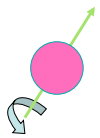
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2012
MRI Lecture 1

Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

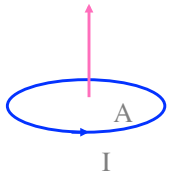
This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where γ is the gyromagnetic ratio and **S** is the spin angular momentum.

Nuclear Spin Rules

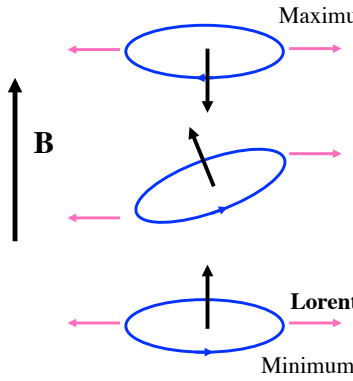
Number of Protons	Number of Neutrons	Spin	Examples
Even	Even	0	^{12}C, ^{16}O
Even	Odd	j/2	^{17}O
Odd	Even	j/2	^{1}H, ^{23}Na, ^{31}P
Odd	Odd	j	^{2}H

Classical Magnetic Moment



$$\vec{\mu} = IA\hat{n}$$

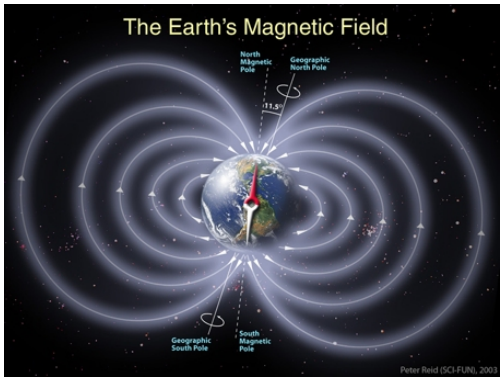
Energy in a Magnetic Field



$$E = -\vec{\mu} \bullet \mathbf{B}$$

$$= -\mu_z B$$

Energy in a Magnetic Field



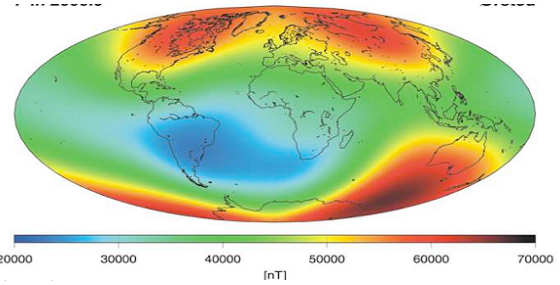
www.qi-whiz.com/images/ Earth-magnetic-field.jpg

Magnetic Field Units

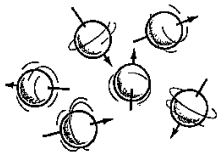
1 Tesla = 10,000 Gauss

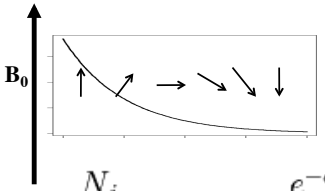
Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT



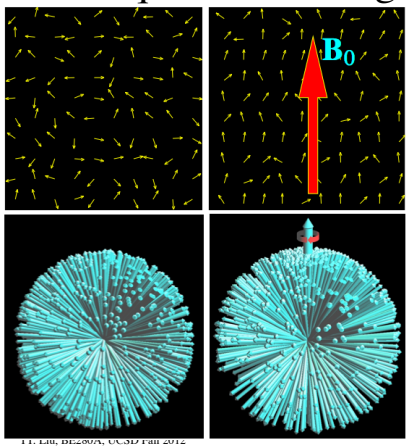
Boltzmann Distribution





$$\frac{N_j}{N} \equiv \mathcal{P}(\epsilon_j) = \frac{e^{-\epsilon_j/(kT)}}{Q}.$$

Equilibrium Magnetization



$$\mathbf{M}_0 = N\langle \mu_z \rangle$$
$$\approx N\mu_z^2 B/(kT)$$
$$= N\gamma^2\hbar^2 B/(4kT)$$

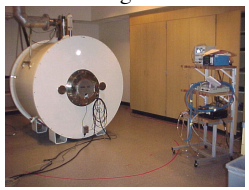
N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

Hansen 2009

Bigger is better

3T Human imager at UCSD.

