Diffusion Tensor Imaging

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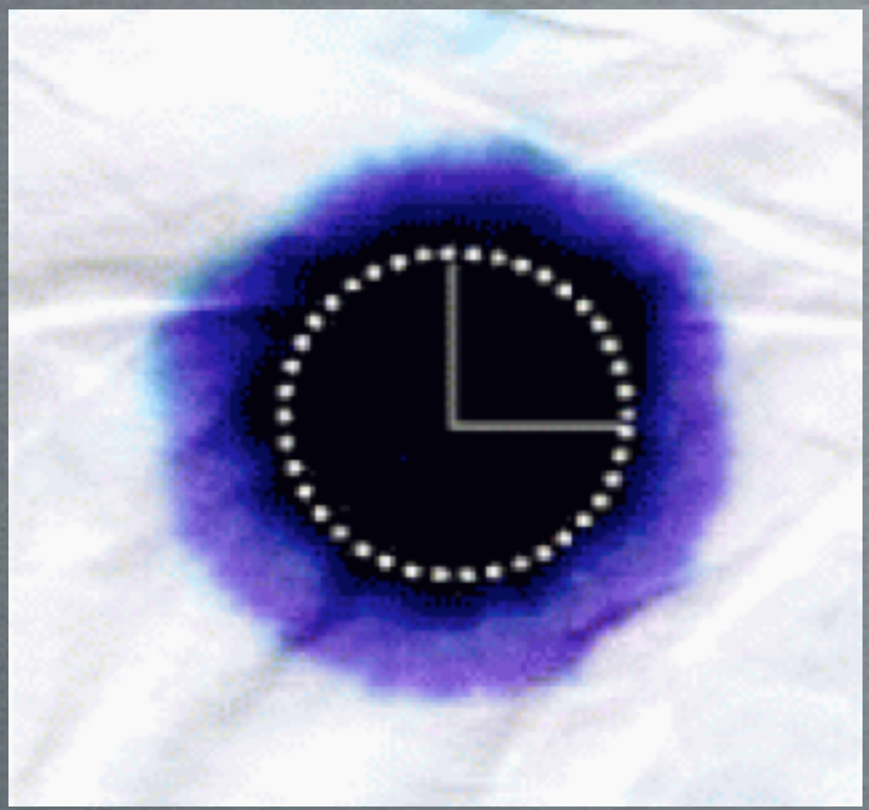
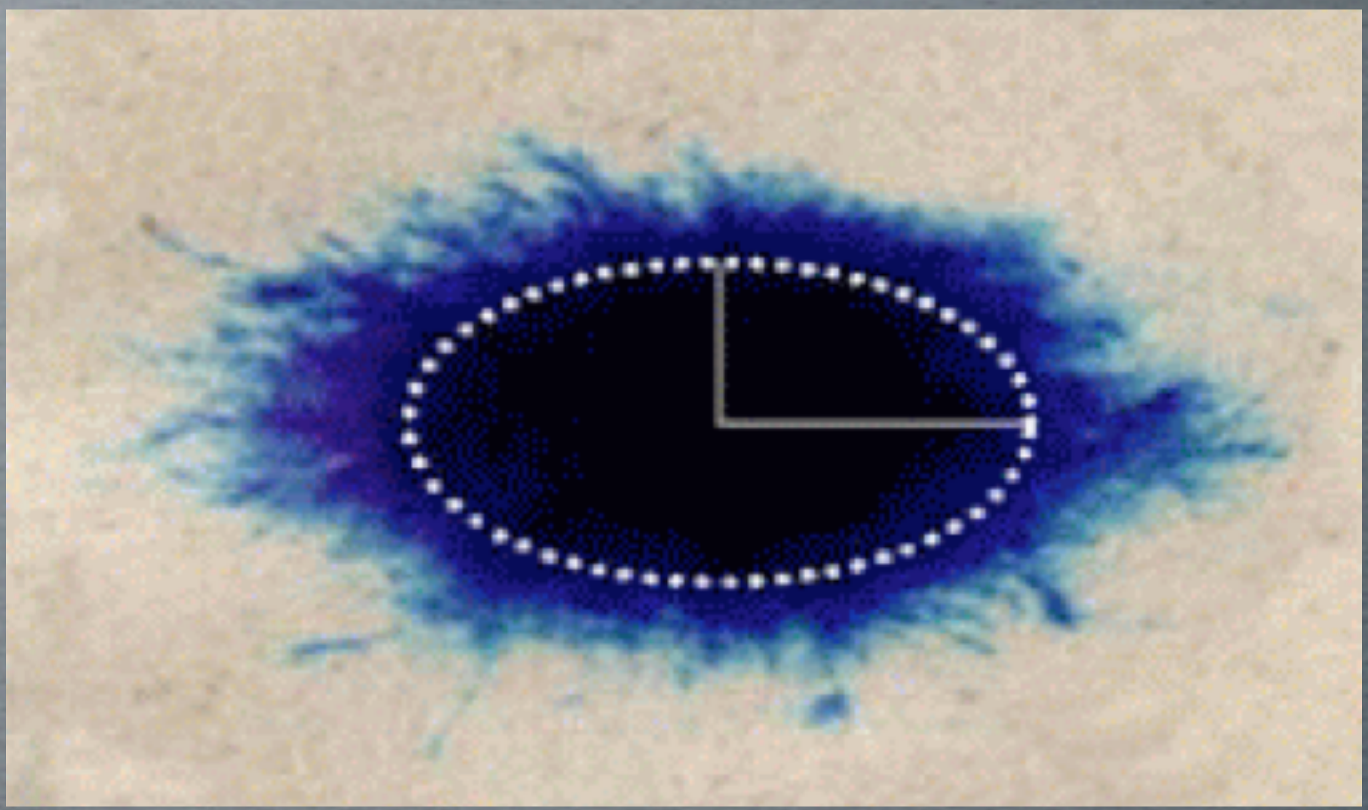
Macroscopic Information from Microscopic Measurements



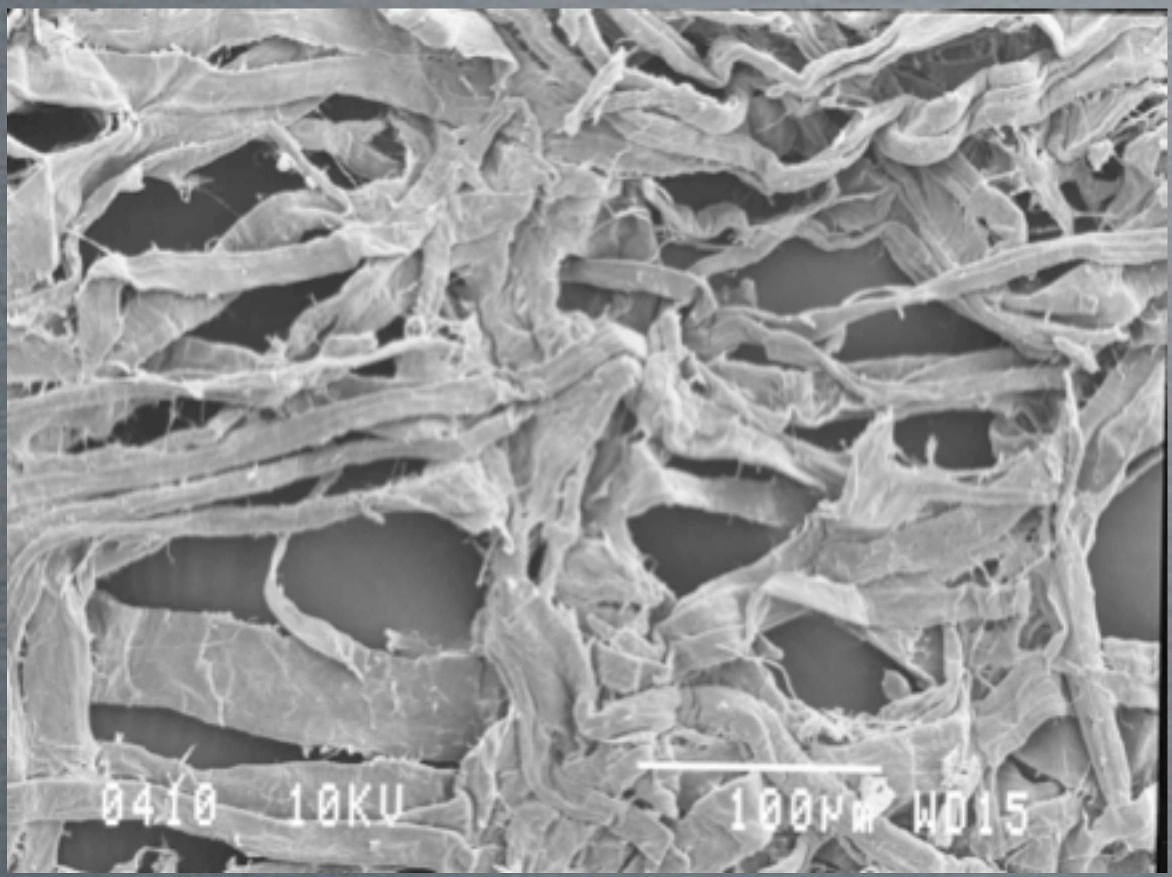
tissue paper

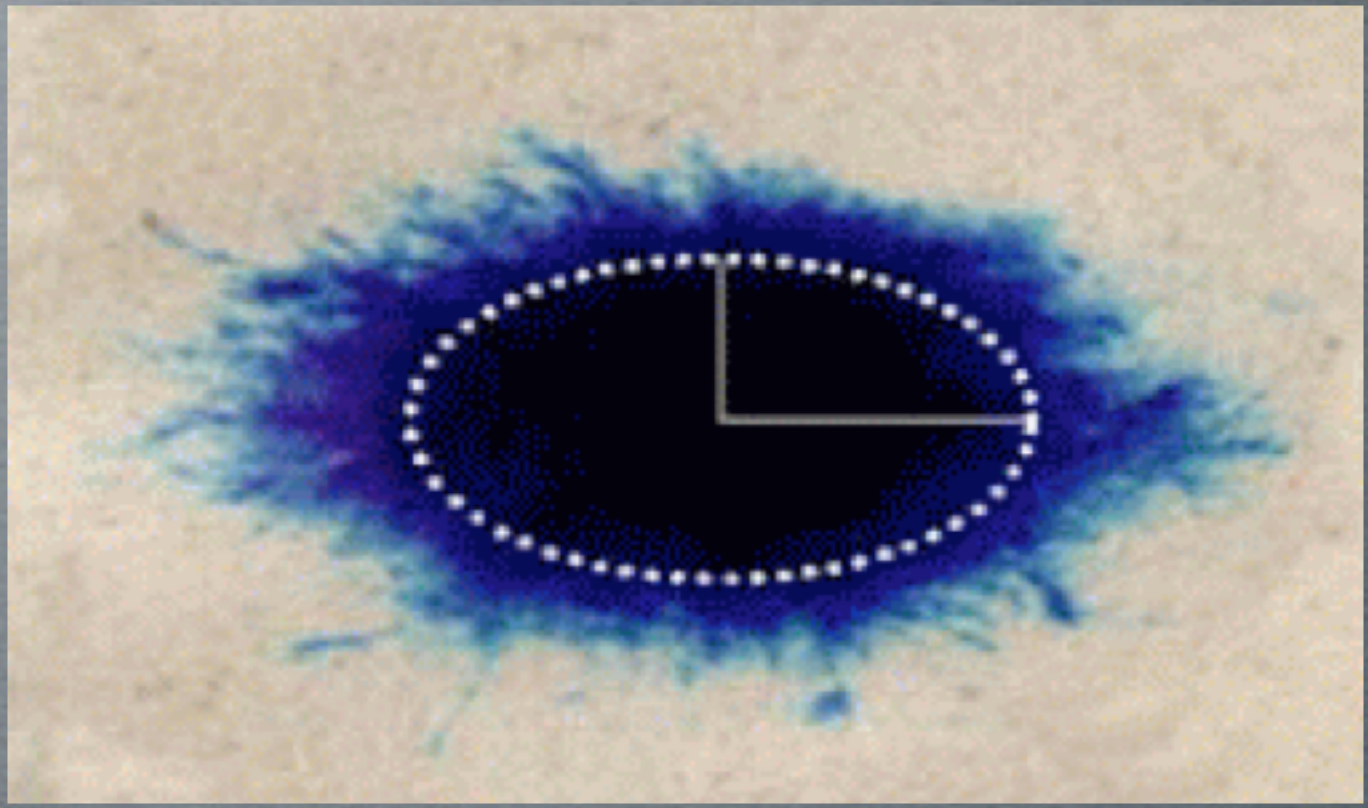


newspaper

diffusing ink in paper

Macroscopic Information from Microscopic Measurements





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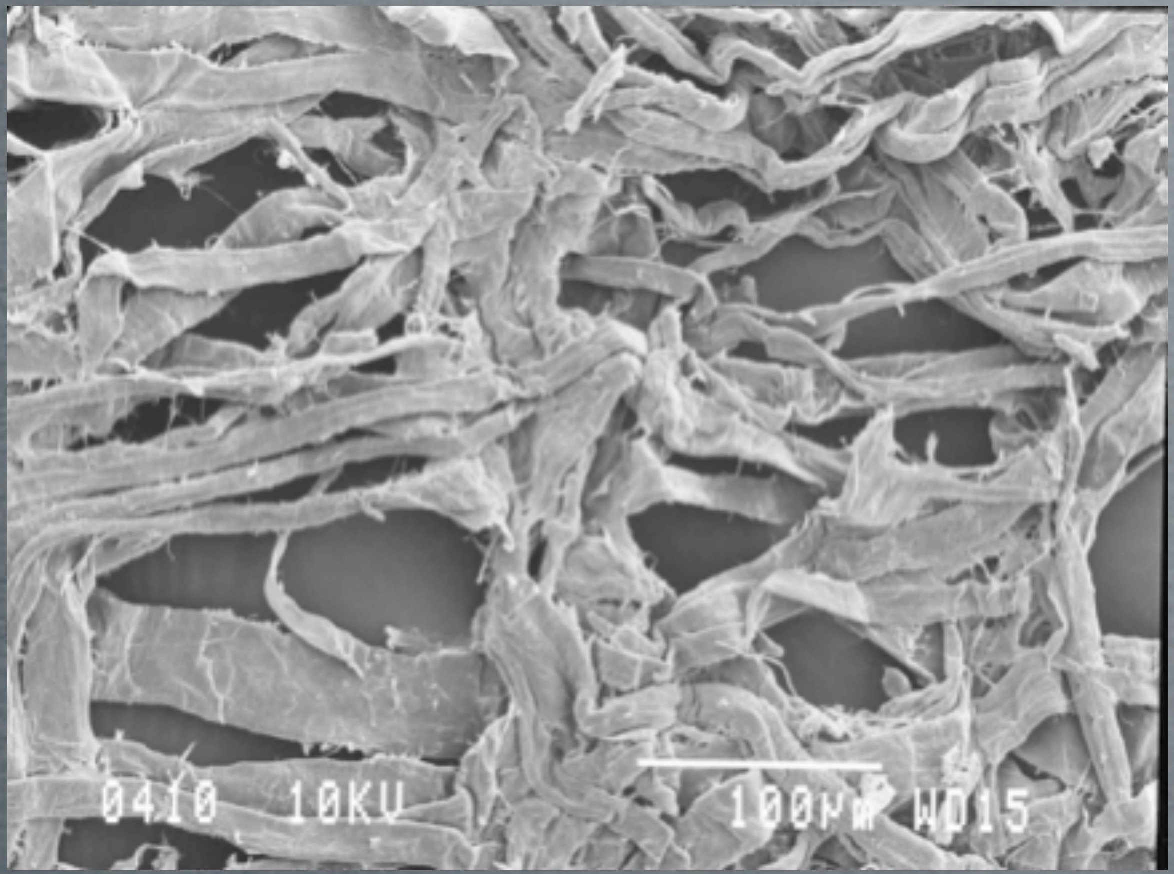
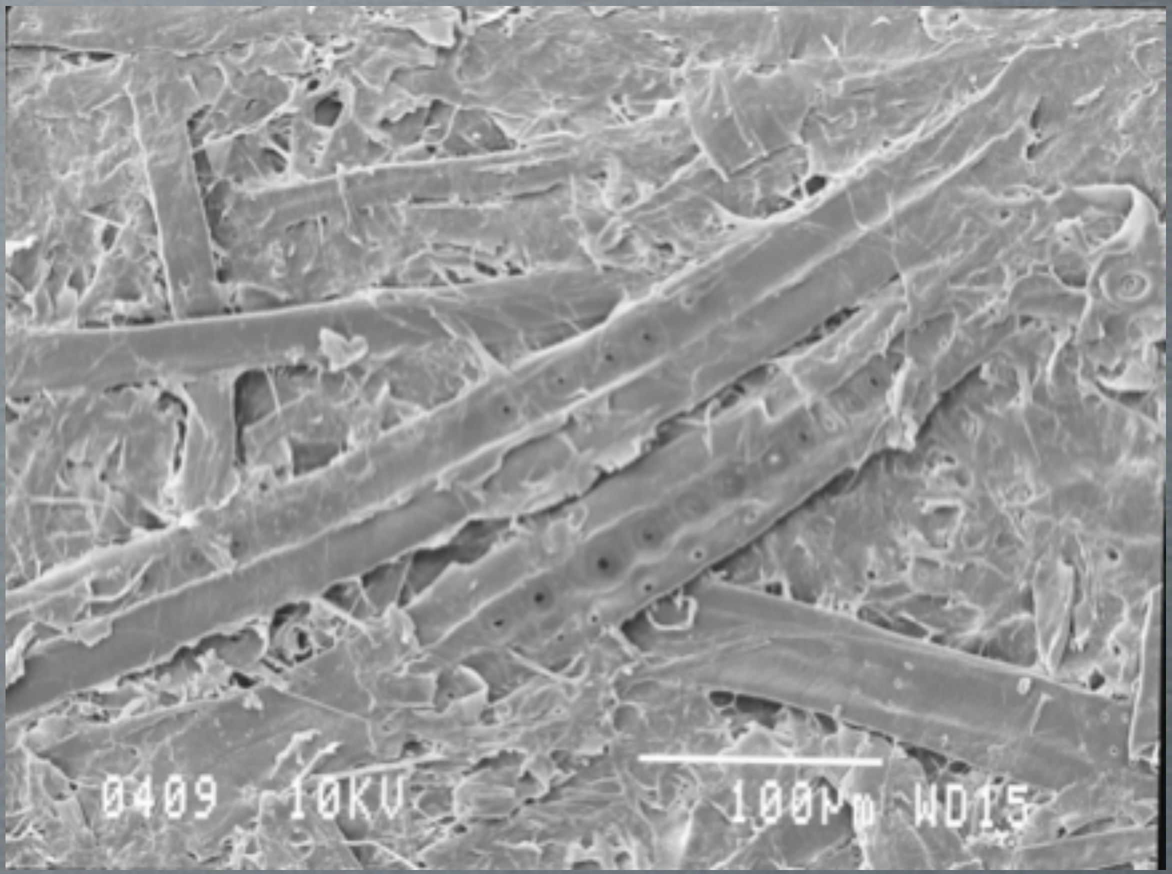
Macroscopic Information from Microscopic Measurements





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What is diffusion and why do we care about it?

