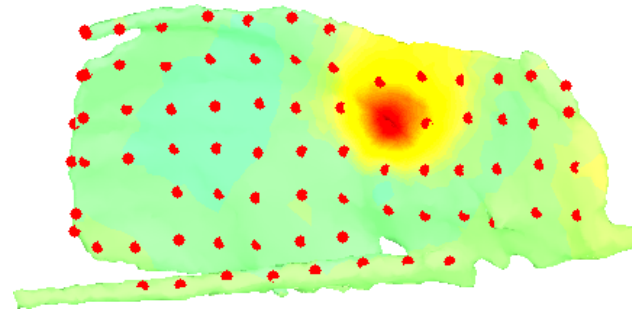


Independent Components

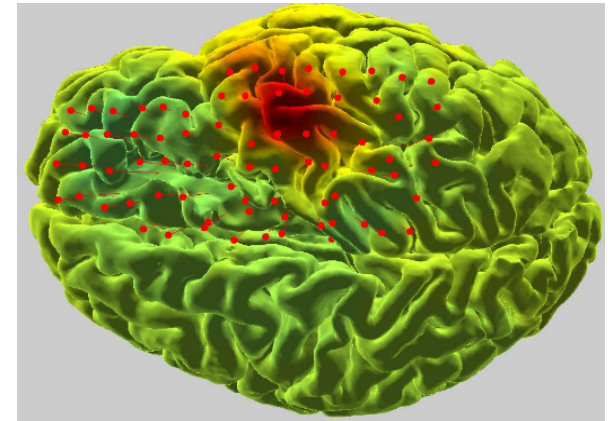
IC 1



Potentials on scalp

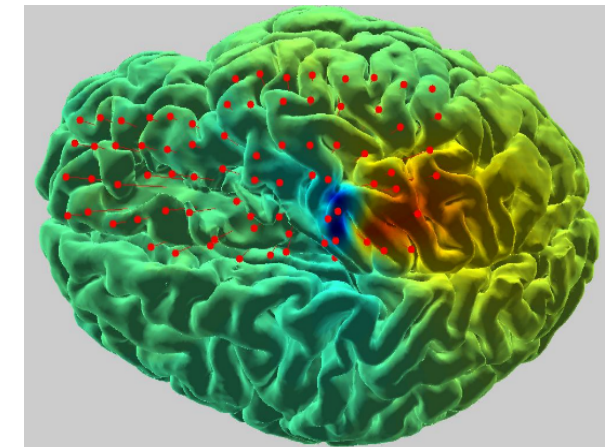
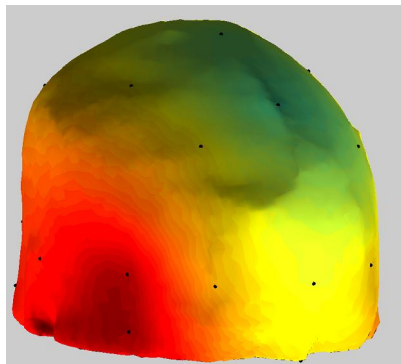
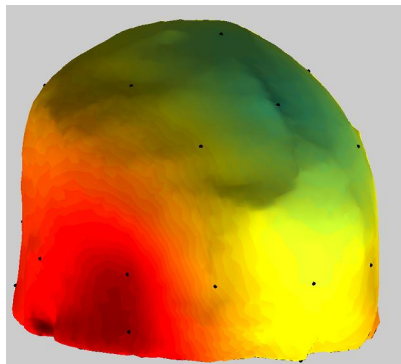