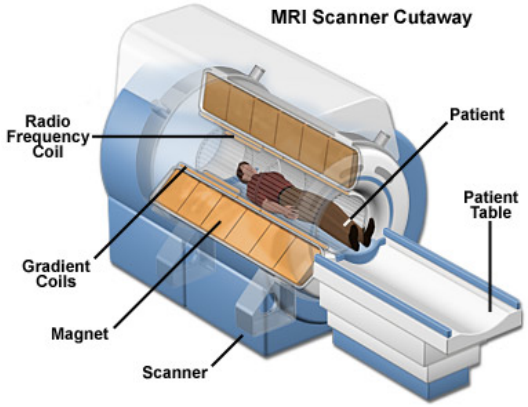
7T Human imager at U. Minn.



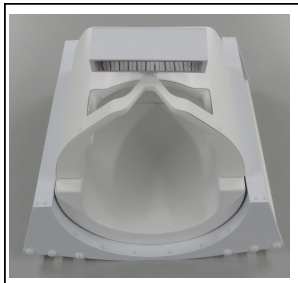
7T Rodent Imager at UCSD

9.4T Human imager at UIC

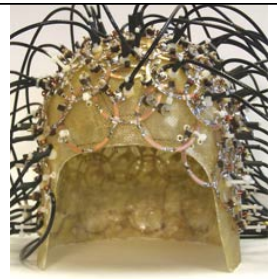
MRI System



http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/fullarticle.html

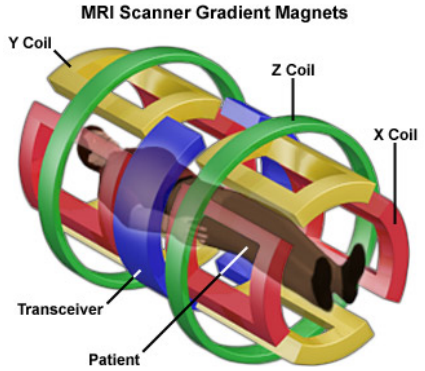


Nova 32 channel



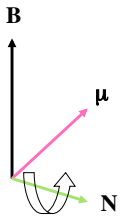
Siemens 32 channel

MRI Gradients



http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/fullarticle.html

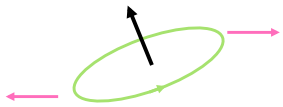
Torque



$\mathbf{N} = \boldsymbol{\mu} \times \mathbf{B}$

Torque

For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)



Precession

Torque

$\mathbf{N} = \boldsymbol{\mu} \times \mathbf{B}$

$\frac{d\mathbf{S}}{dt} = \mathbf{N}$

Change in Angular momentum

$\frac{d\mathbf{S}}{dt} = \boldsymbol{\mu} \times \mathbf{B}$

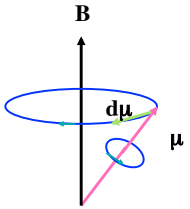
$\boldsymbol{\mu} = \gamma \mathbf{S}$

Relation between magnetic moment and angular momentum

$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \mathbf{B}$

Precession

$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \mathbf{B}$

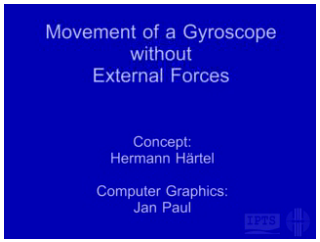


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$\omega = \gamma B$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertelmpg_e/gyros_free.mpg

Magnetization Vector

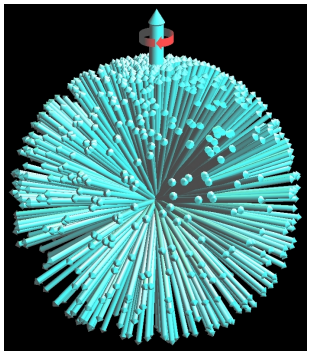
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

$$\mathbf{M} = \frac{1}{V} \sum_{\text{protons in V}} \mu_i$$

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$



Hansen 2009

Gyromagnetic Ratios

Nucleus	Spin	Magnetic Moment	$\gamma/(2\pi)$ (MHz/Tesla)	Abundance
^{1}H	1/2	2.793	42.58	88 M
^{23}Na	3/2	2.216	11.27	80 mM
^{31}P	1/2	1.131	17.25	75 mM

Source: Haacke et al., p. 27

Larmor Frequency

$\omega = \gamma B$ — Angular frequency in rad/sec

$f = \gamma B / (2\pi)$ — Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Notation and Units