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Self-diffusion is the thermally driven random motions of molecules that occurs in the absence of a concentration gradient

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The self-diffusion of water is ongoing in the human body and its characteristics depend on the local tissue architecture and physiology

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Self-diffusion is the thermally driven random motions of molecules that occurs in the absence of a concentration gradient

The self-diffusion of water is ongoing in the human body and its characteristics depend on the local tissue architecture and physiology

Therefore the ability to measure self-diffusion offers the possibility of non-invasively measuring tissue structure and physiology

A Brief History of Diffusion Measurement

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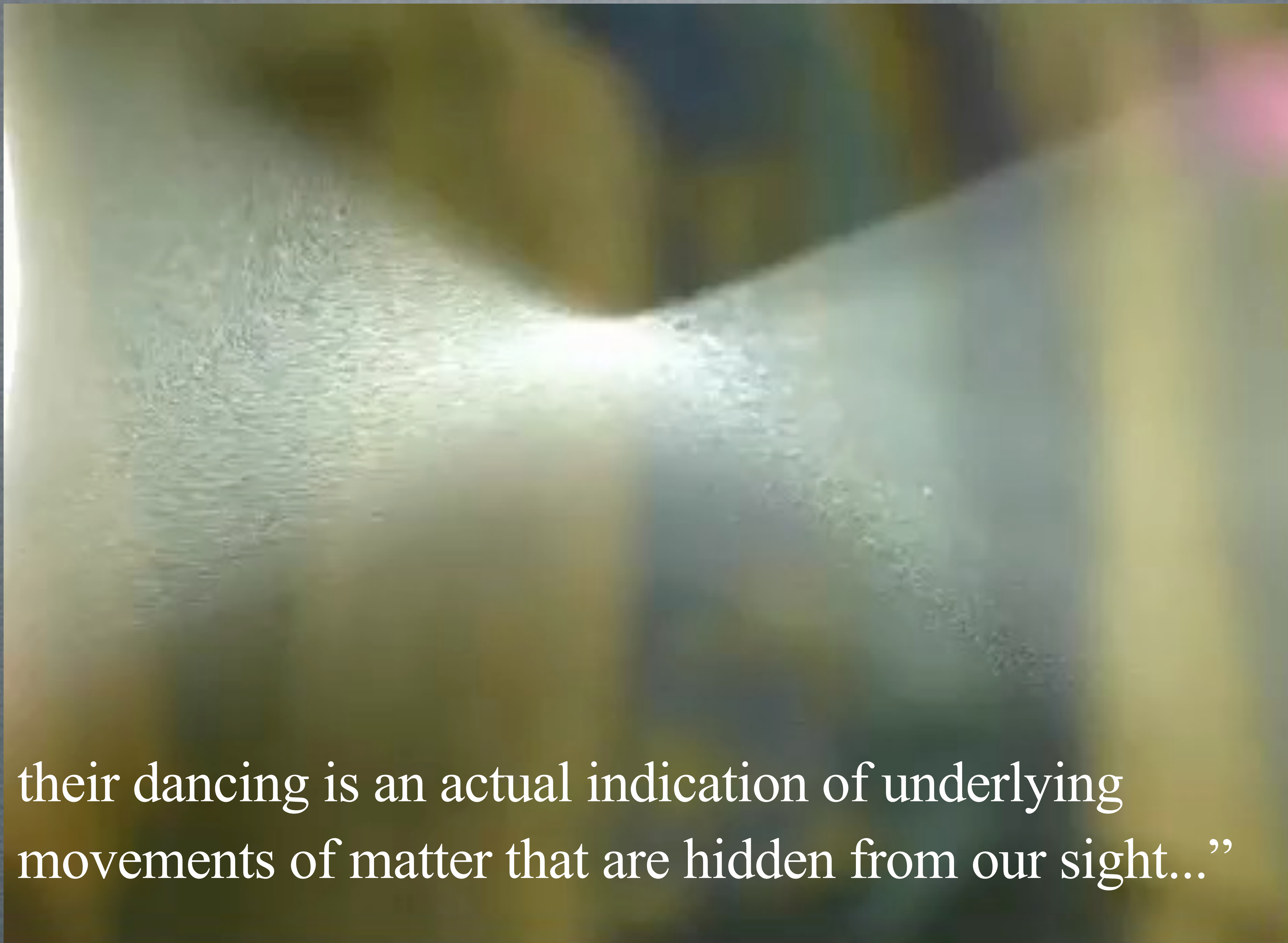
Lucretius (ca. 99BC-55BC)
Roman philosopher and poet

A Brief History of Diffusion Measurement



http://www.youtube.com/eYeFractal

Convection vs Diffusion
A Cautionary Note

Convection vs Diffusion
A Cautionary Note

The large scale swirling of the dust particles is primarily due to air currents (convection) but the *much* smaller scale jittery movements are diffusion

Convection vs Diffusion
A Cautionary Note

Convection

A Brief History of Diffusion Measurement

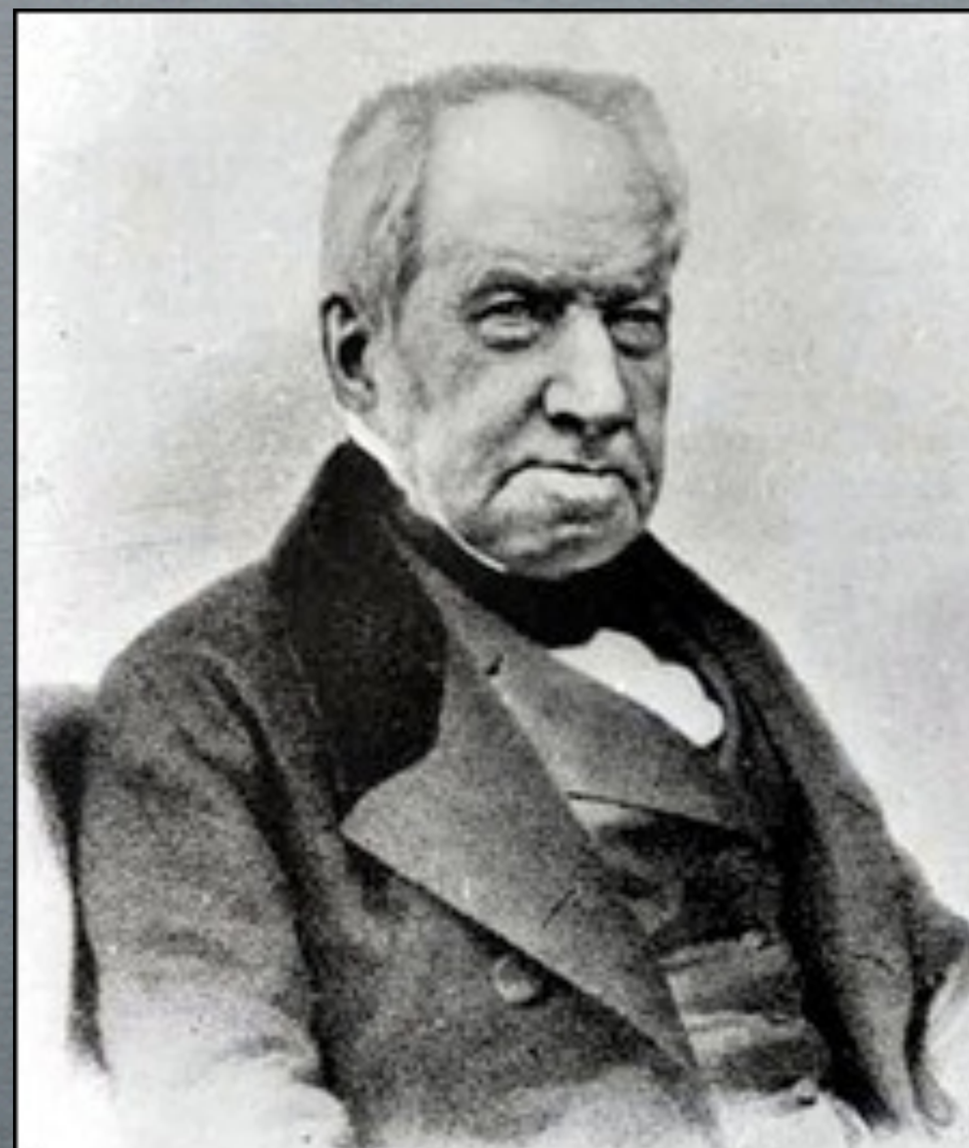
A Brief History of Diffusion Measurement

Jan Ingenhousz (1730 – 1799)
Dutch botanist and physiologist

Described the "irregular movements" of coal dust on the surface of alcohol

A Brief History of Diffusion Measurement

A Brief History of Diffusion Measurement



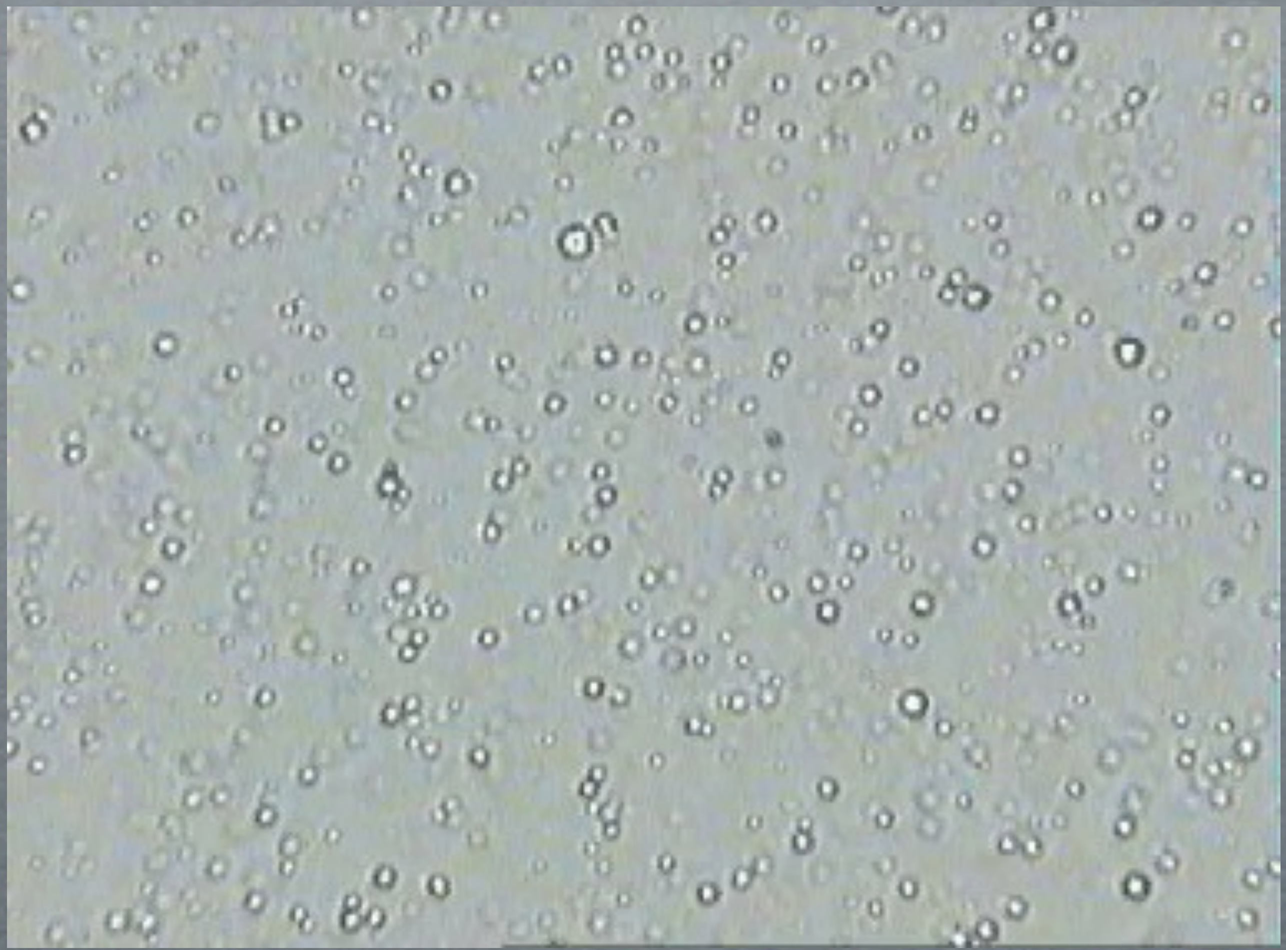
Robert Brown (1773 – 1858)
British botanist and surgeon

Observation:

irregular movement of pollen granules in water

A Brief History of Diffusion Measurement

"Brownian Motion"



Experiment: Repeat pollen experiment using tiny shards of window glass

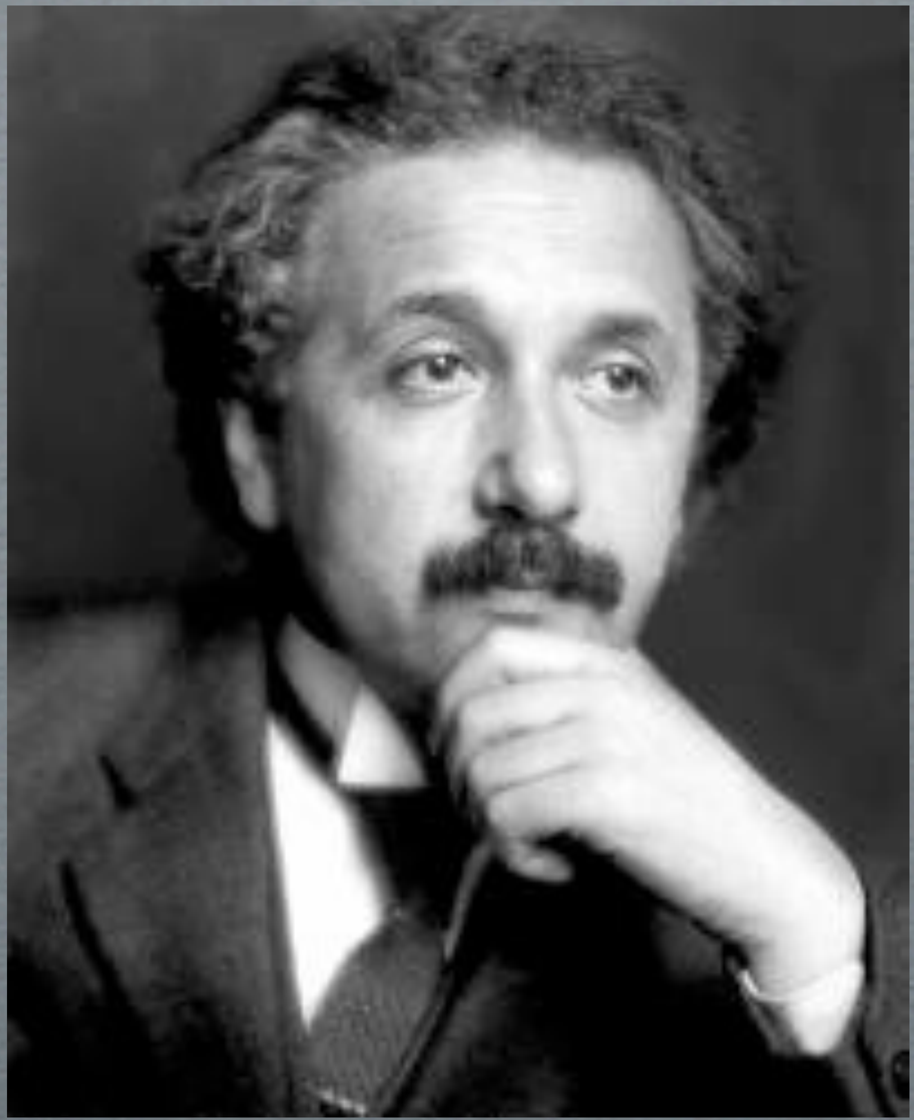
Result: Same!

Conclusion: Not alive

Theory: ???

