BEENG280A, Principles of Biomedical Imaging 10/10/12
Fall Quarter 2012

HOMEWORK #2
Due at 5 pm on Wednesday 10/17/12

Homework policy: Homeworks can be turned in during class or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material.

Readings: Skim sections 2.1, 2.2, and 3.1-3.5 (read this for general understanding, you can skip pages 32-34). Read sections: 4.1-4.5, 5.1-5.5, 5.6.2. View the MRI safety video on the website.

Problem 1
From the safety video, answer the following questions: (a) What are helium and nitrogen used for in the MRI system? (b) What does the term quench mean? (c) Why is it dangerous to smoke near an MRI system? Find a example (on the web) of a large object that's been pulled into the magnet and include a copy of the image.

Problem 2

a) Prove the 1D version of the Hermitian Symmetry property (Equation 2.8 in the book).
b) Explain why this property makes sense (Hint: think about how much information is in each domain).
c) Prove the 2D version of the Hermitian symmetry property $F^*(-k_x,-k_y) = F(k_x,k_y)$ where the underlying object *f(x,y)* is real and * denotes complex conjugation.
d) Show that the Hermitian symmetry property holds for the function $\cos(2\pi(x+2y)+5\pi/4)$. In addition, sketch the function and its Fourier transform

Problem 3

a) Derive the Fourier shift property $F[g(x-a,y-b)] = G(k_x,k_y)e^{-j2\pi(k_x a+k_y b)}$
b) Sketch the object $m(x,y) = rect(x+1,y+1) - rect(x-1,y-1)$
c) Use the Fourier shift property to derive the Fourier transform of the object. Make sure to simplify your expression.
d) Sketch the Fourier transform – indicate key features (e.g. peaks and valleys) and find the spacing between the peaks.
e) Evaluate the expression for the Fourier transform at the following coordinates in k-space: $(0,0)$, $(1,-1)$, and $\left(\frac{1}{8},\frac{1}{8}\right)$. For each point, draw quiver diagrams (you can do this by hand or with MATLAB) and explain how these are consistent with the value of the Fourier transform.

MATLAB exercise on the next page

Problem 4 – MATLAB exercise
In this exercise we will examine the Fourier transforms of some simple test objects.

Part 1. Create a script with the following code

```
%Define a test object
span = -16:15;
nvox = length(span);
[x,y] = meshgrid(span,span);
obj = zeros(nvox,nvox);
obj([1:8 17:24],:) = 1;

%Define k-space coverage
dk = 1/32;kmax = 0.5;
kspan = -kmax:dk:(kmax-dk);
Nk = length(kspan);
[kx,ky] = meshgrid(kspan,kspan);

%Brute-force computation of the Fourier Transform
% This is not the best way to compute it, but is helpful for showing the
% process
j = sqrt(-1);

for ix = 1:Nk
    for iy = 1:Nk;
        g = exp(-j*2*pi*(kx(ix,iy)*x + ky(ix,iy)*y));
        f = sum(sum(g.*obj)); % Fourier Transform
        fmat(ix,iy) = f;  %Store the Fourier transform values in fmat
    end;
end
```

Part 2. Use imagesc to plot images of the object and its Fourier transform.

Part 3. Find the 3 highest absolute values in the Fourier transform.

a) Where in k-space do these occur? (i.e. provide the values of kx and ky).
b) Draw the quiver diagrams showing orientation of phasors corresponding to each of these points in k-space.
c) Create a second set of quiver diagrams that takes into account the knowledge of the object (i.e. where the object is equal to zero, the product of the object and the phasor is zero). Based on the orientation of the phasors in the quiver diagrams and the knowledge of the object, provide an explanation of why the 3 highest values are observed at these points in k-space.
d) Compute and plot the vector sum of the phasors in the quiver diagrams from part c for each of the 3 points in k-space. Verify that the vector sum is equal to the value of the Fourier transform at the corresponding points in k-space.
e) Using the code loop above as a starting point, write MATLAB code to do a brute force computation of the inverse 2D Fourier transform and verify that the transform generates the original object. (HINTS: You may need to take the real part of your answer to account for numerical precision effects. You will also want to divide the sum by the total number of points (which is 1024 for this object)).

Part 4.

a) Now design an object where the highest absolute value occurs at the center of k-space (kx = 0, ky = 0) and the next 4 highest absolute values occur at the following points in k-space (-1/16,-1/16), (-1/16,1/16), (1/16,-1/16), and (1/16,1/16).
b) For each of the points in k-space: (1) plot the quiver diagrams showing orientation of phasors; (2) plot a second set of quiver diagrams that also take into account knowledge of the object; (3) compute and plot

the vector sum of the phasors from the second set of quiver diagrams and verify that it that it is equal to the Fourier transform at the corresponding k-space location.

c) Use your code to compute the inverse transform of the object and verify that you obtain the original object.