1 Tesla = 10,000 Gauss

Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

$\gamma = 26752$ radians/second/Gauss

$\gamma = \gamma / 2\pi = 4258$ Hz/Gauss

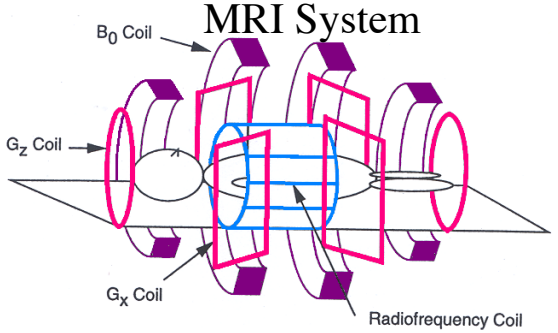
$= 42.58$ MHz/Tesla

Gradients

Spins precess at the Larmor frequency, which is proportional to the local magnetic field. In a constant magnetic field $B_z=B_0$, all the spins precess at the same frequency (ignoring chemical shift).

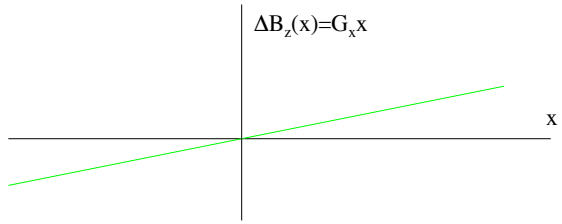
Gradient coils are used to add a spatial variation to B_z such that $B_z(x,y,z) = B_0+\Delta B_z(x,y,z)$. Thus, spins at different physical locations will precess at different frequencies.

MRI System



Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_0,
gradient fields (two of three shown),
and radiofrequency field B_1.

Interpretation



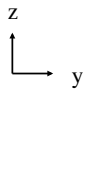
Spins Precess at $\gamma B_0 - \gamma G_x x$ (slower)

Spins Precess at γB_0

Spins Precess at at $\gamma B_0 + \gamma G_x x$ (faster)

Gradient Fields

$$B_z(x,y,z) = B_0 + \frac{\partial B_z}{\partial x}x + \frac{\partial B_z}{\partial y}y + \frac{\partial B_z}{\partial z}z$$
$$= B_0 + G_x x + G_y y + G_z z$$



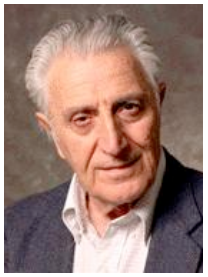
$$G_z = \frac{\partial B_z}{\partial z} > 0$$

$$G_y = \frac{\partial B_z}{\partial y} > 0$$

Rotating Frame of Reference

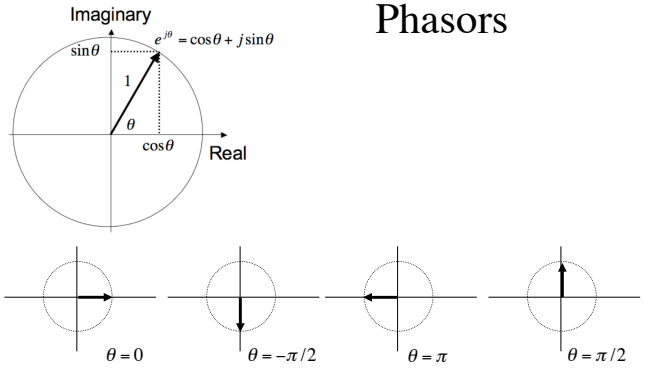
Reference everything to the magnetic field at isocenter.

Spins



There is nothing that nuclear spins will not do for you, as long as you treat them as human beings.
Erwin Hahn