Einstein's Theory of Brownian Motion

Einstein's Theory of Brownian Motion



Albert Einstein (1879 – 1955)
German physicist

Einstein's Theory of Brownian Motion

Einstein's Theory

Part 1: Equation describing motion of a Brownian particle

Part 2: Relate diffusion to experimentally measurable quantities

Einstein Theory of Brownian Motion
Part I

Einstein Theory of Brownian Motion
Part I

The particle density $\rho(x,t)$ at a position x at time t obeys

$$\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2}$$

change with time

diffusion coefficient

change with space

The Diffusion Equation

Einstein Theory of Brownian Motion
Part I

Einstein Theory of Brownian Motion
Part I

The solution to the Diffusion Equation

for particles initially at location x_0

$$\rho(x,t) = \frac{1}{\sqrt{4\pi D t}}\, e^{-(x-x_0)^2/4Dt}$$

This is a Gaussian (or Normal) distribution
with mean position

$$\bar{x} = x_0$$

and variance in the position

$$\sigma_x^2 = \overline{(x - x_0)^2} = 2Dt$$

Einstein Theory of Brownian Motion
Part I

Einstein Theory of Brownian Motion
Part I

What does this mean?

$$\bar{x} = x_0$$

implies that, on *average*,
the particles do not move from their initial position

$$\sigma_x^2 = 2Dt$$

implies that the *variance* of a Brownian particle's position is proportional to the diffusion coefficient D and time t

Einstein Theory of Brownian Motion
Part I

Einstein Theory of Brownian Motion
Part I

Einstein argued that the *displacement* of a Brownian particle is thus the RMS distance

$$\Delta x = \sqrt{\overline{(x - x_0)^2}} = \sqrt{2Dt}$$

and thus *not* linearly proportional to time (like flow), but to the *square root of time*

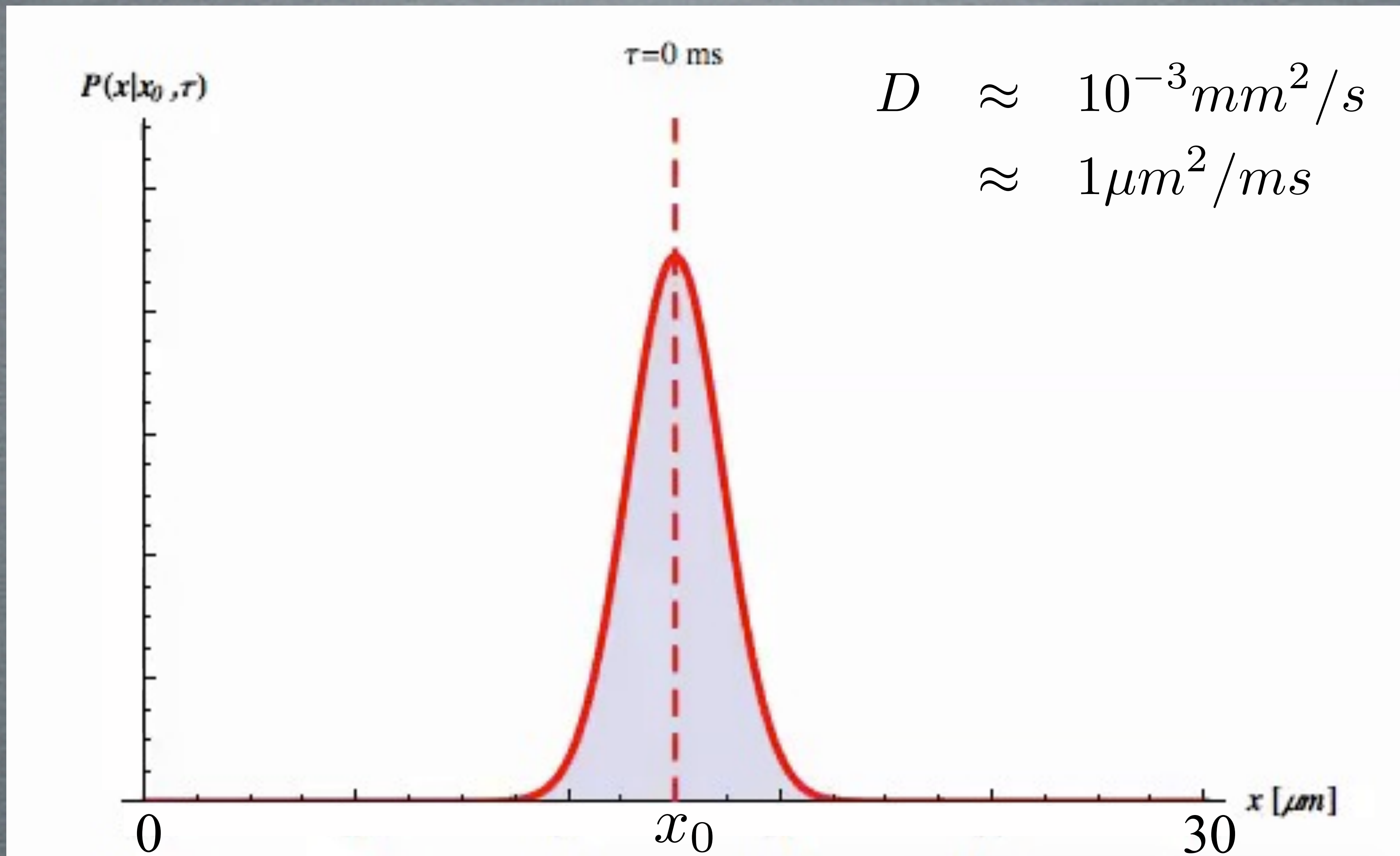
Diffusion in Brain Tissue:

$D \cong 1\ \mu^2/\ ms = (0.001\ mm^2/s)$

For t=100 msec, $\Delta x \cong 14\ \mu$

Gaussian Diffusion

Gaussian Diffusion



$$P(x|x_o, \tau) = \frac{1}{\sqrt{4\pi D\tau}} e^{-\frac{(x-x_o)^2}{4\pi D\tau}}$$

$$\sigma = \sqrt{2D\tau}$$

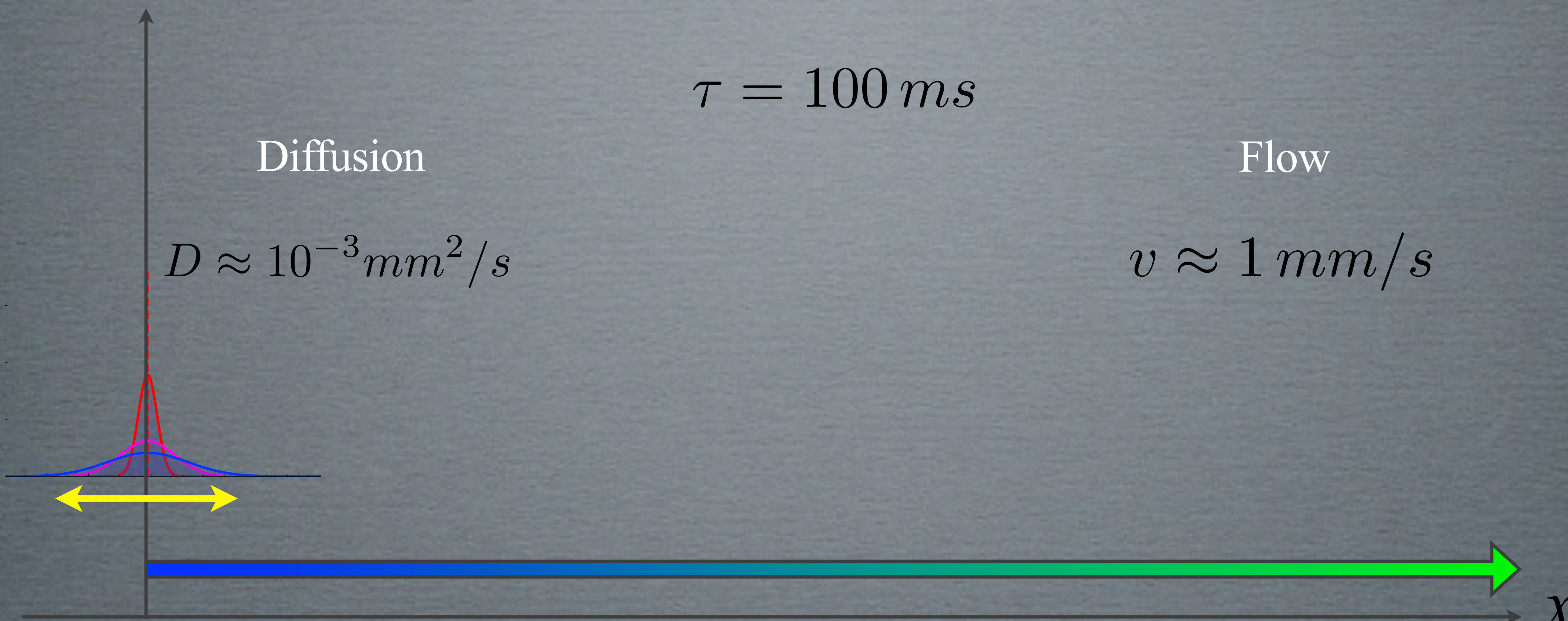
Einstein Theory of Brownian Motion
Part I

Einstein Theory of Brownian Motion
Part I

Diffusion vs Flow

$\tau = 100\, ms$

Diffusion	Flow
$D \approx 10^{-3} mm^2/s$	$v \approx 1\, mm/s$



$\Delta x \approx 14\, \mu m$	$\Delta x \approx 100\, \mu m$

Einstein Theory of Brownian Motion
Part II

Einstein Theory of Brownian Motion
Part II

The diffusion coefficient is

$$D = \alpha \frac{T}{\eta r}$$

Diffusion coefficient goes up with temperature and down with viscosity and particle radius

It's sensitive to the local environment!

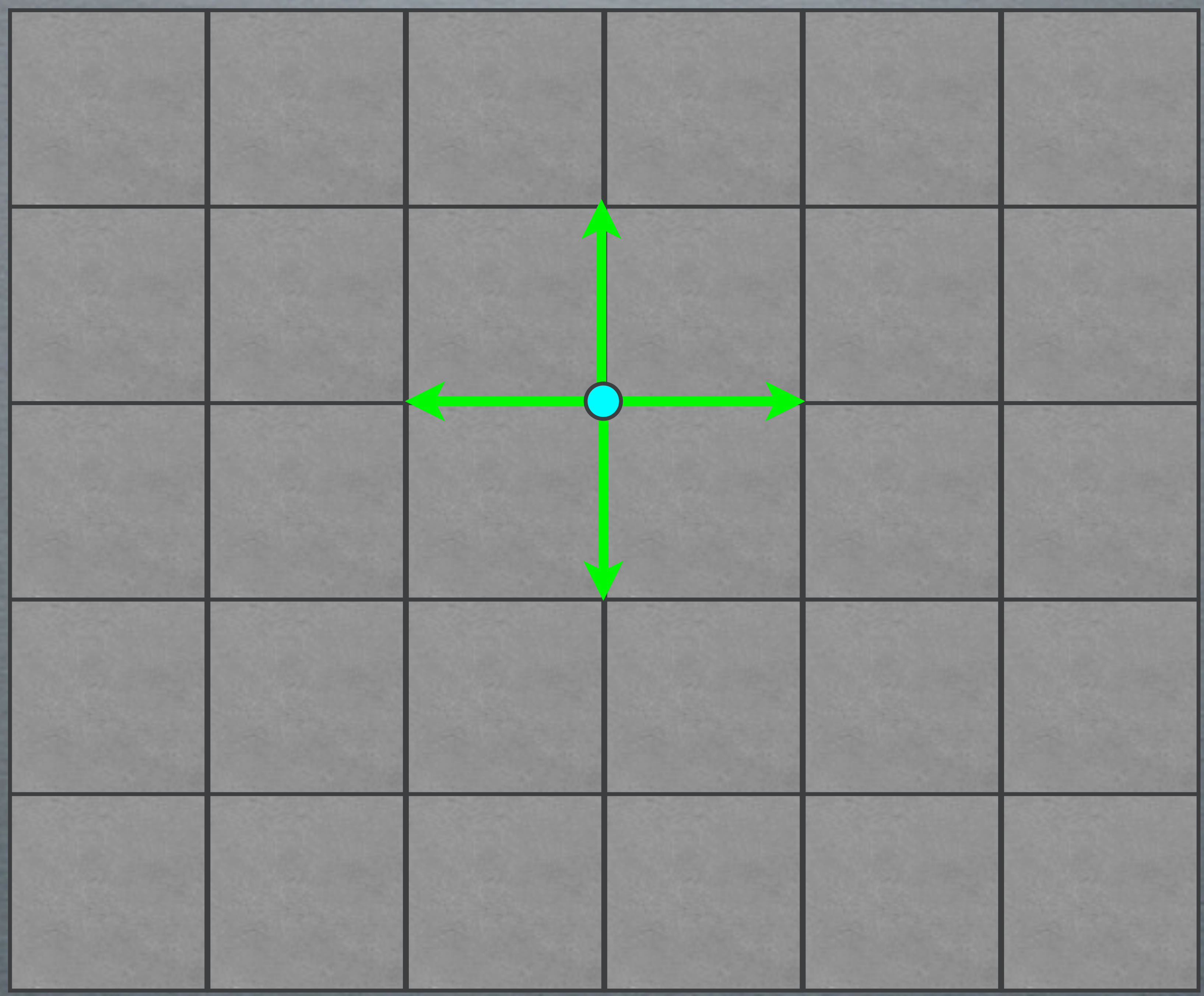
Modeling diffusion: Random Walk

Modeling diffusion: Random Walk

MRI is all about mapping the locations of molecules ...

... we need a way to model the spatial locations
of Brownian molecules as a function of time

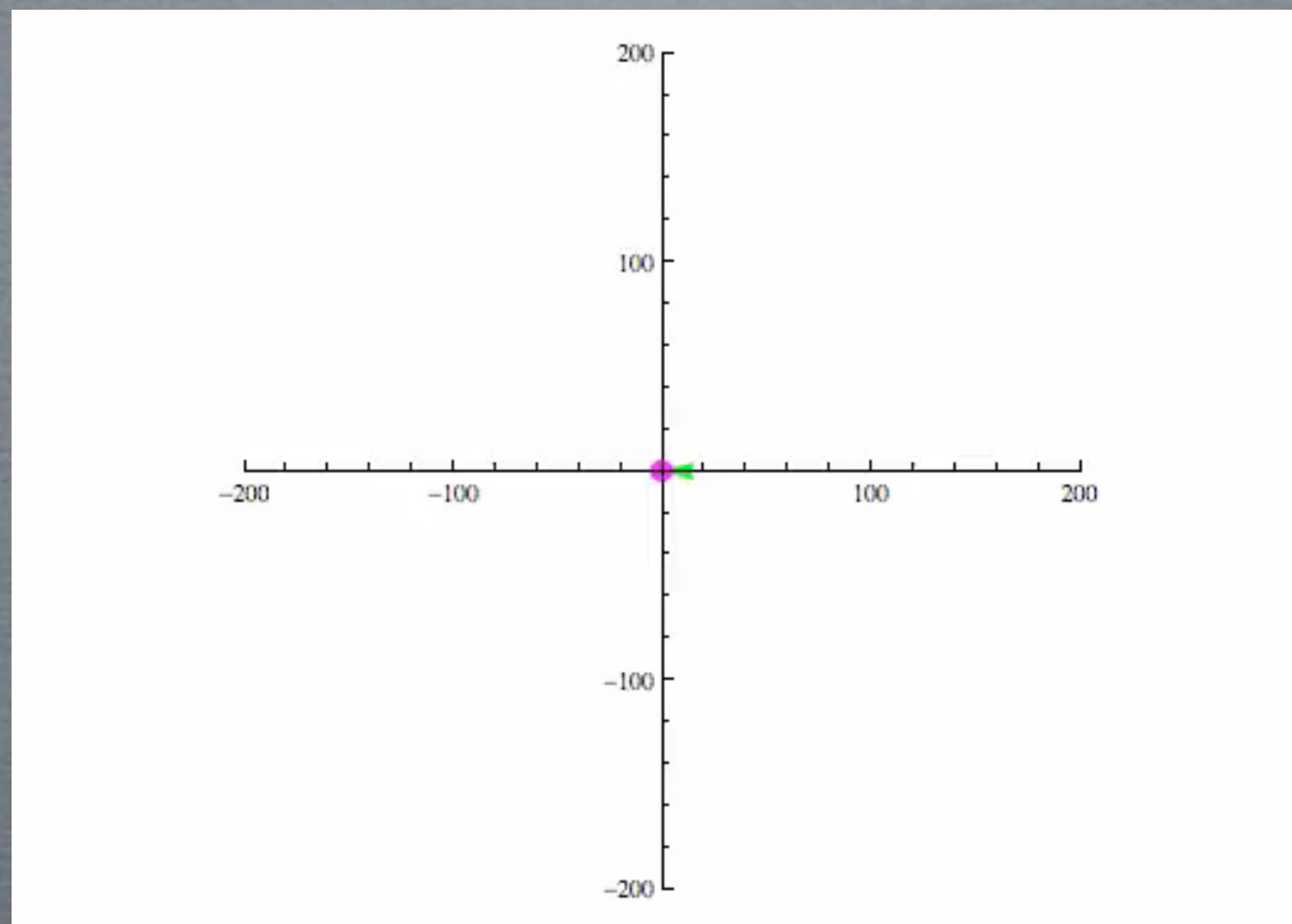
Modeling diffusion: Random Walk



Modeling diffusion: Random Walk

$$\tau = \text{constant}$$

Modeling diffusion: Random Walk



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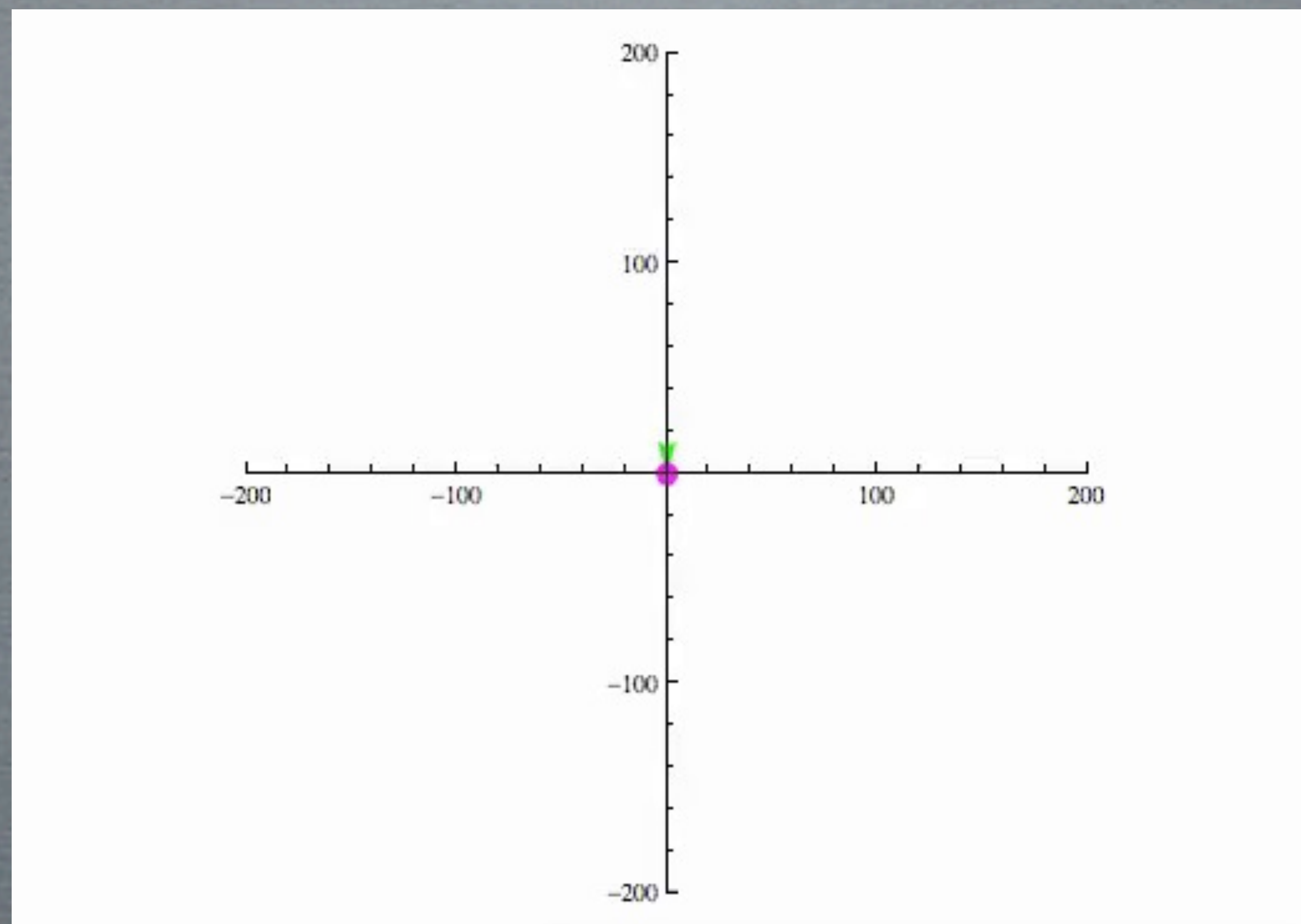