Potentials on plastic sheet



On the brain surface

IC 2

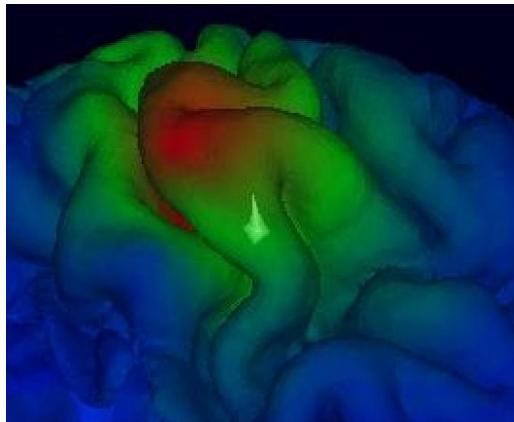


Source Localization Results

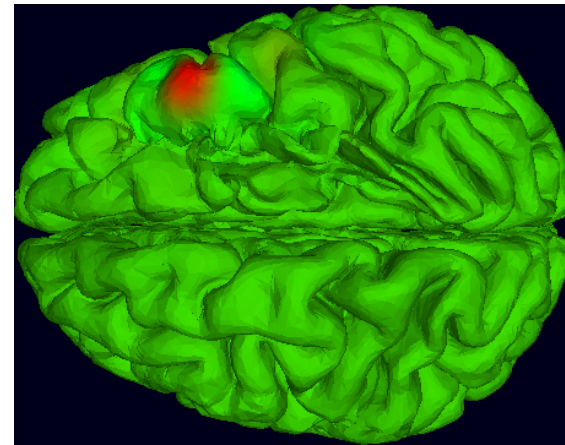
Dipole source localization

Distributed source localization - SBL

IC 1

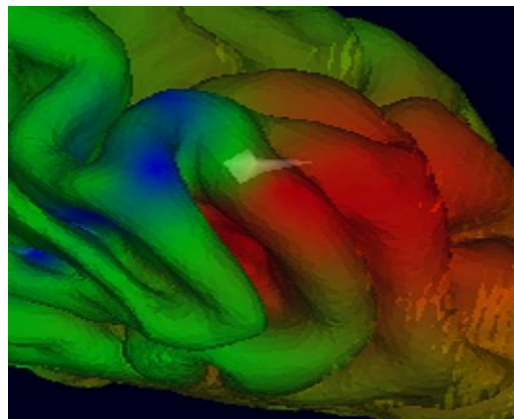


Radial source

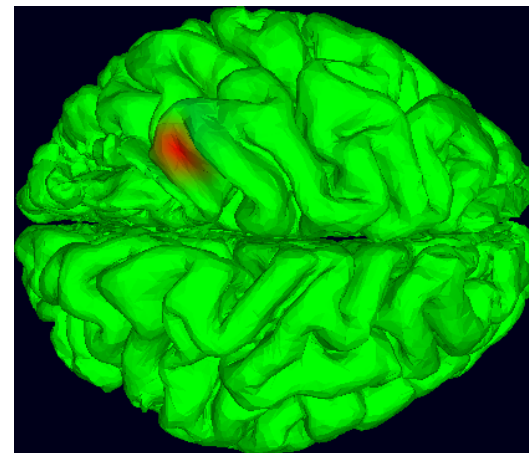


Gyral source

IC 2



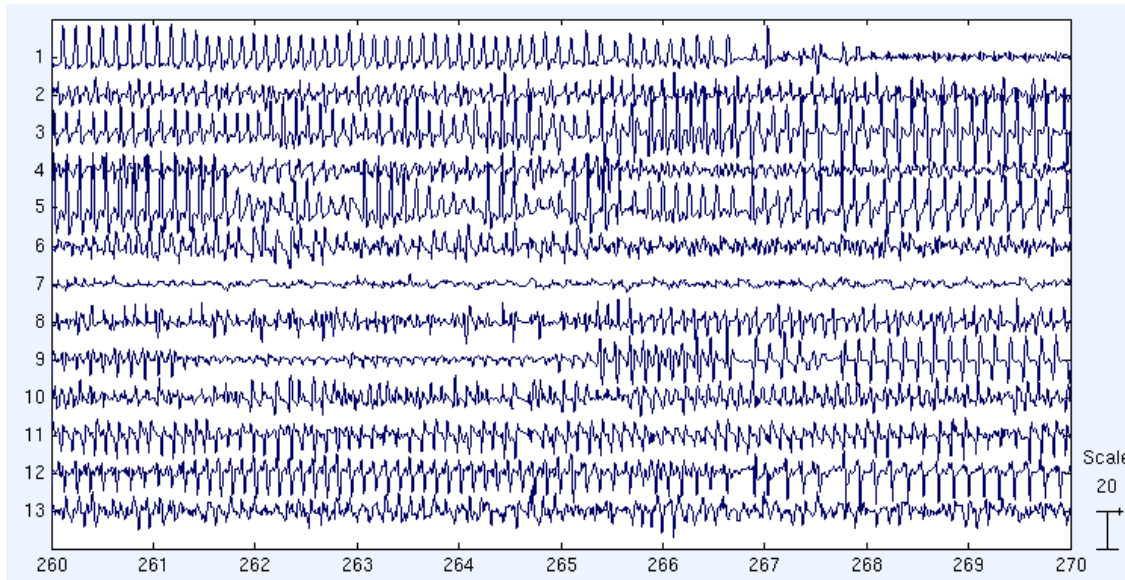
Tangential source



Sulcal source

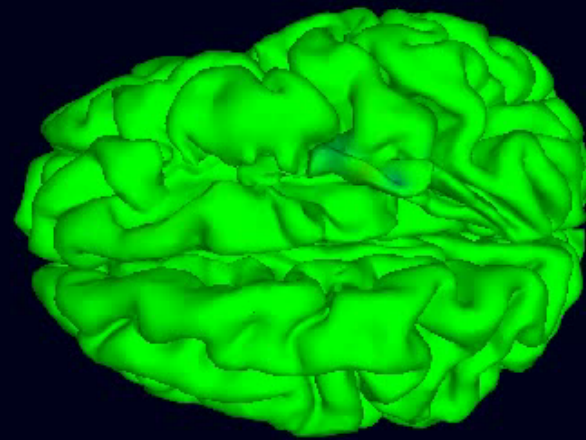
Cortical activity of seizure components

Activations of 13 seizure components



Cortical activity of
Seizure components

$$Movie(t) = \sum_{i=1}^{13} S_i \times Act_i(t)$$



Thank you...

Swartz Center for Computational Neuroscience



Algebraic formulation of the FP

Scalp potentials for **N electrodes** and **p dipoles**:

$$V(r) = \sum_i^p g(r, r_{dip}, d_i) = \sum_i^p g(r, r_{dip}, e_{d_i}) d_i$$

$$V = \begin{bmatrix} V(r_1) \\ \vdots \\ V(r_N) \end{bmatrix} = \begin{bmatrix} g(r_1, r_{dip}, e_{d1}) & \cdots & g(r_1, r_{dip}, e_{dp}) \\ \vdots & \ddots & \vdots \\ g(r_N, r_{dip}, e_{d1}) & \cdots & g(r_N, r_{dip}, e_{dp}) \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix} = G(\{r_j, r_{dip_i}, e_{d_i}\}) \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}$$

For **N electrodes** and **p dipoles** and **T discrete time samples**:

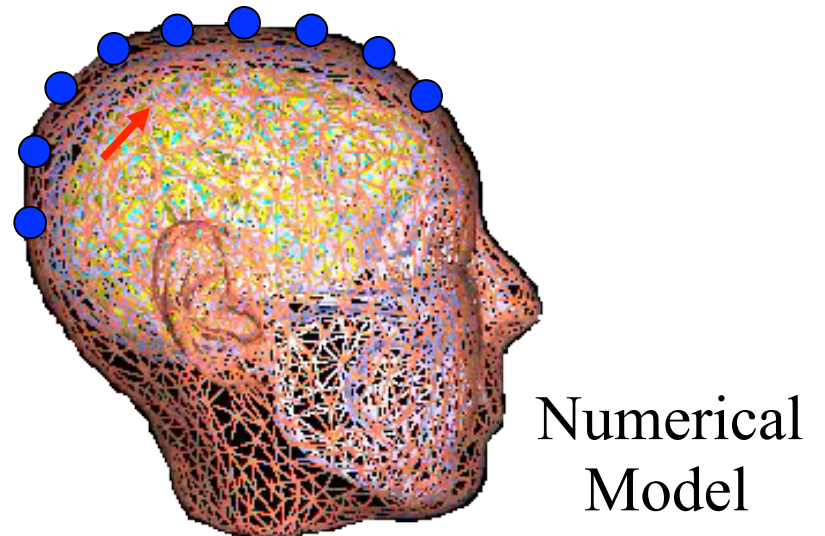
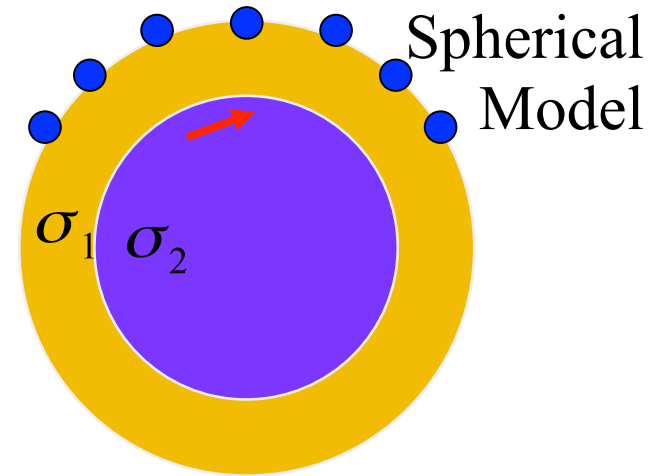
$$V = \begin{bmatrix} V(r_1, 1) & \cdots & V(r_1, T) \\ \vdots & \ddots & \vdots \\ V(r_N, 1) & \cdots & V(r_N, T) \end{bmatrix} = G(\{r_j, r_{dip_i}, e_{d_i}\}) \begin{bmatrix} d_{1,1} & \cdots & d_{1,T} \\ \vdots & \ddots & \vdots \\ d_{p,1} & \cdots & d_{p,T} \end{bmatrix}$$