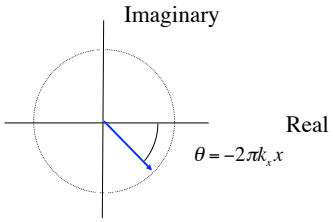
Phasors



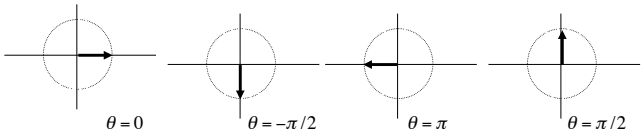
Phasor Diagram

$$G(k_x) = \int_{-\infty}^{\infty} g(x)\exp(-j2\pi k_x x)dx$$

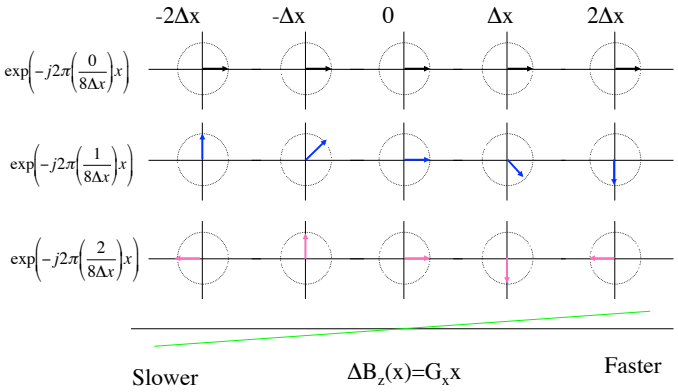
$\theta = -2\pi k_x x$

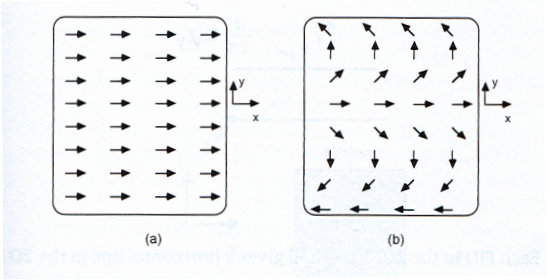


$k_x = 1;\ x = 0$	$x = 1/4$	$x = 1/2$	$x = 3/2$
$2\pi k_x x = 0$	$2\pi k_x x = \pi/2$	$2\pi k_x x = \pi$	$2\pi k_x x = 3\pi/4$