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Modeling diffusion: Random Walk

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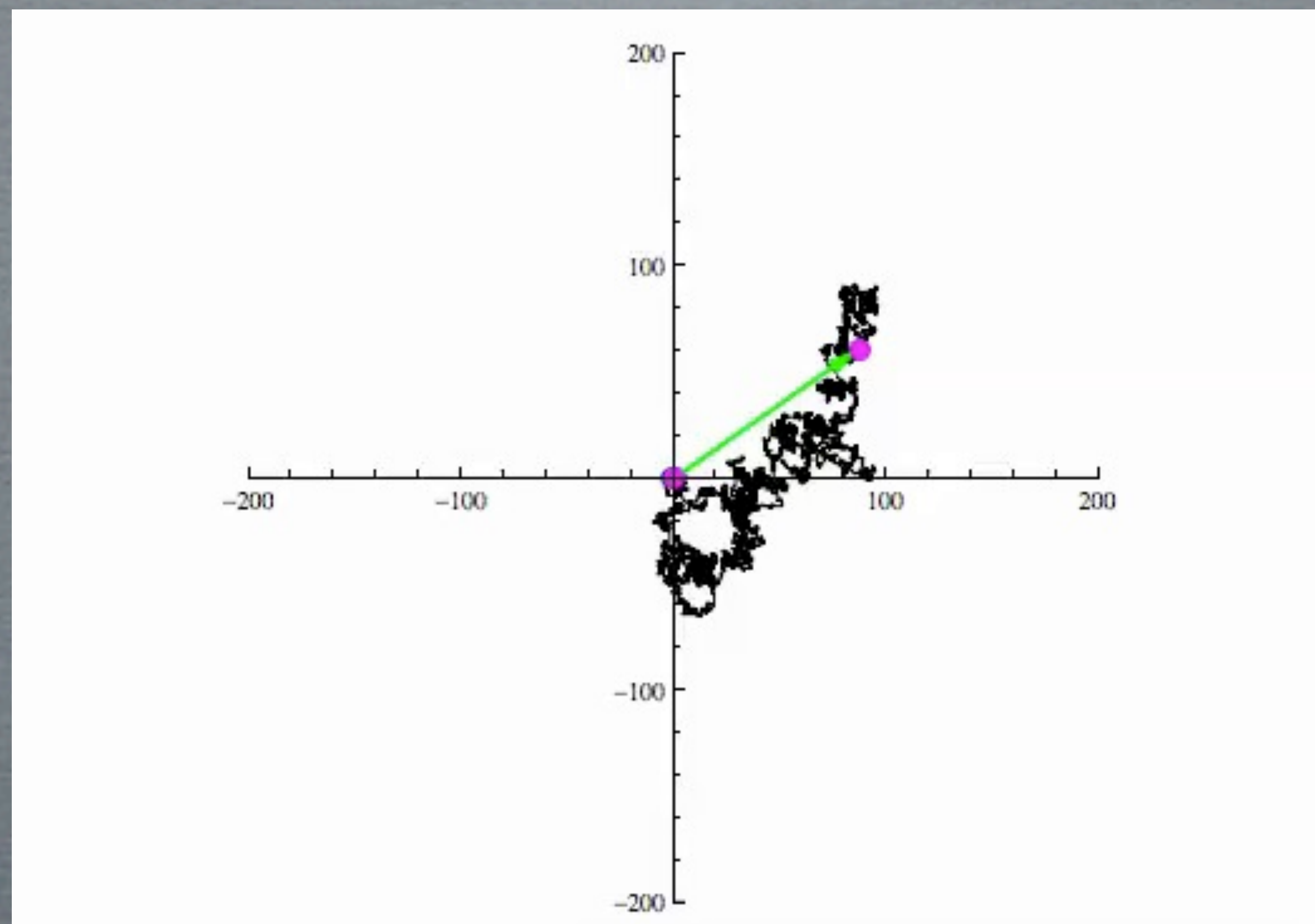

$$\tau = \text{constant}$$

Modeling diffusion: Random Walk

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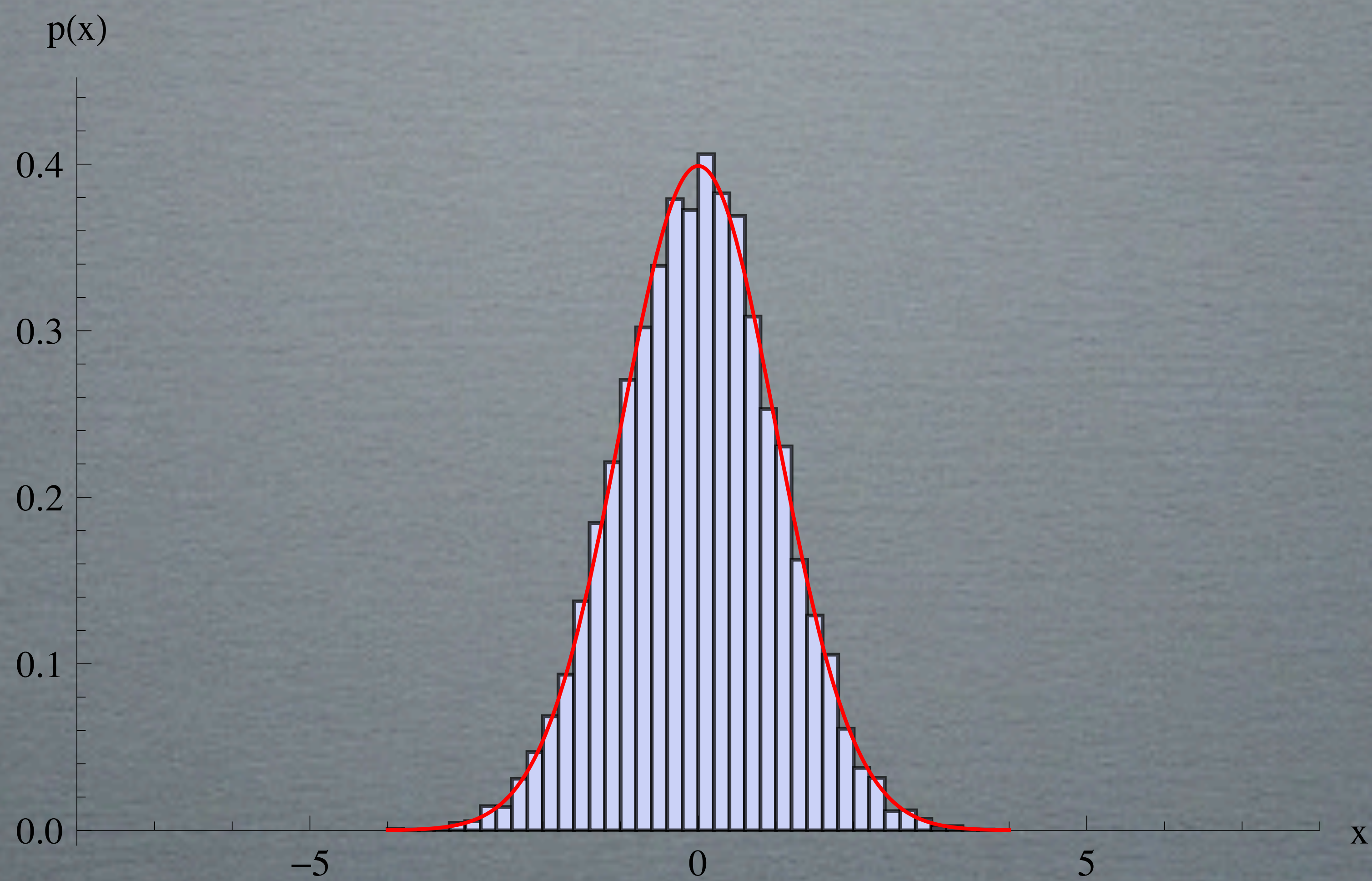
The distribution of particles after a time τ

Modeling diffusion: Random Walk

The distribution of particles after a time τ

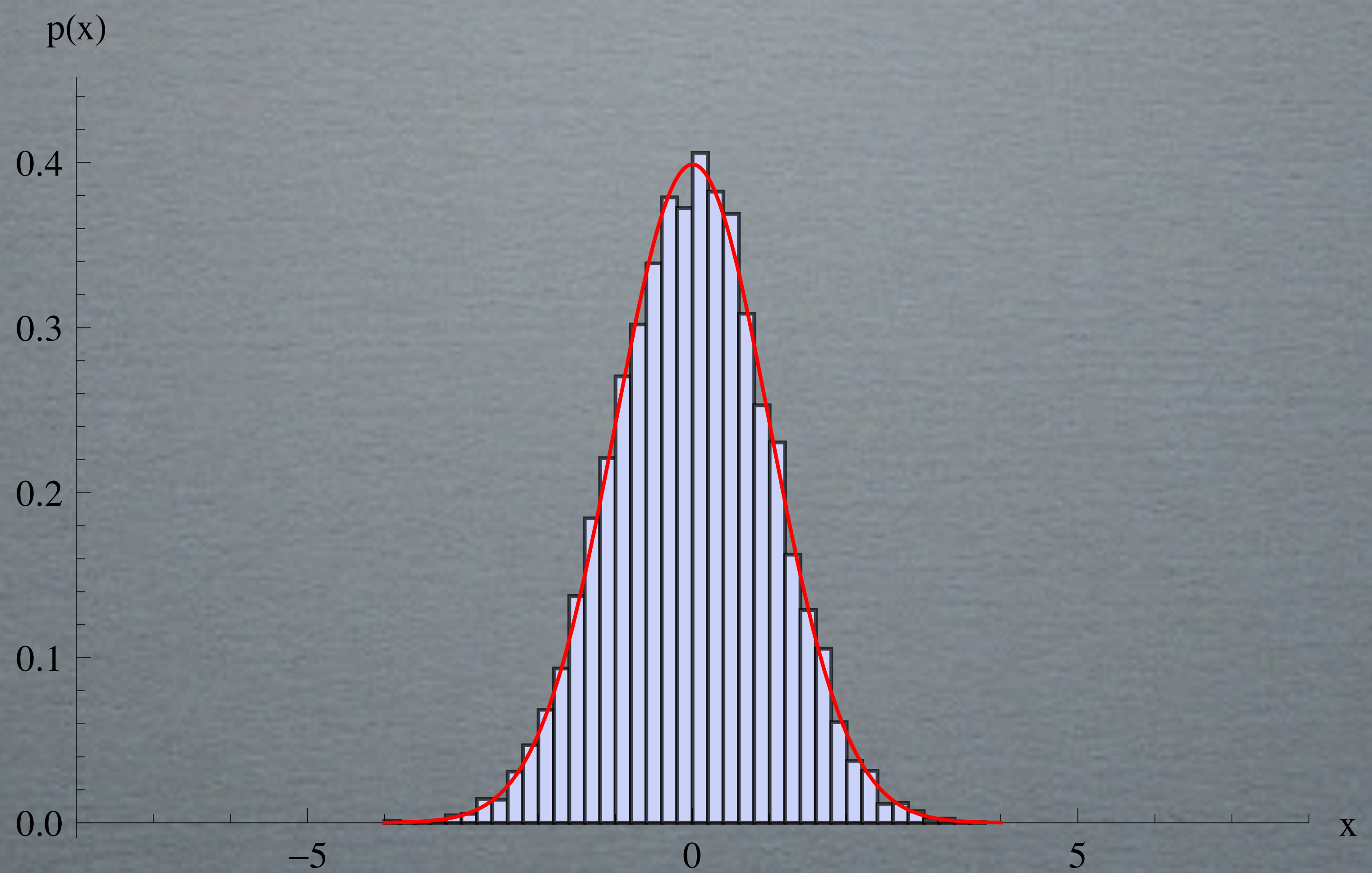


Gaussian diffusion

$$P(x|x_o,\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-\frac{(x-x_o)^2}{4\pi D\tau}}$$

Modeling diffusion: Random Walk

The distribution of particles after a time τ

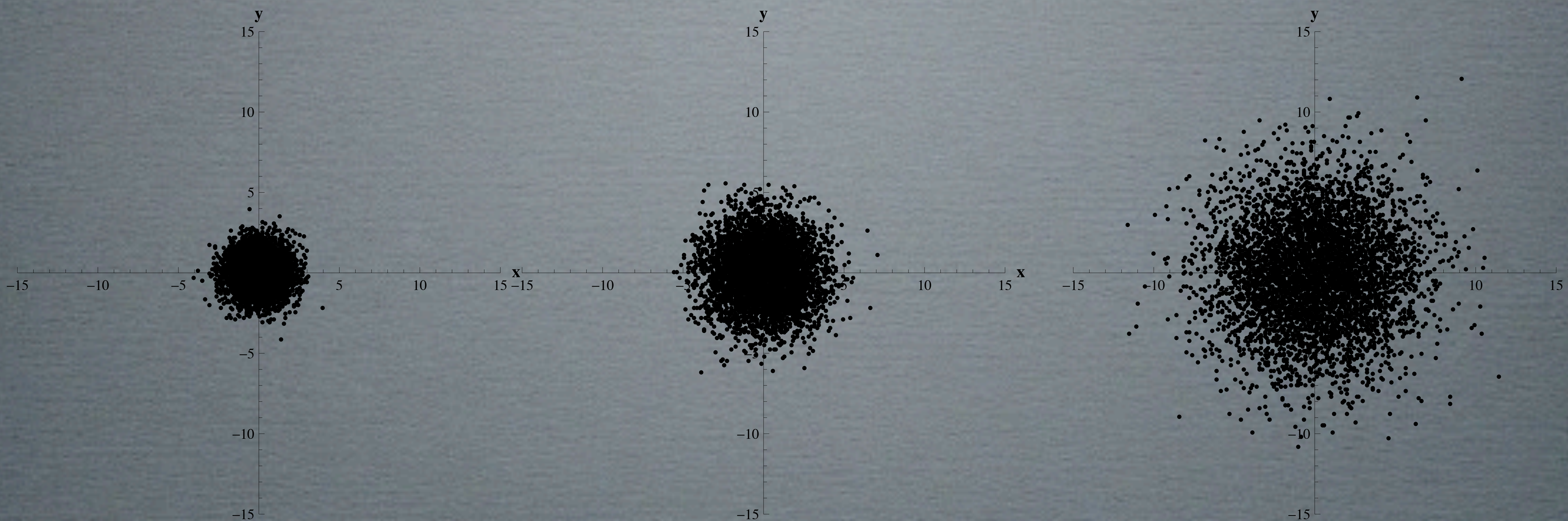


Gaussian diffusion

$$P(x|x_0, \tau) \sim N(x_0, \sigma^2)$$

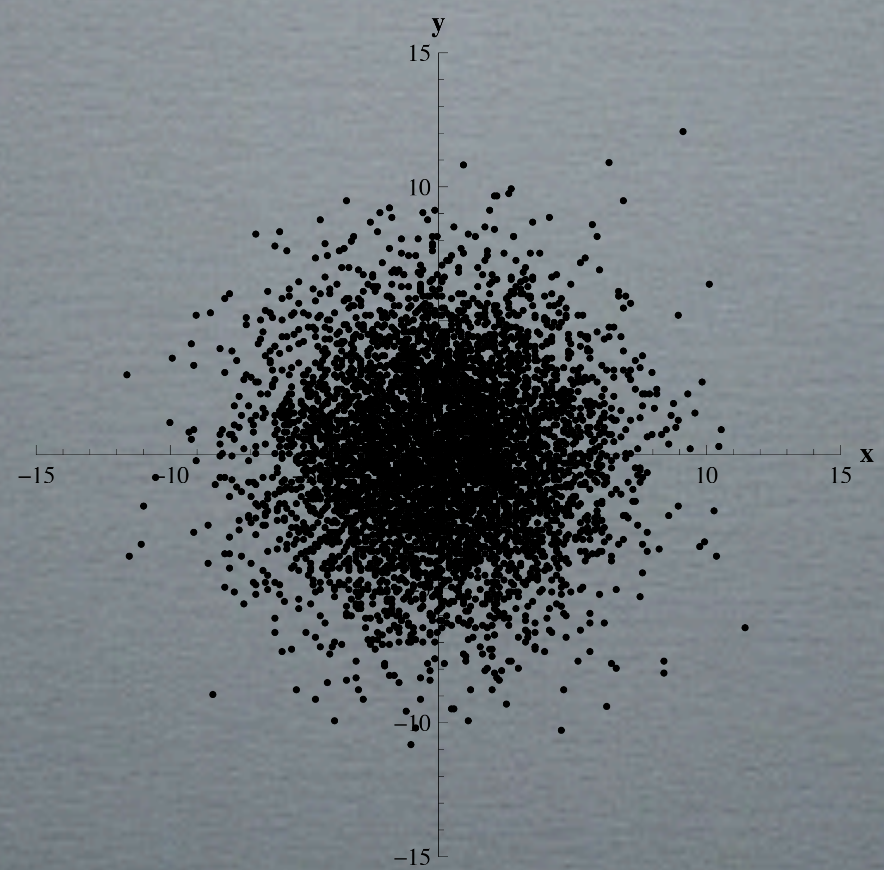
isotropic diffusion in 2D

ISOTROPIC DIFFUSION IN 2D



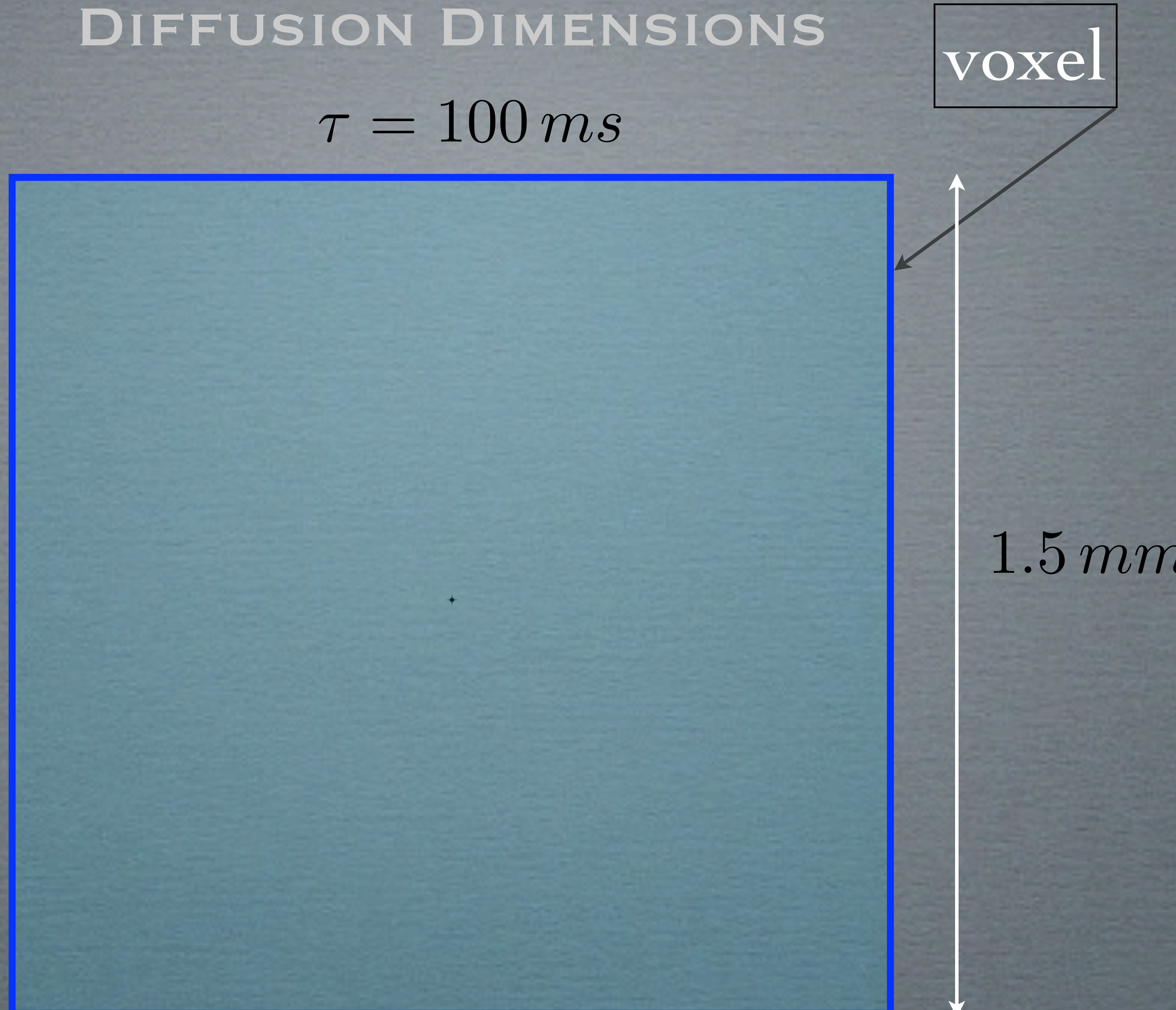
$\tau = 1\,ms$ $\tau = 10\,ms$ $\tau = 100\,ms$