$$V = GD + n$$

To Solve the Forward Problem

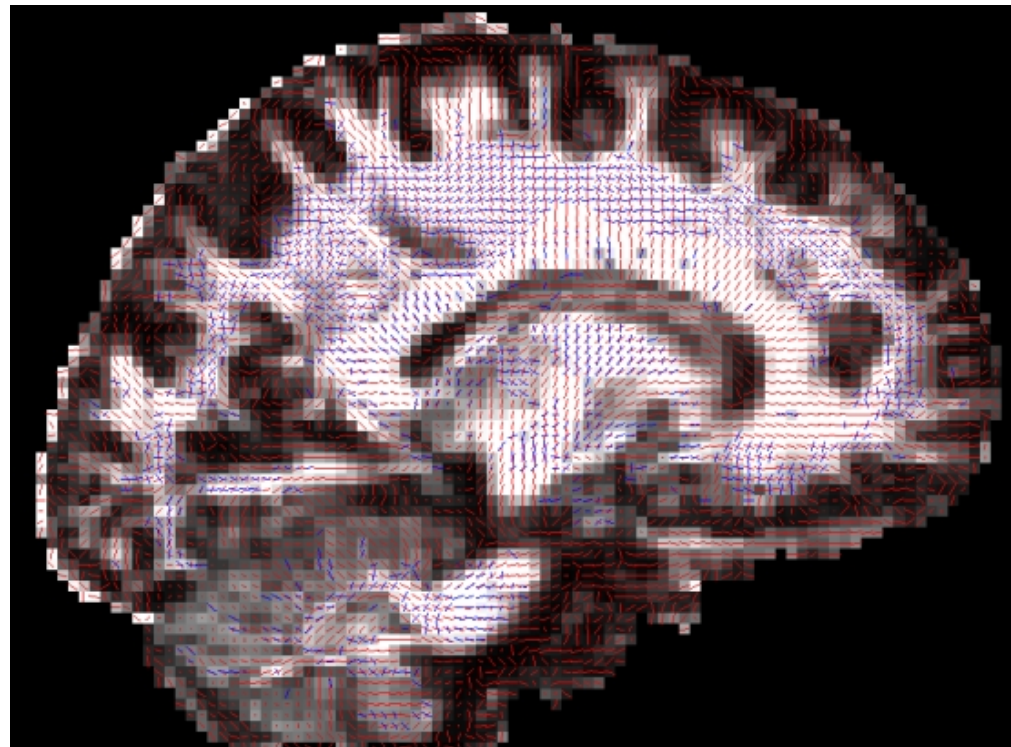
WE NEED

- ◆ Head Model
 - Conductivity values
 - Geometry
- ◆ Source distribution
 - Magnitude
 - Location
 - Direction
- ◆ Field Locations
- ◆ Solver