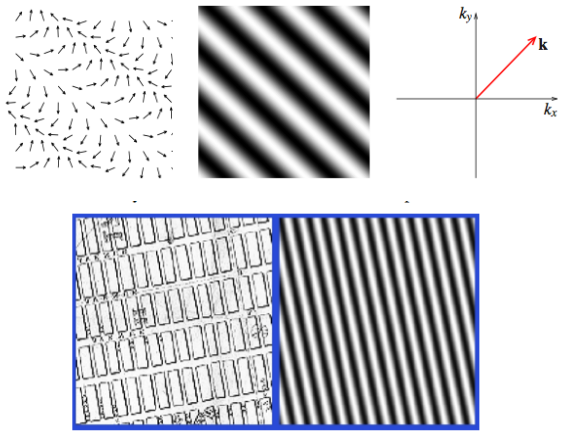
Interpretation



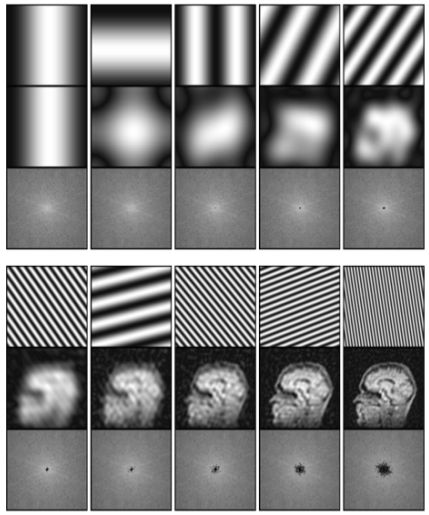


$k_x=0;\ k_y=0$ $\qquad$ $k_x=0;\ k_y\neq 0$

Fig 3.12 from Nishimura

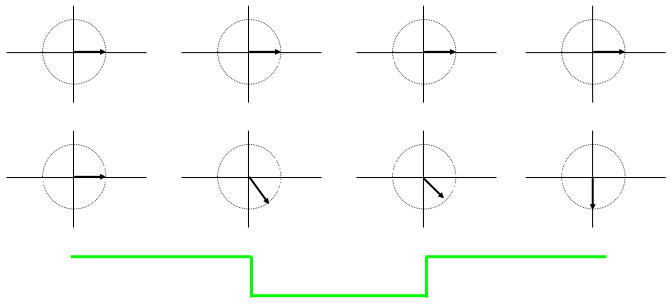


Hanson 2009

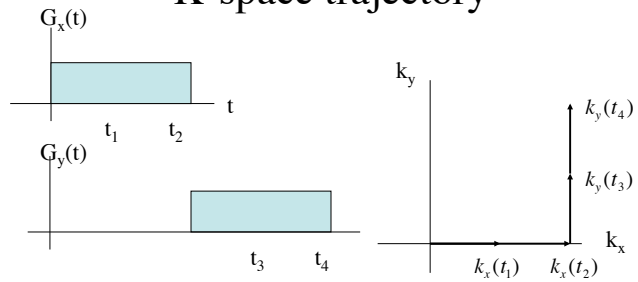


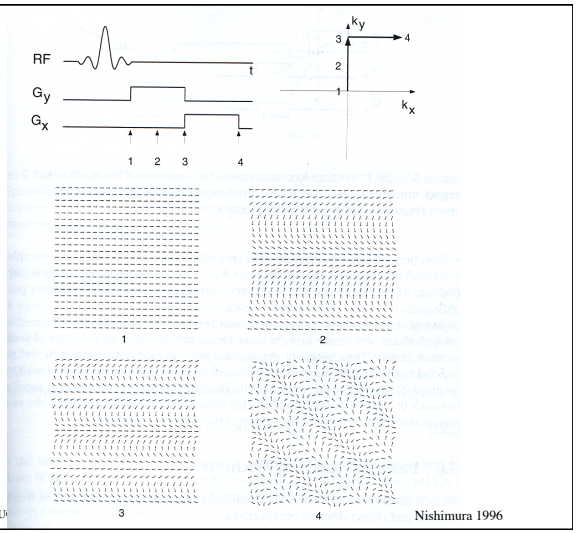
Hanson 2009

Phase with time-varying gradient



K-space trajectory

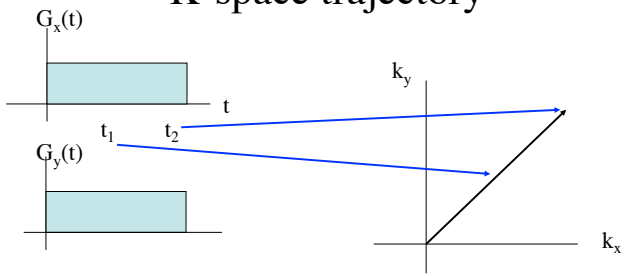




TT. Liu, BE280A, U

Nishimura 1996

K-space trajectory



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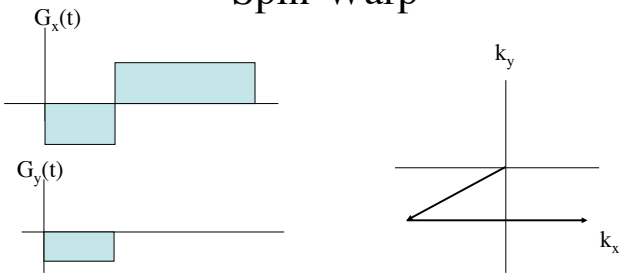
Spin-Warp



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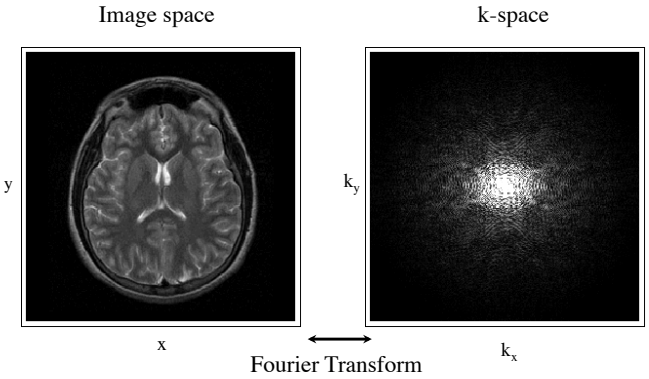
k-space