Diffusion Dimensions



Diffusion Dimensions

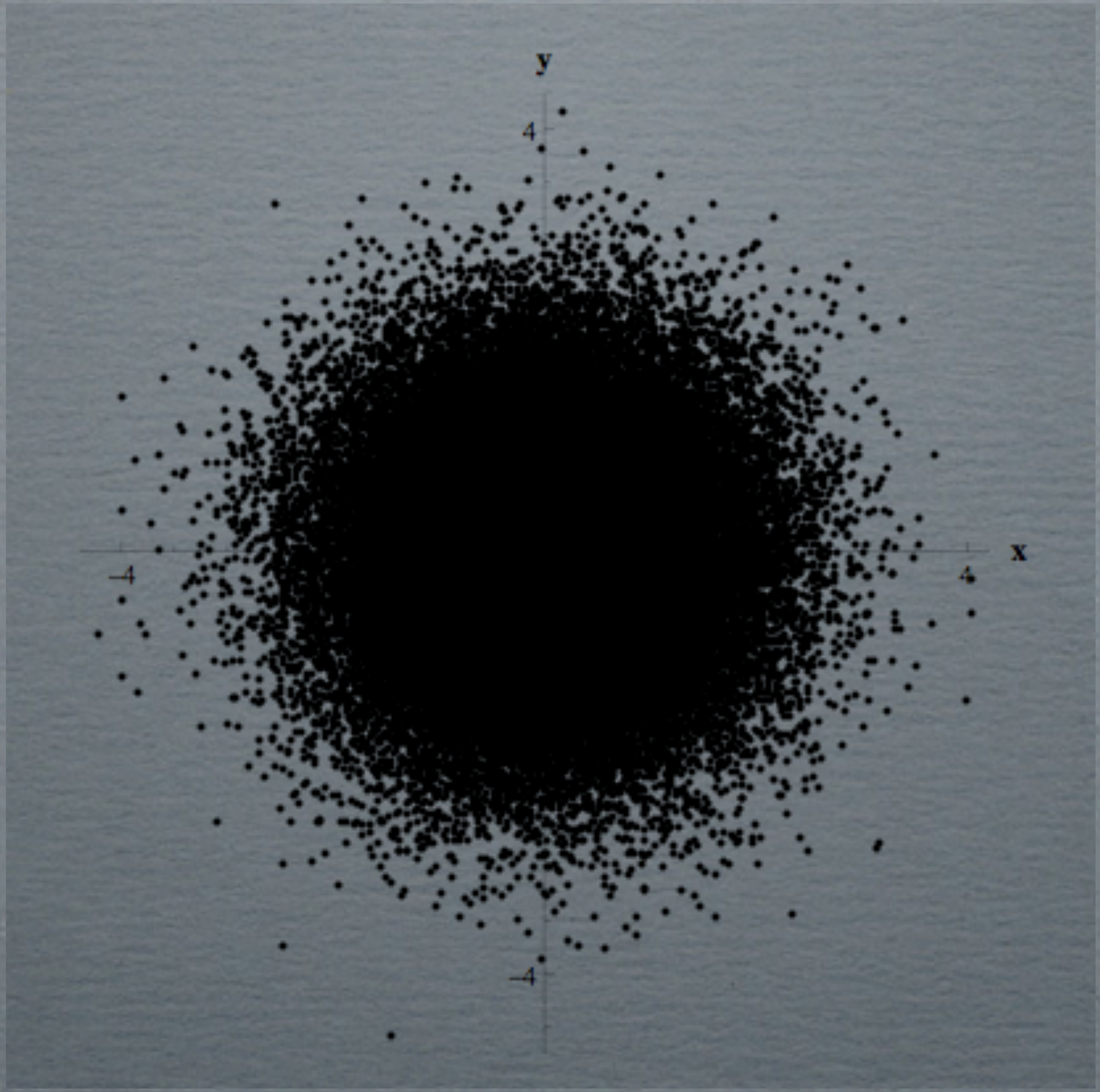
$$\tau = 100\, ms$$



$$\Delta x \approx \left(\frac{1}{1000}\right) \text{a typical imaging voxel dimension}$$

Probability Contours
(isotropic diffusion)

Probability Contours (isotropic diffusion)



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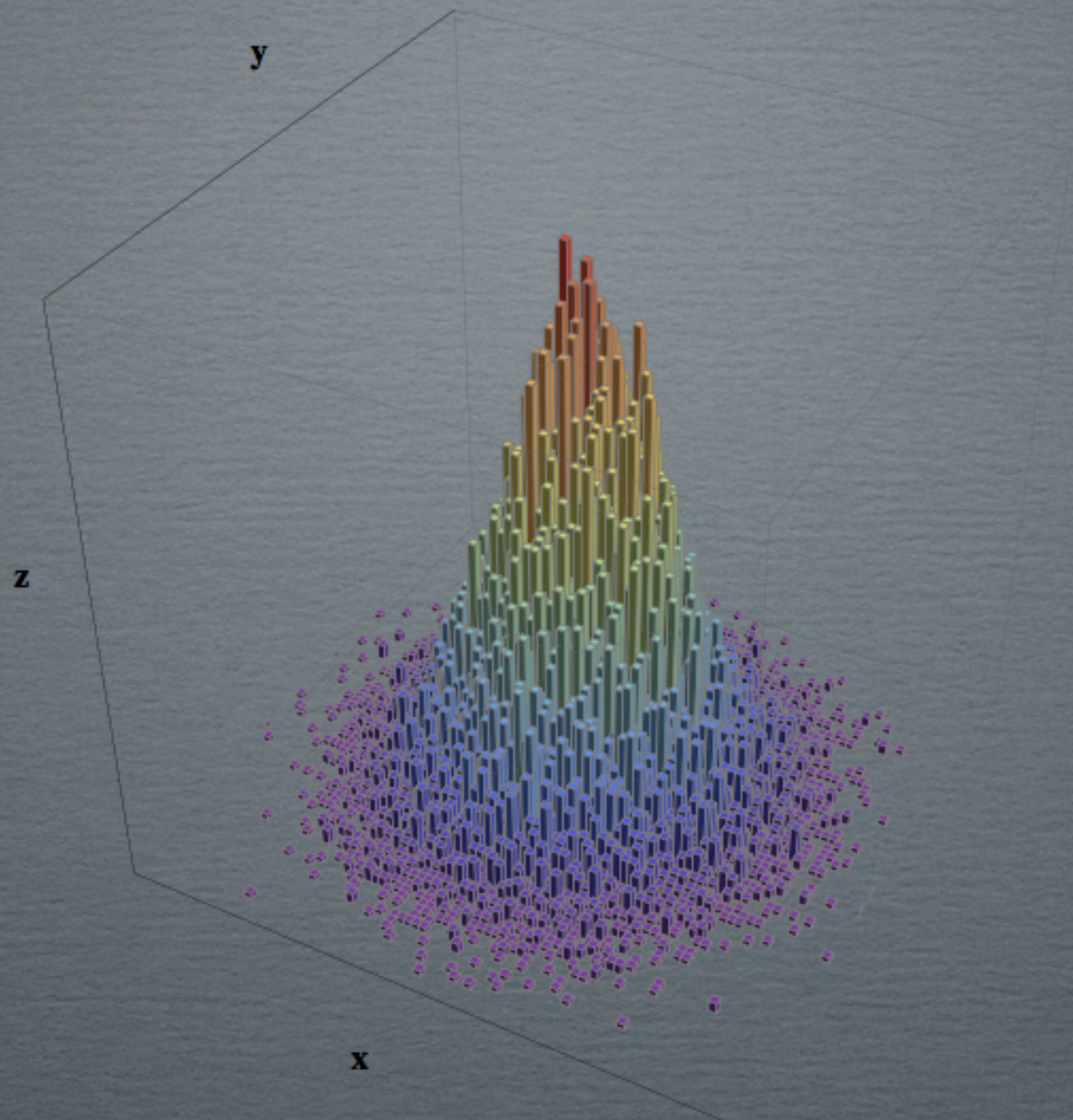

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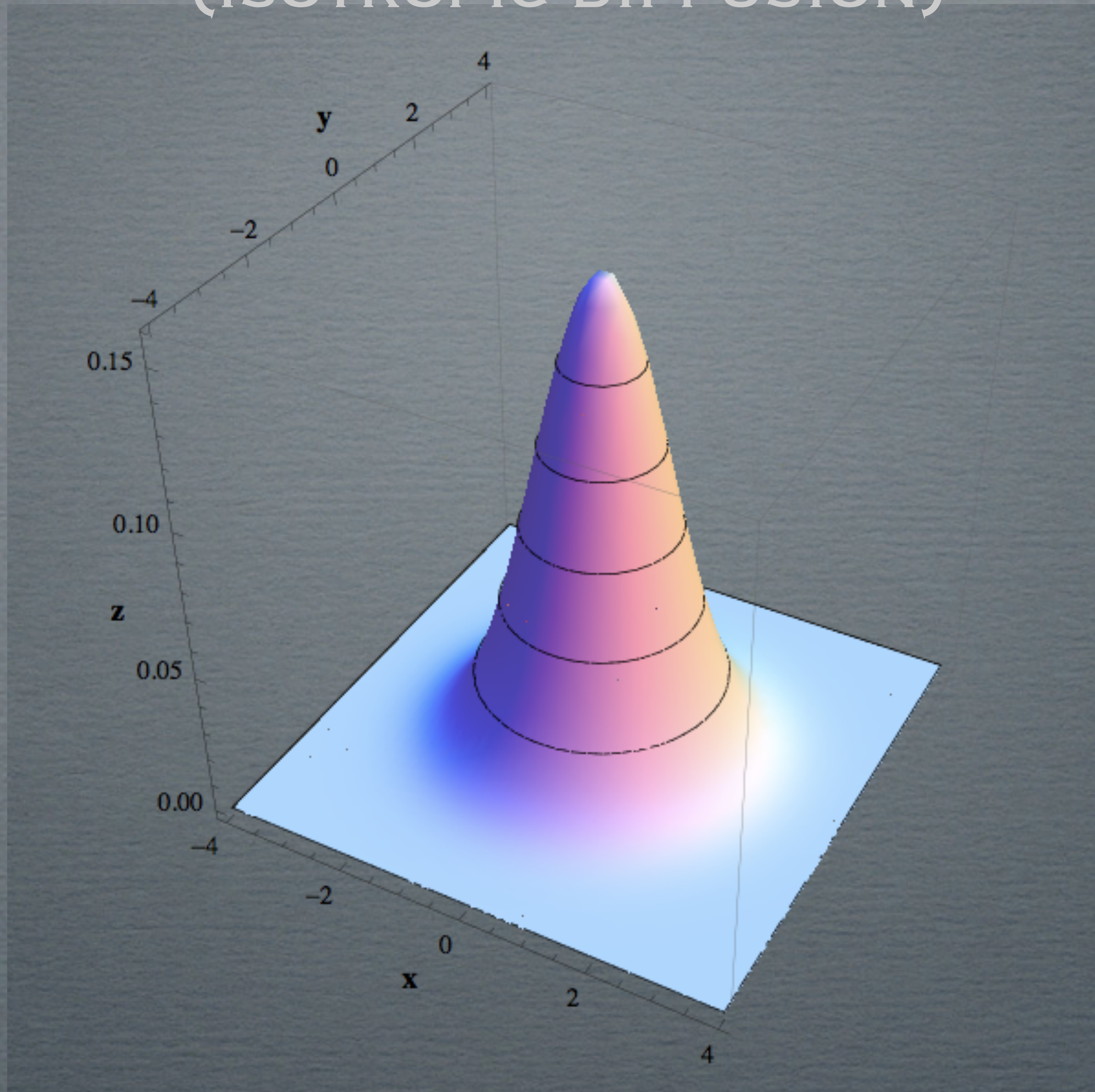

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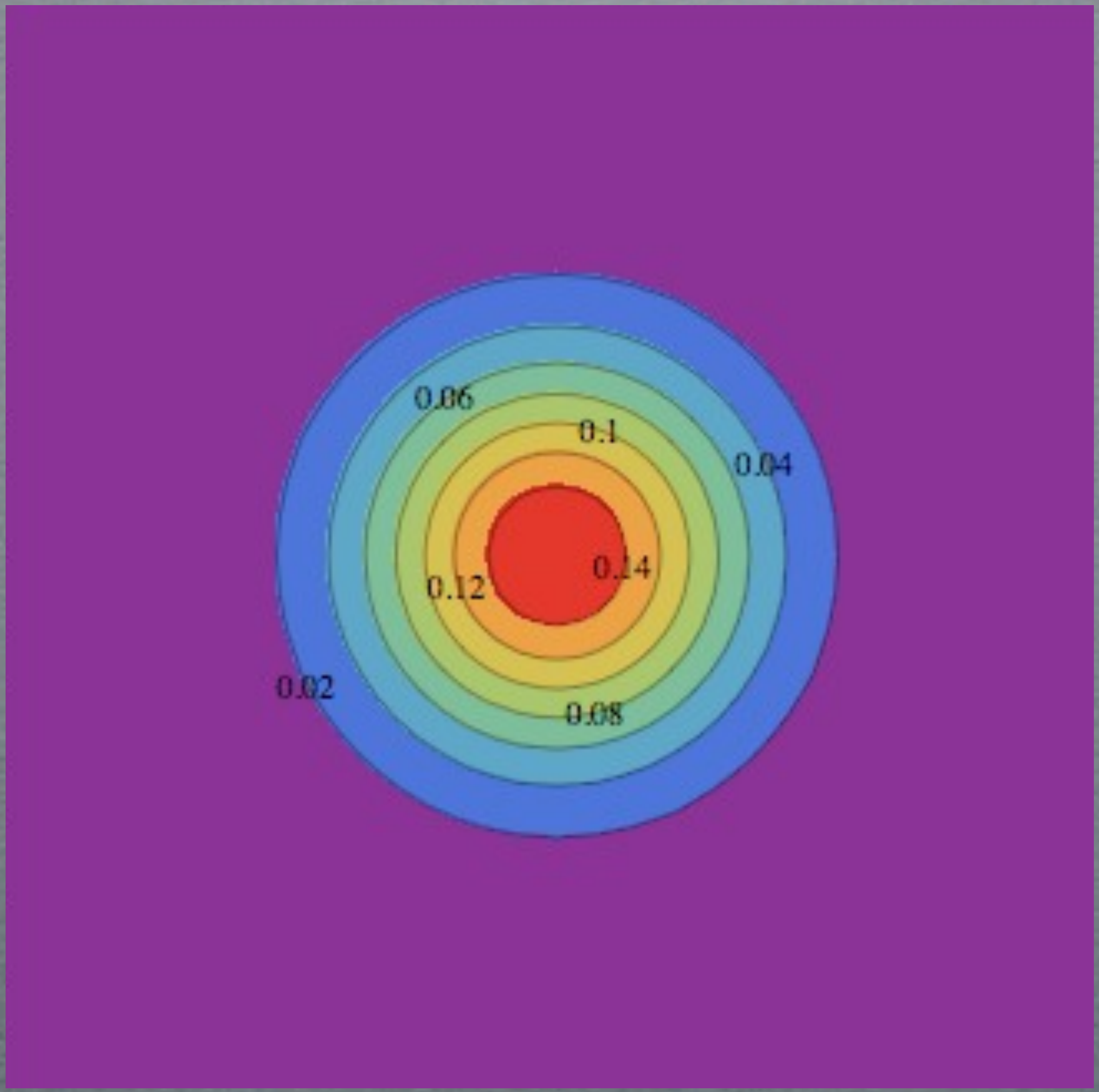
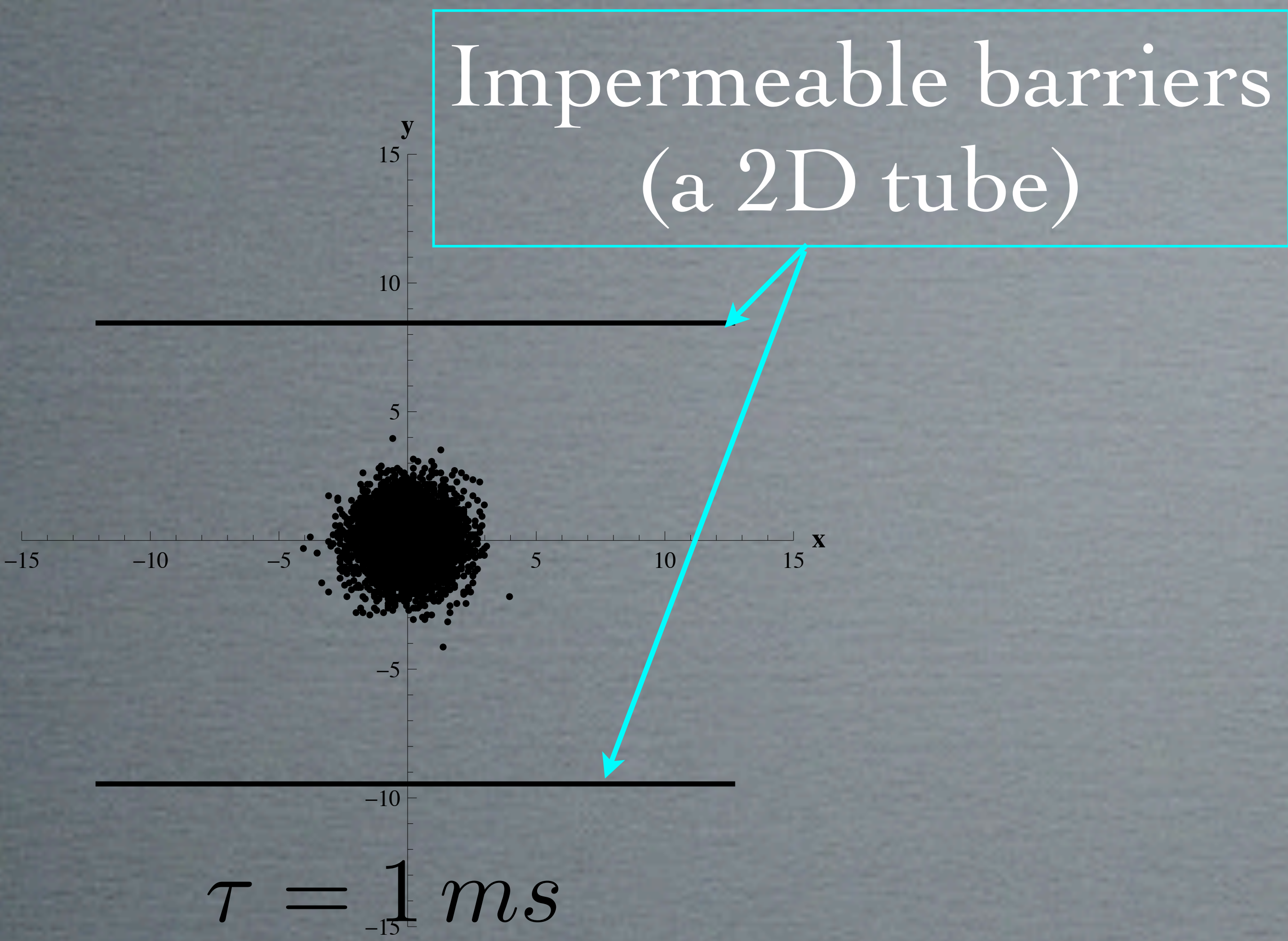