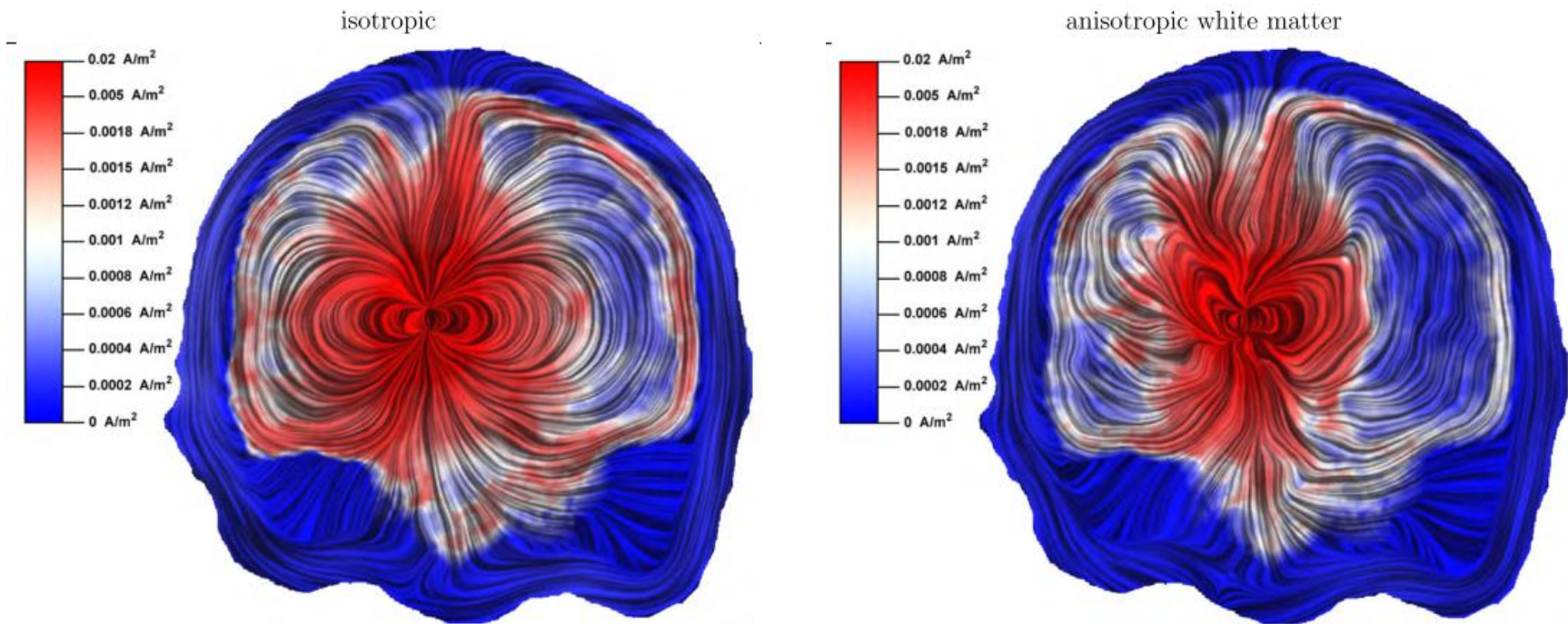


Anisotropy

- ◆ Directional conductivity for skull and WM.
- ◆ WM anisotropy can be obtained from diffusion tensor imaging (DTI).
- ◆ WM
anisotropy
ratio = 9:1
- ◆ Skull
ratio = 10:1