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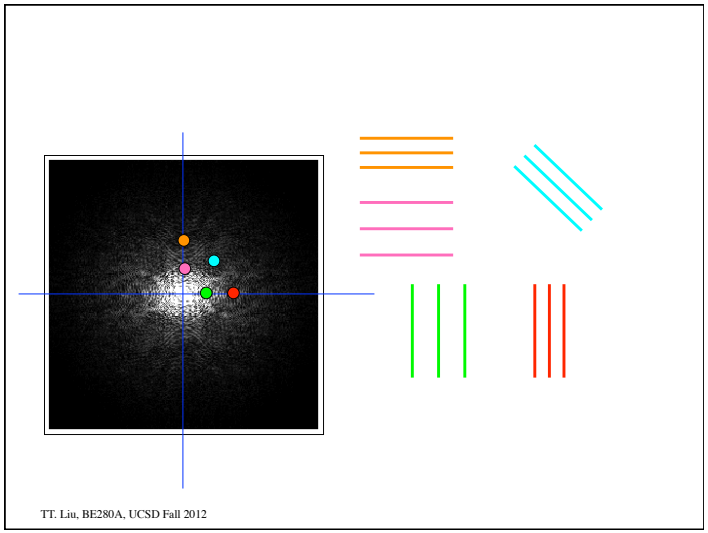
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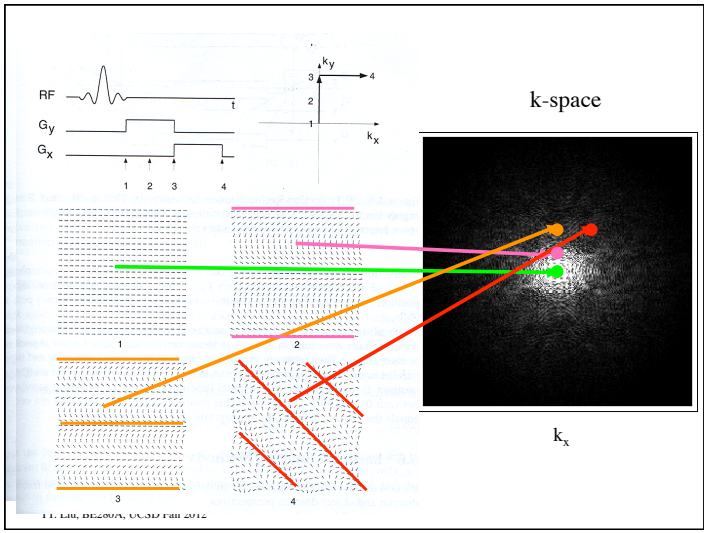
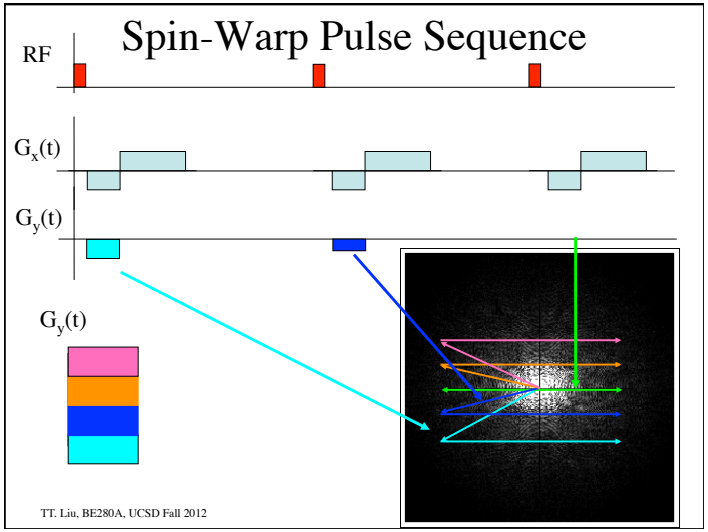

RF
G_y
G_x
t
k_y
k_x
1 2 3 4
k-space
k_x


Spin-Warp Pulse Sequence
RF
$G_x(t)$
$G_y(t)$
$G_y(t)$
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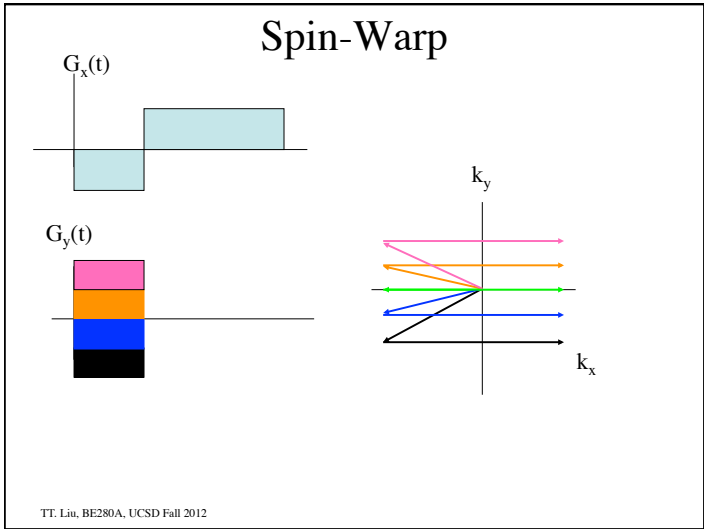
Spin-Warp
$G_x(t)$
$G_y(t)$
k_y
k_x
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