Anisotropic diffusion in 2D

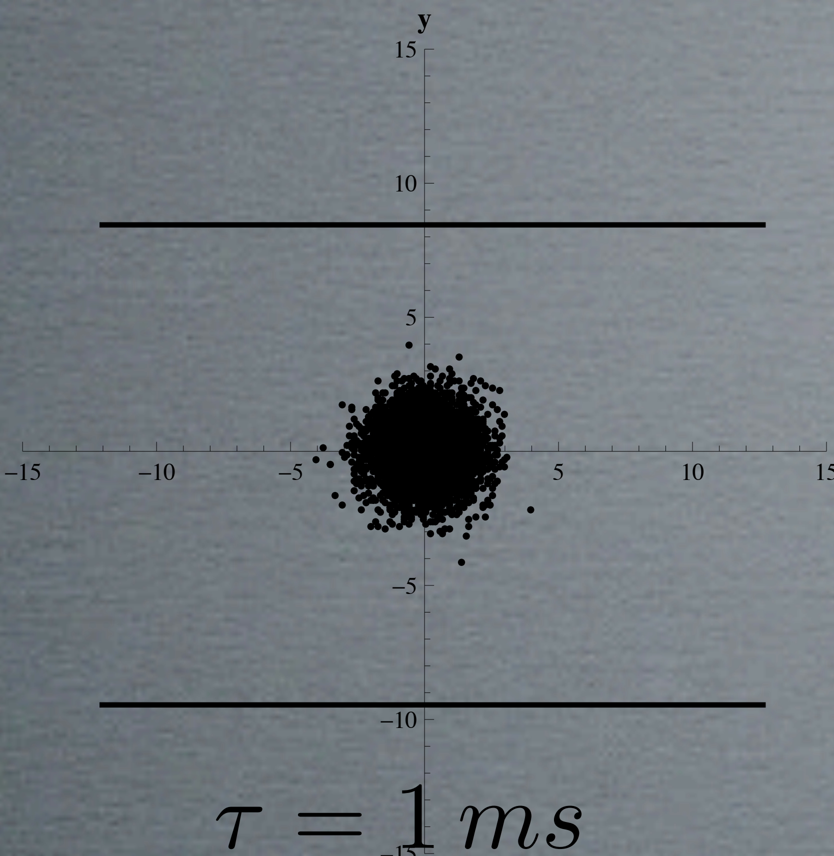
Anisotropic diffusion in 2D



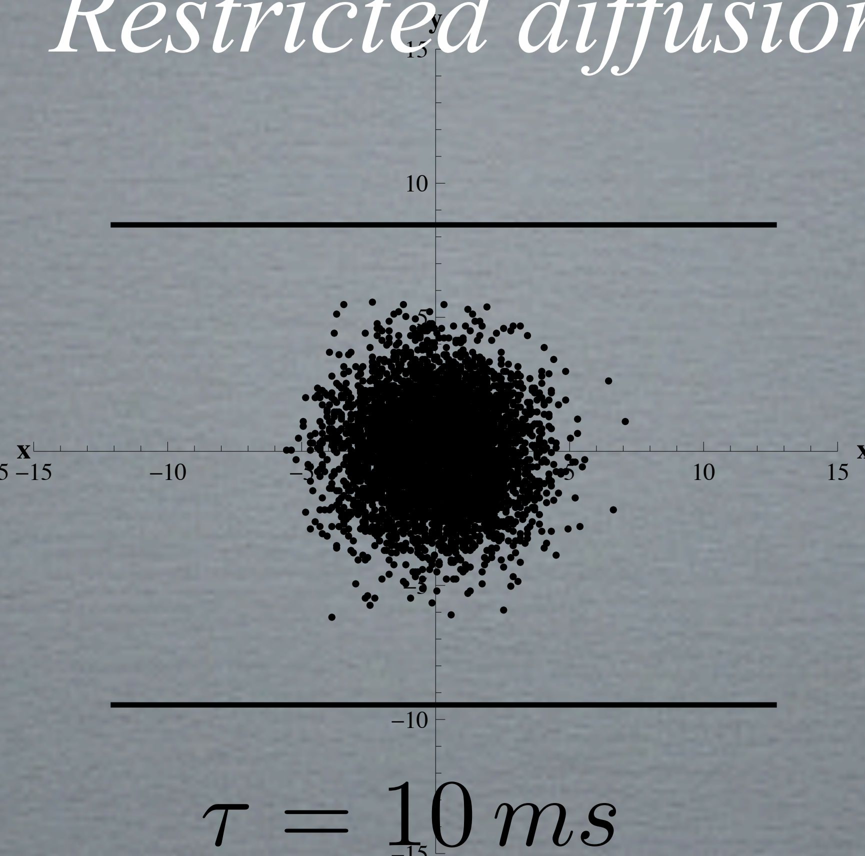
$\tau = 1\, ms$

Anisotropic diffusion in 2D

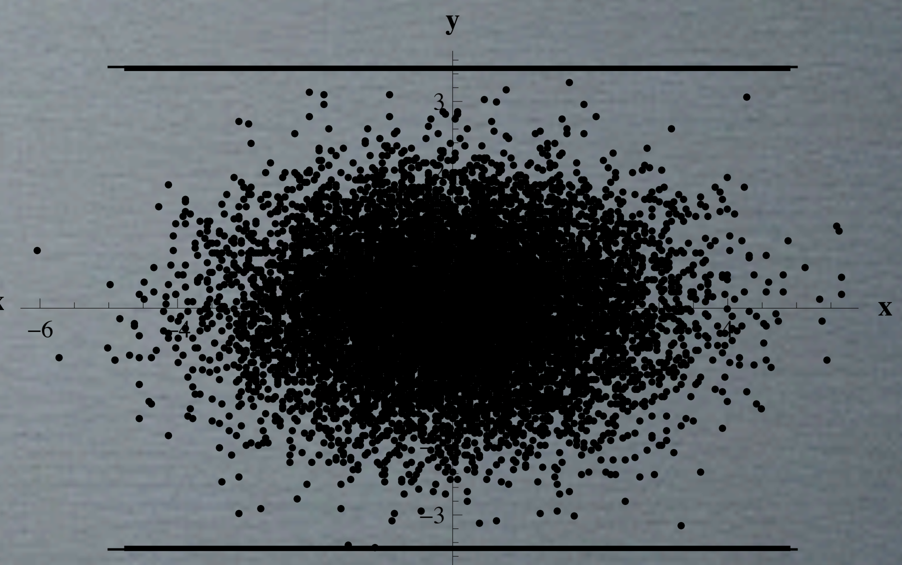
Restricted diffusion



$\tau = 1\, ms$



$\tau = 10\, ms$

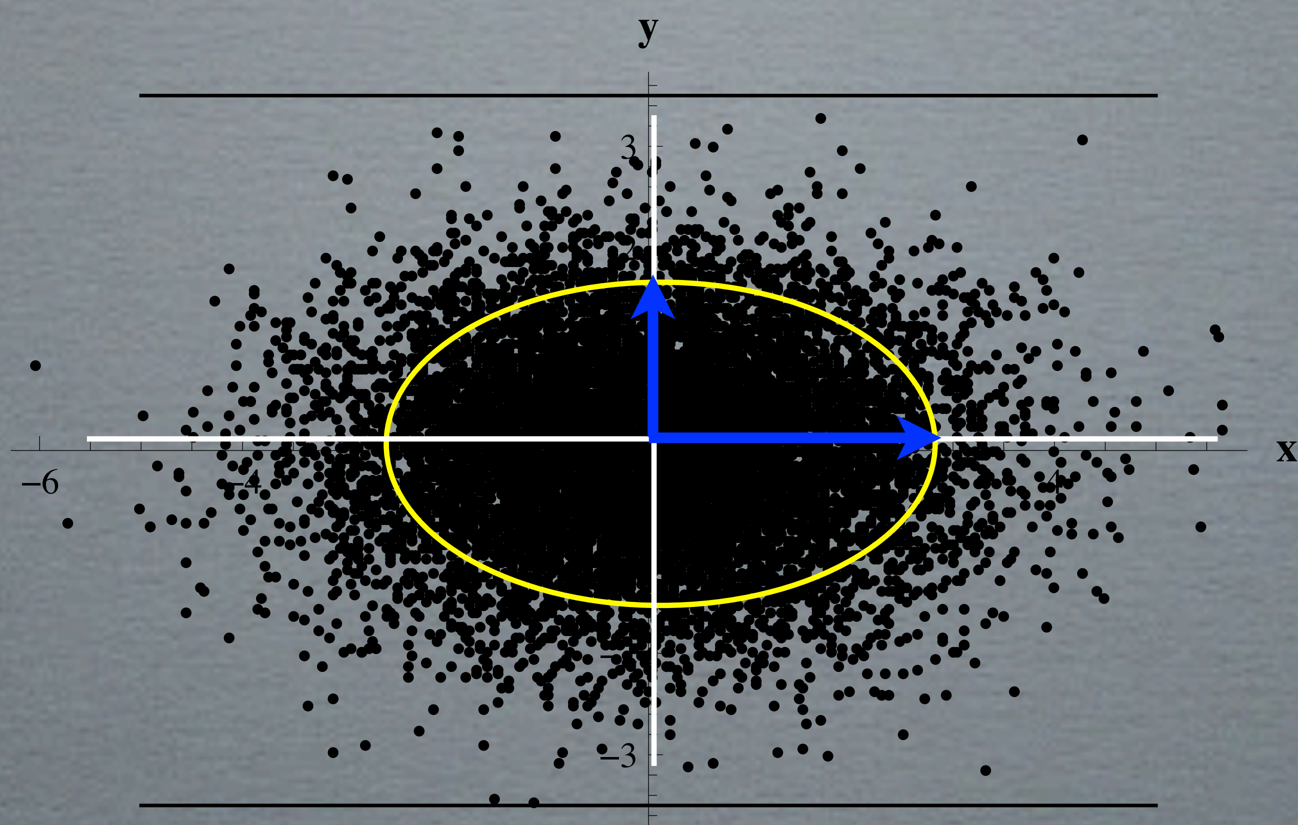


$\tau = 100\, ms$

Anisotropic diffusion in 2D

Anisotropic diffusion in 2D

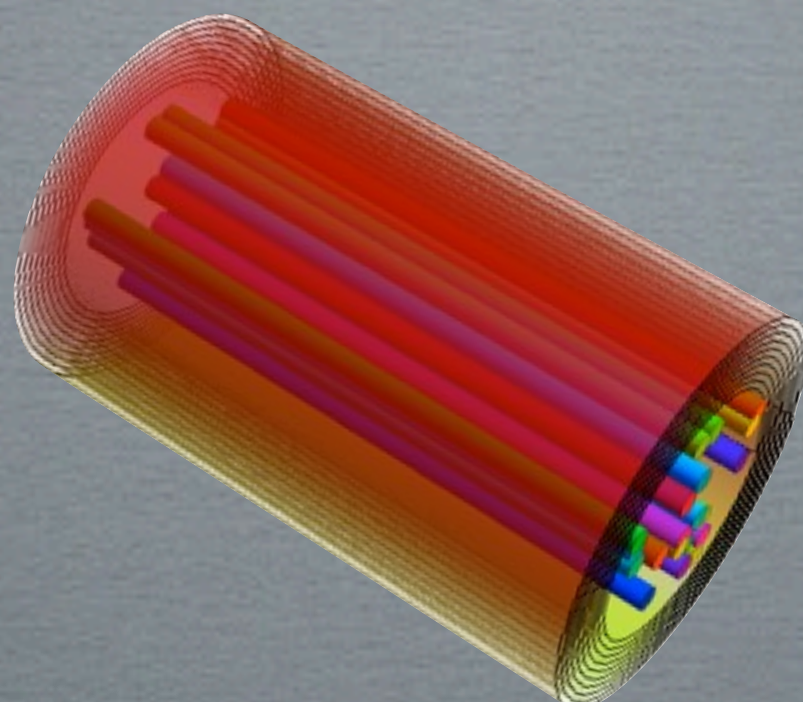
$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$



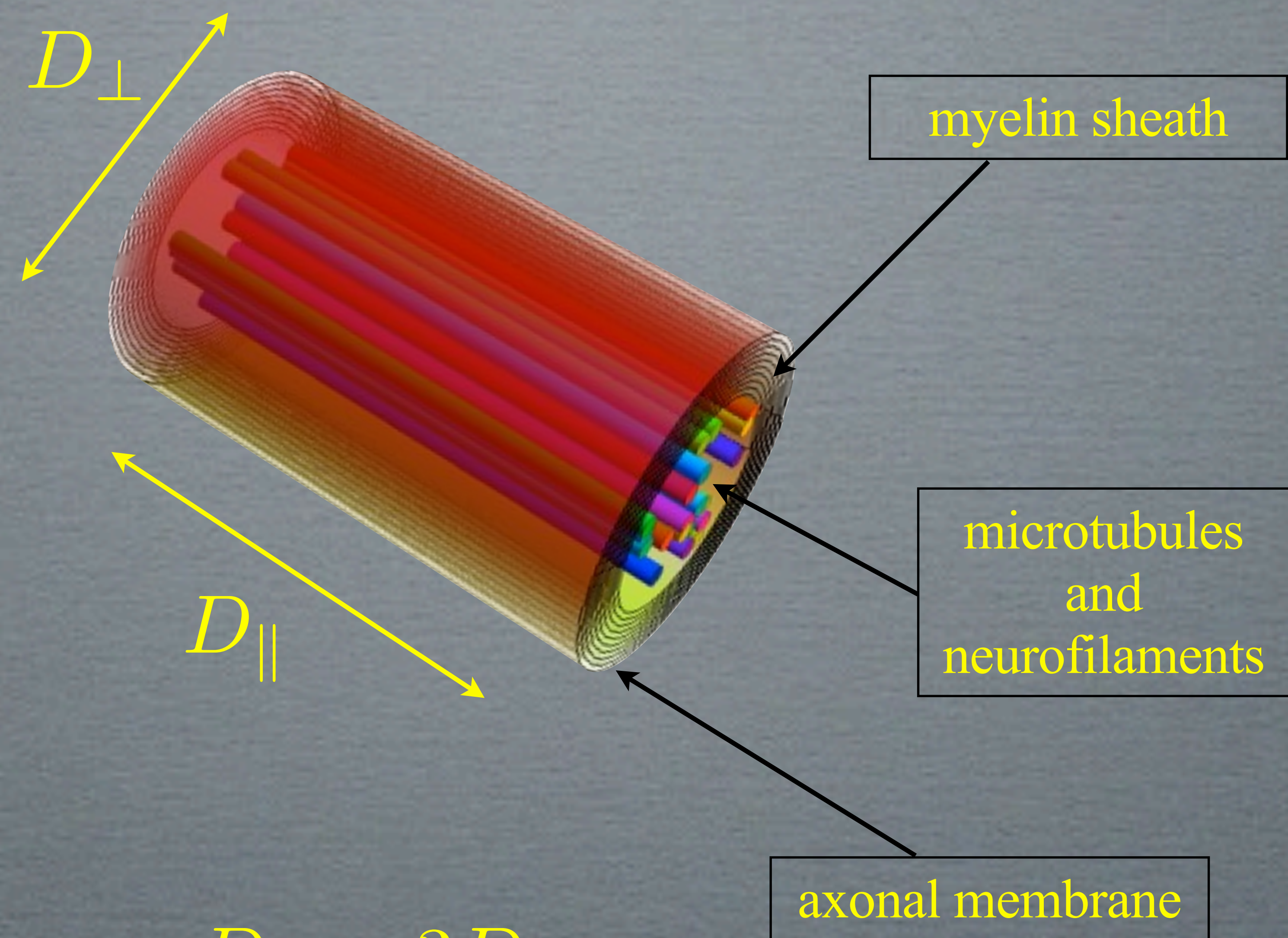
Covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}$$