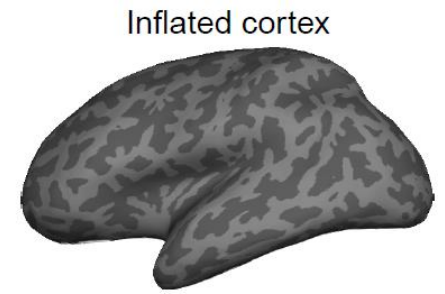


Anisotropy



Return currents for a left thalamic source on a coronal cut
Wolters et al, 2006

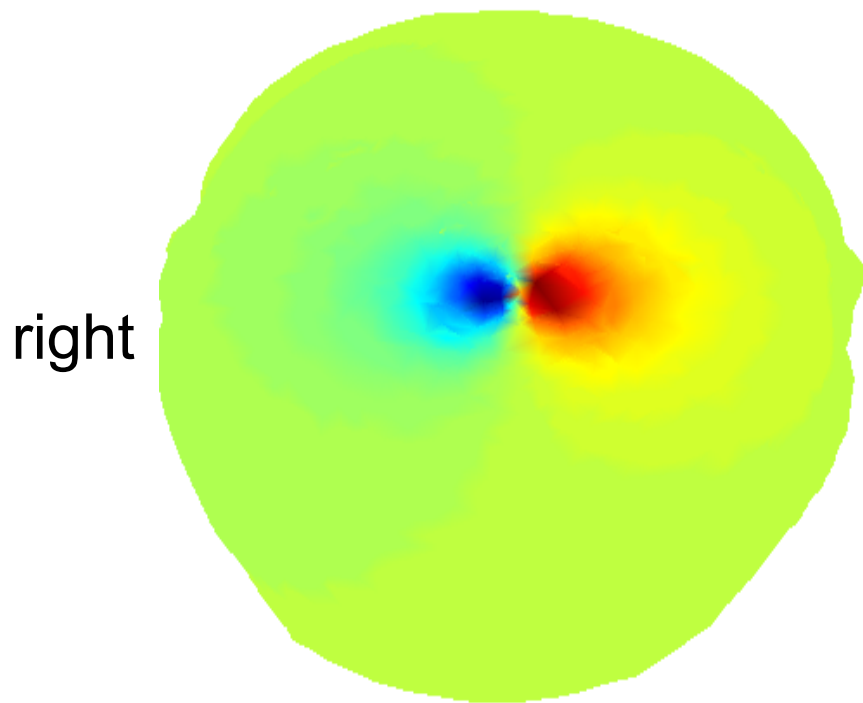
Potential fields on the scalp



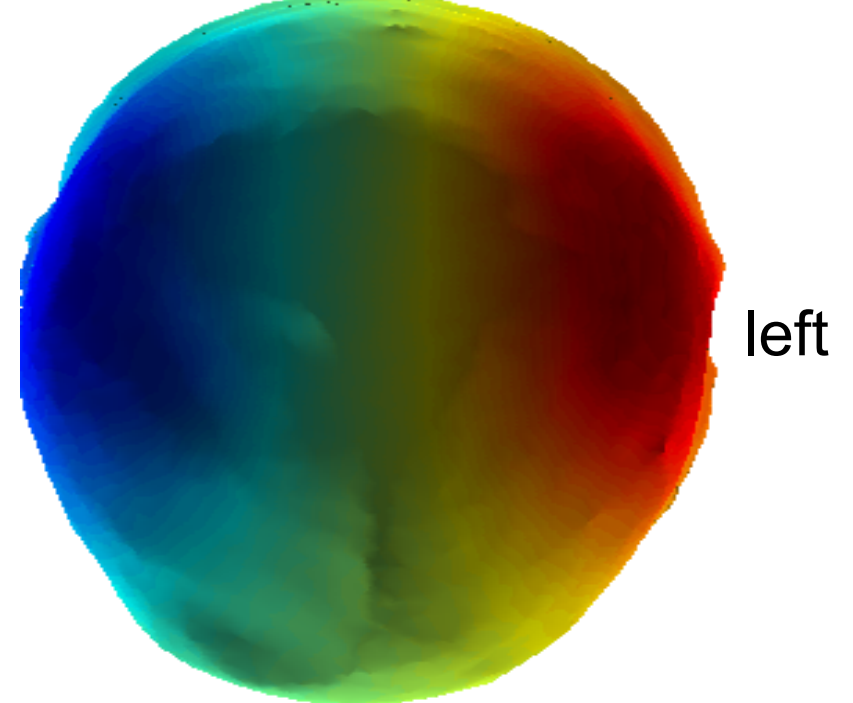
Shallow tangential source



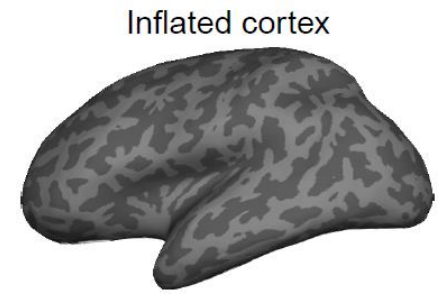
Deep tangential source



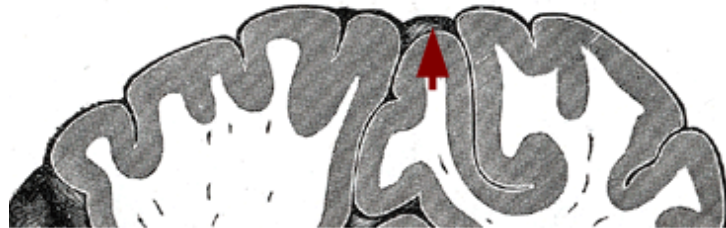
front top view of head



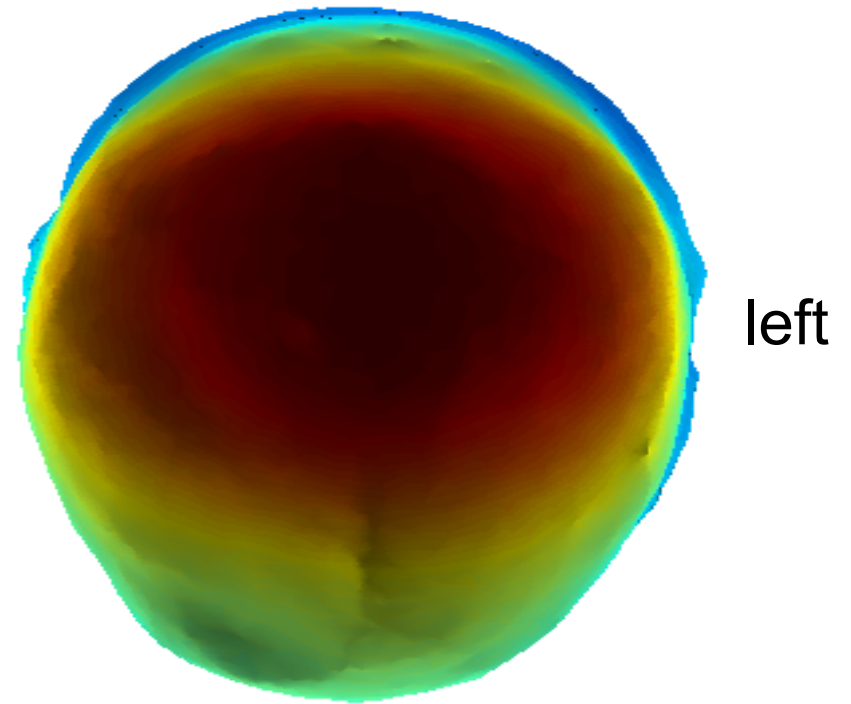
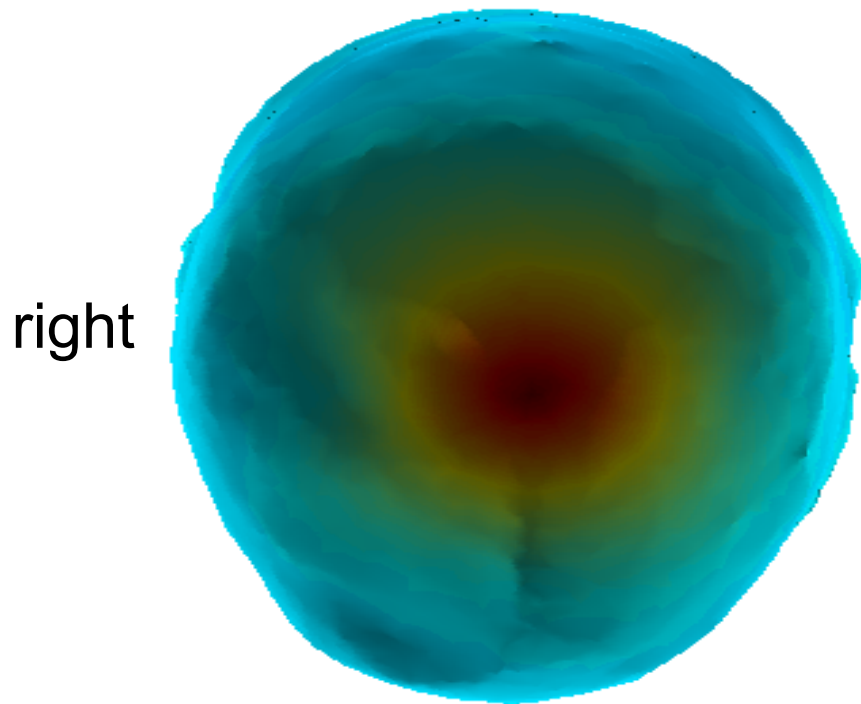
Potential fields on the scalp



Shallow radial source



Deep radial source



top view of head