Diffusion Anisotropy in neural tissues



Diffusion Anisotropy in neural tissues



$$D_\parallel \approx 3D_\perp$$

$$(1.2\mu^2/ms) \qquad (0.4\mu^2/ms)$$

Diffusion Anisotropy in 3D

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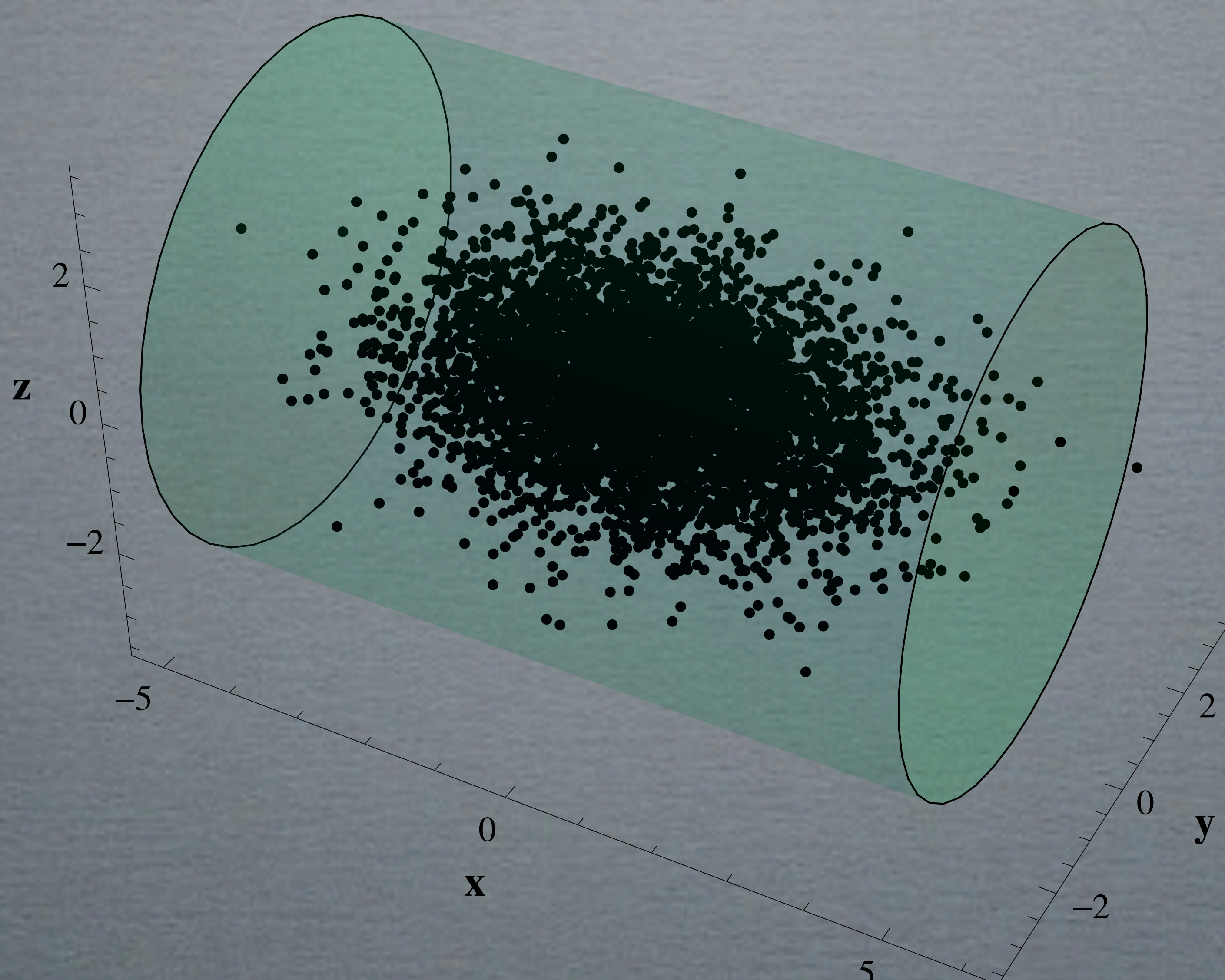
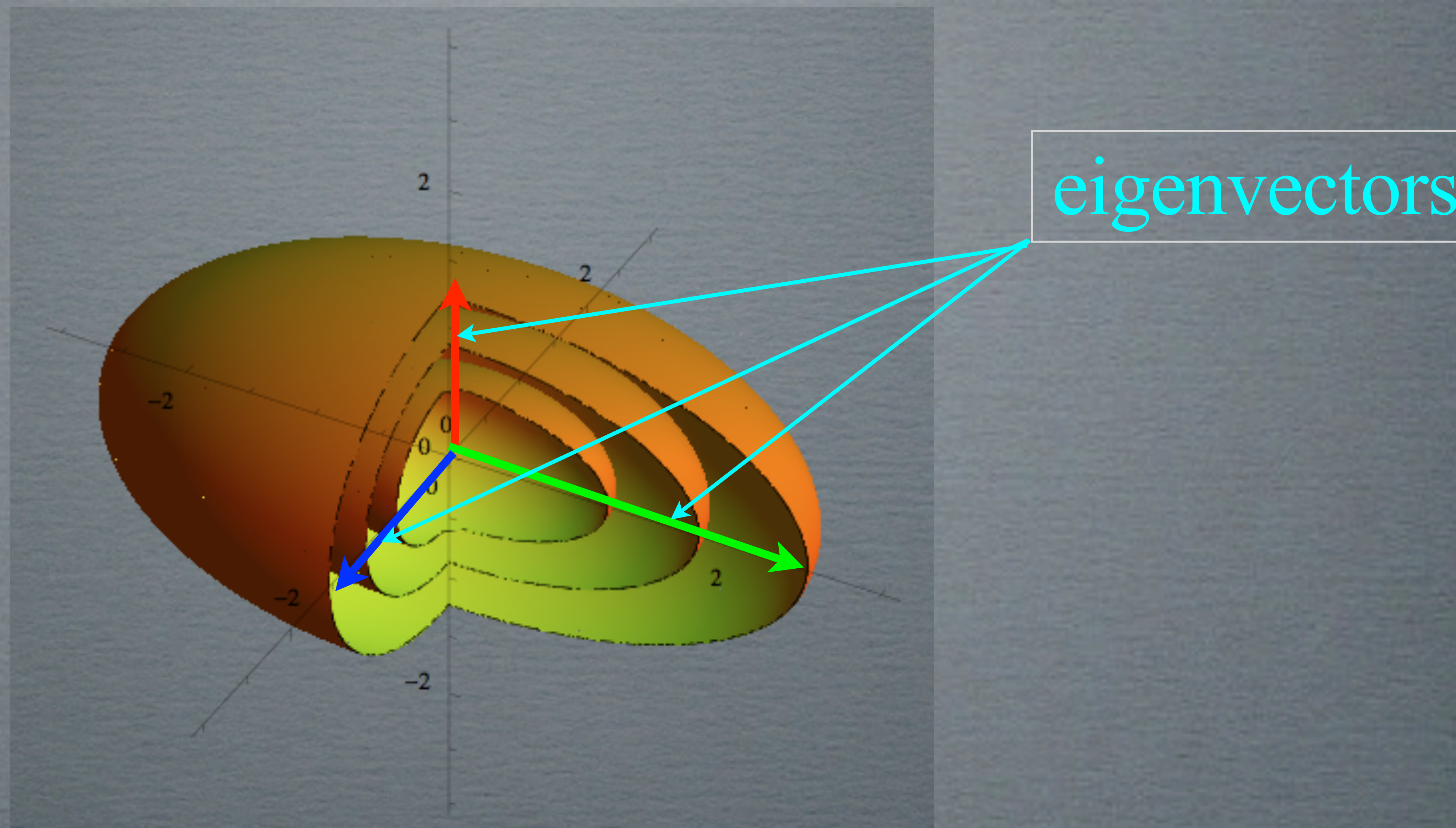


Diffusion Anisotropy in 3D



probability contours in 3D

The Sensitivity of MRI to Diffusion

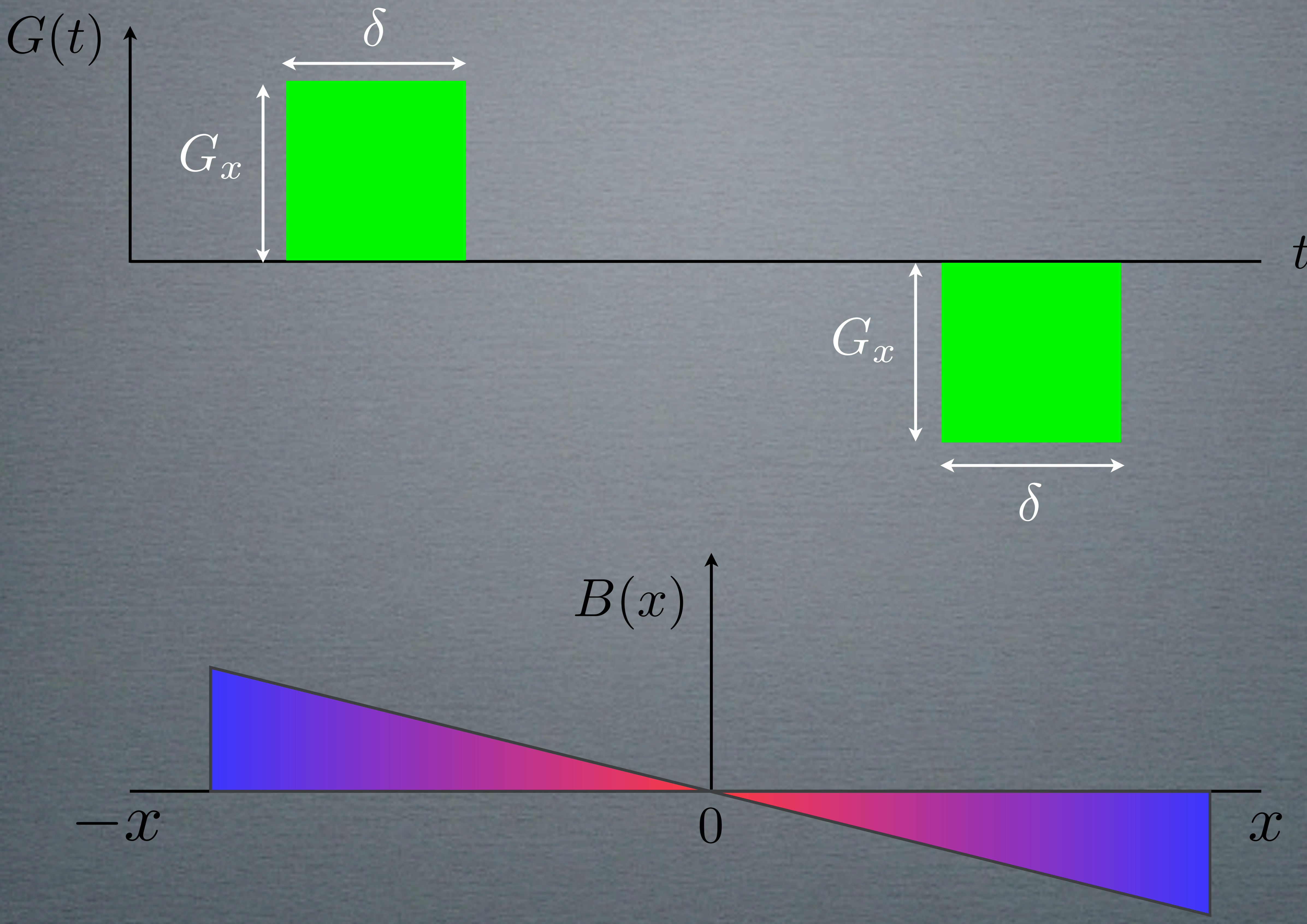
The Sensitivity of MRI to Diffusion

We've described the spatial and Temporal characteristics of the molecules.

What is the influence of this on the MRI signal?

The Bipolar Gradient Pulse (gradient echo)

The Bipolar Gradient Pulse (gradient echo)



The Bipolar Gradient Pulse (spin echo)

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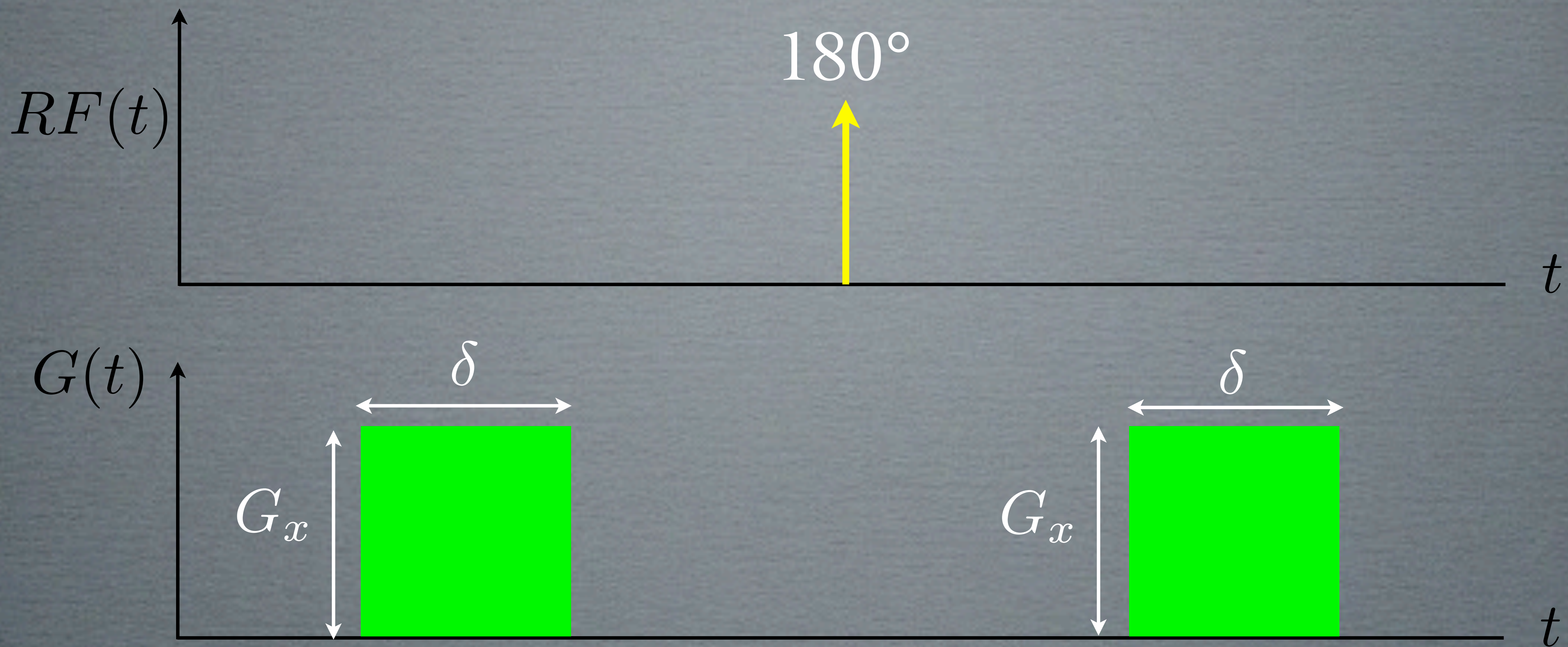
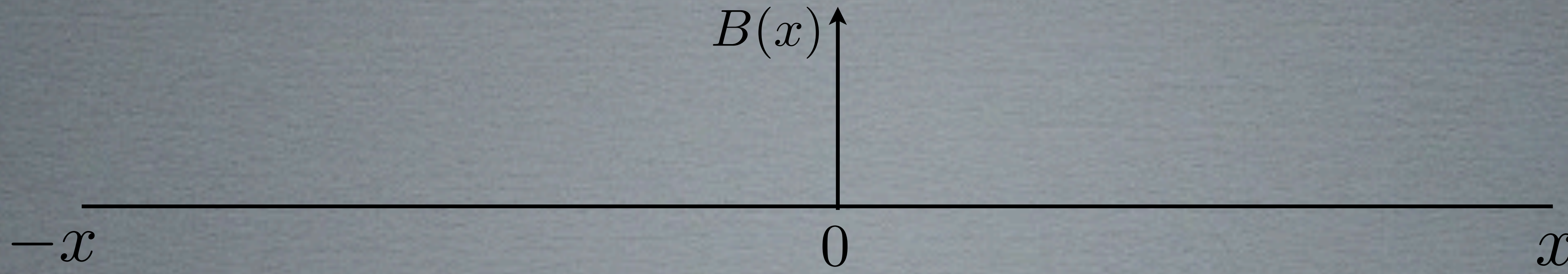


Stationary Spins in Bipolar Pulse



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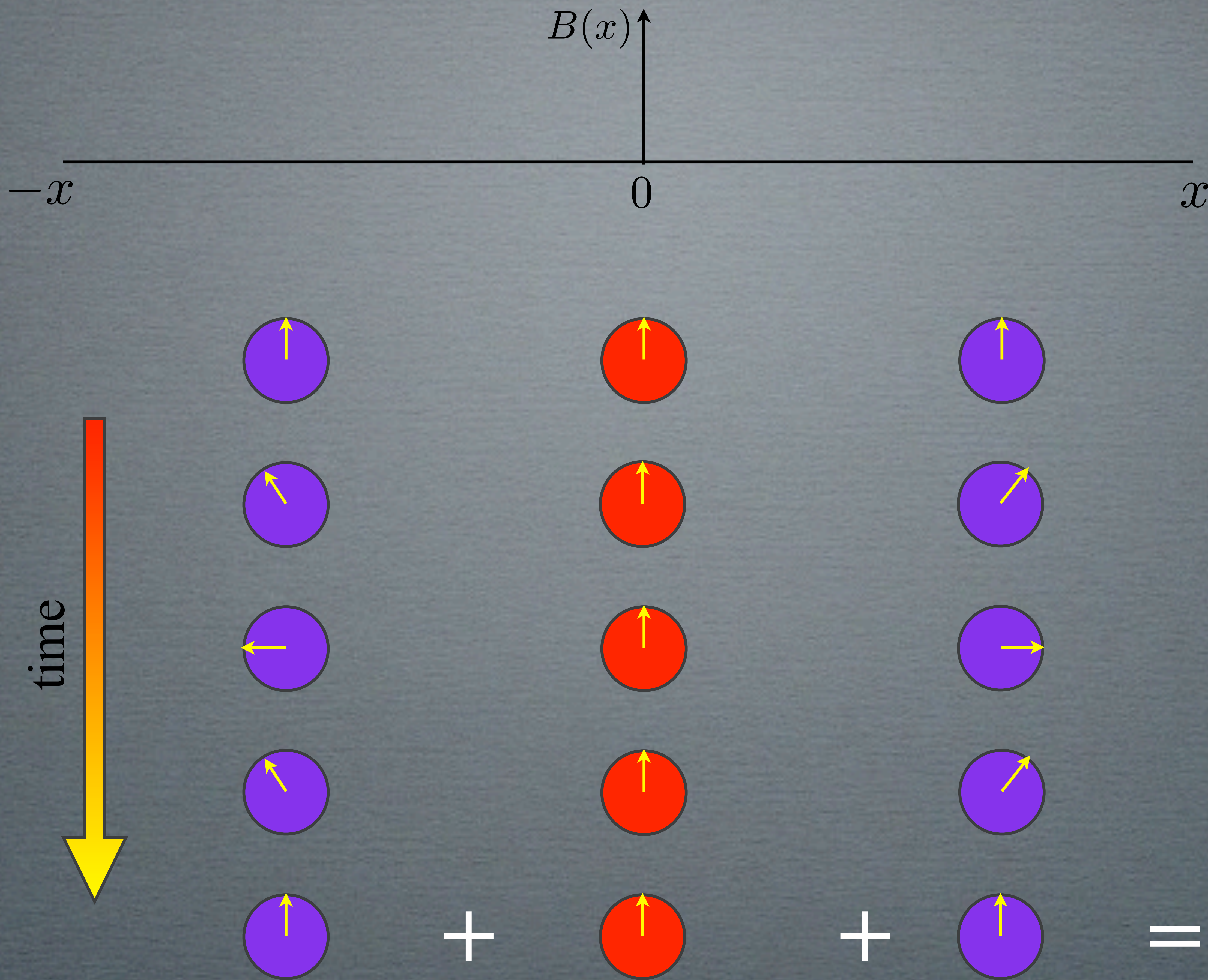
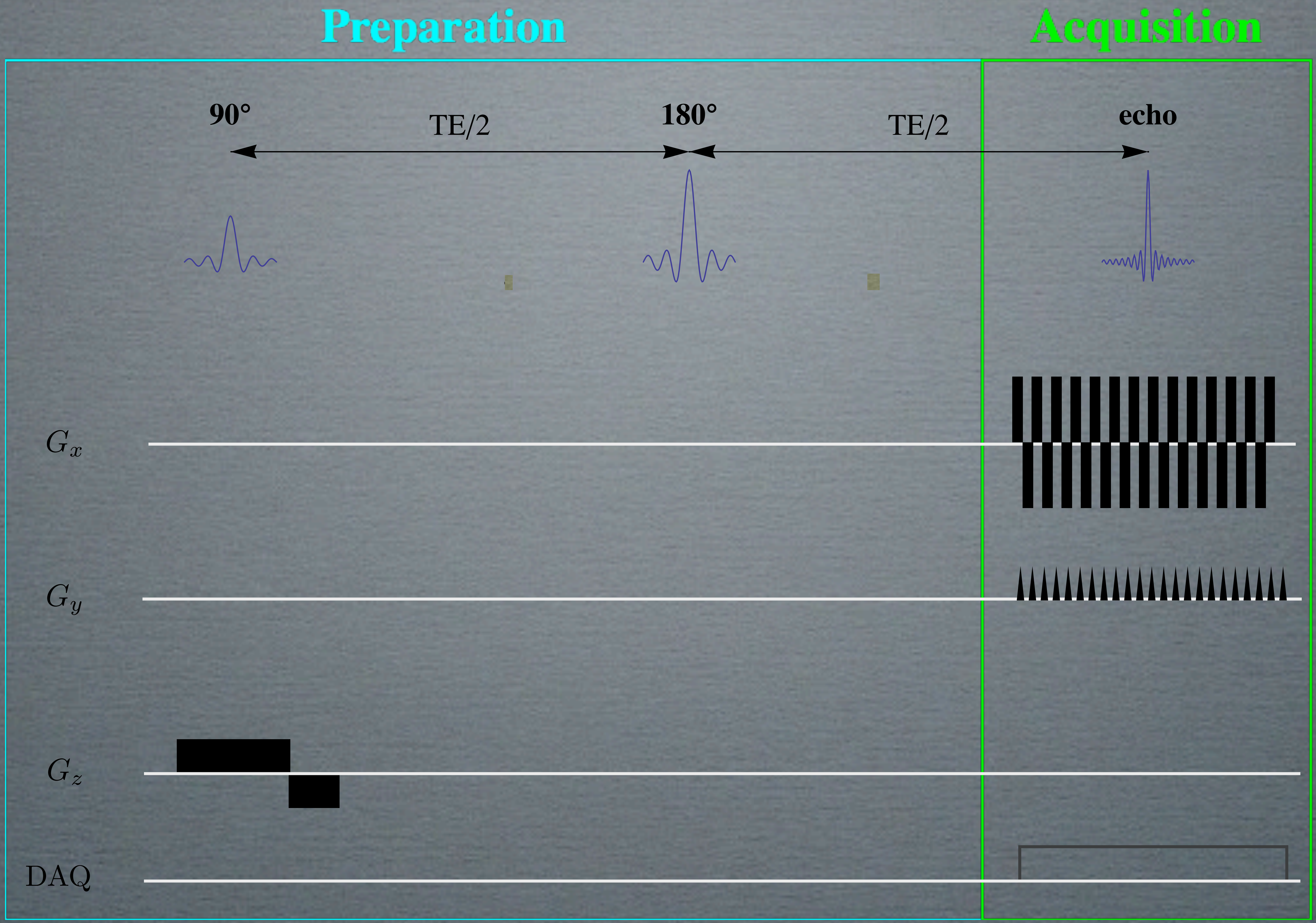
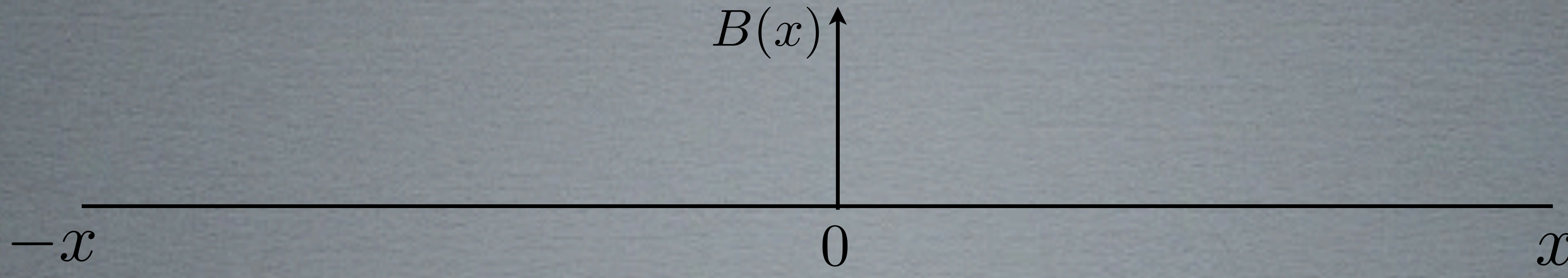


Monday, November 25, 13

Echo-Planar Imaging



Diffusing Spins in Bipolar Pulse



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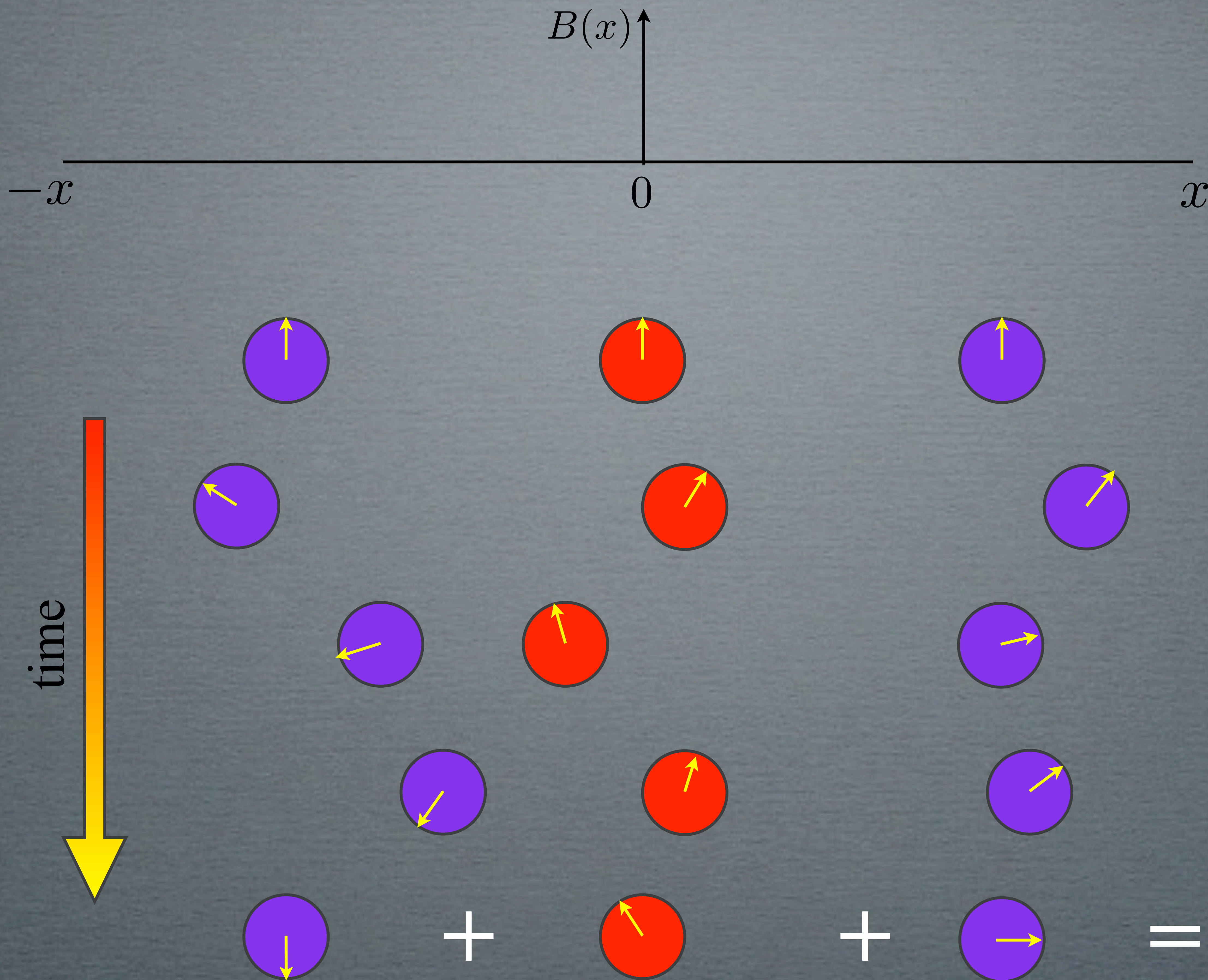


Key Fact

Key Fact

Only diffusion along the direction of the applied gradient has an effect

early NMR measurements of Diffusion

early NMR measurements of Diffusion

PHYSICAL REVIEW VOLUME 80, NUMBER 4 NOVEMBER 15, 1950

Spin Echoes*†

E. L. HAHN‡
Physics Department, University of Illinois, Urbana, Illinois

(Received May 22, 1950)

Intense radiofrequency power in the form of pulses is applied to an ensemble of spins in a liquid placed in a large static magnetic field H_0. The frequency of the pulsed r-f power satisfies the condition for nuclear magnetic resonance, and the pulses last for times which are short compared with the time in which the nutating macroscopic magnetic moment of the entire spin ensemble can decay. After removal of the pulses a non-equilibrium configuration of isochromatic macroscopic moments remains in which the moment vectors precess freely. Each moment vector has a magnitude at a given precession frequency which is determined by the distribution of Larmor frequencies imposed upon the ensemble by inhomogeneities in H_0. At times determined by pulse sequences applied in the past the constructive interference of these moment vectors gives rise to observable spontaneous nuclear induction signals. The properties and underlying principles of these spin echo signals are discussed with use of the Bloch theory. Relaxation times are measured directly and accurately from the measurement of echo amplitudes. An analysis includes the effect on relaxation measurements of the self-diffusion of liquid molecules which contain resonant nuclei. Preliminary studies are made of several effects associated with spin echoes, including the observed shifts in magnetic resonance frequency of spins due to magnetic shielding of nuclei contained in molecules.

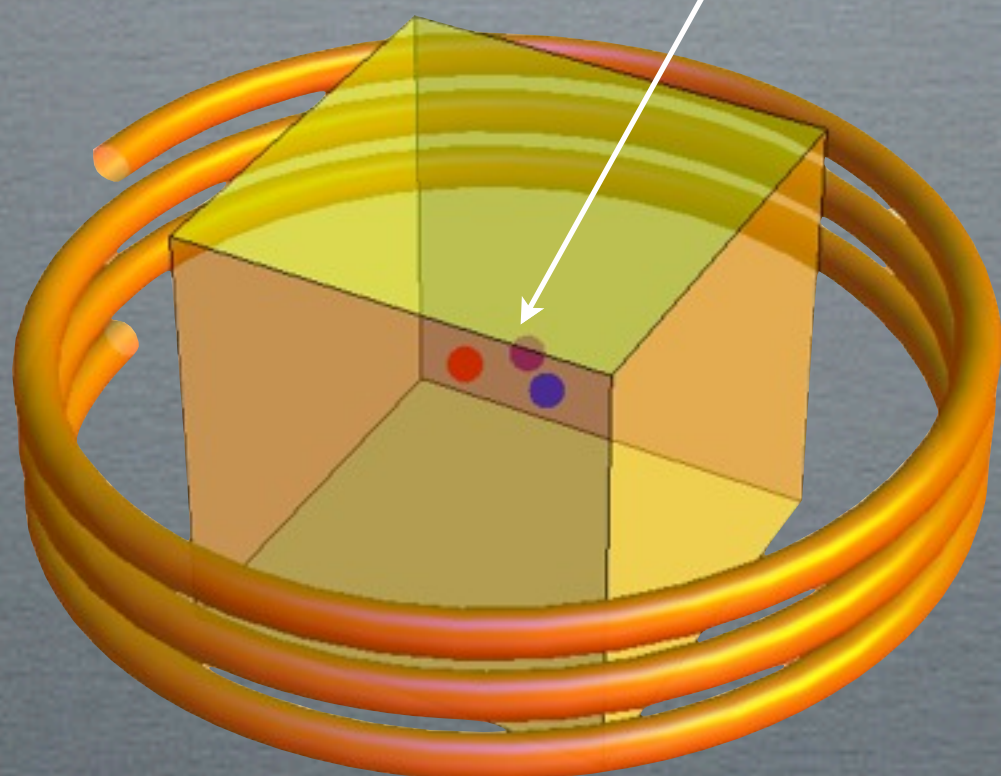
Since there is an established gradient of the magnetic field over the volume of the sample, a molecule whose nuclear moment has been flipped initially in a field H_0, may, in the course of time 2τ, drift by Brownian motion into a randomly differing field H_0. Therefore, as τ is increased, a lesser number of moments participate in the generation of in-phase nuclear radio-frequency signals.

The MRI Signal

The MRI Signal

signal = Sum over all spins

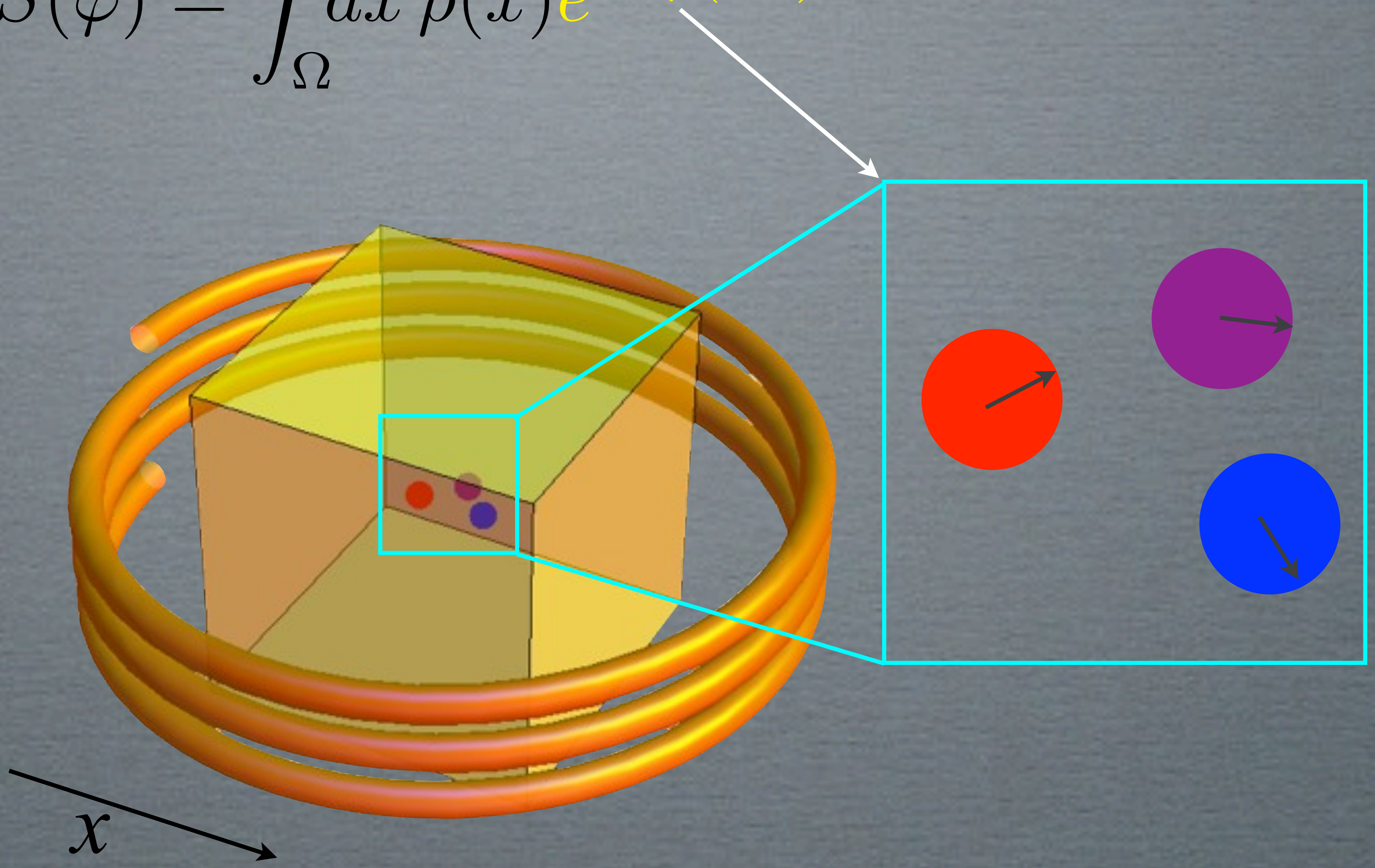
$$S(\varphi) = \int_{\Omega} dx \text{ spins(location, phase)}$$



The MRI Signal

signal = Sum over all spins

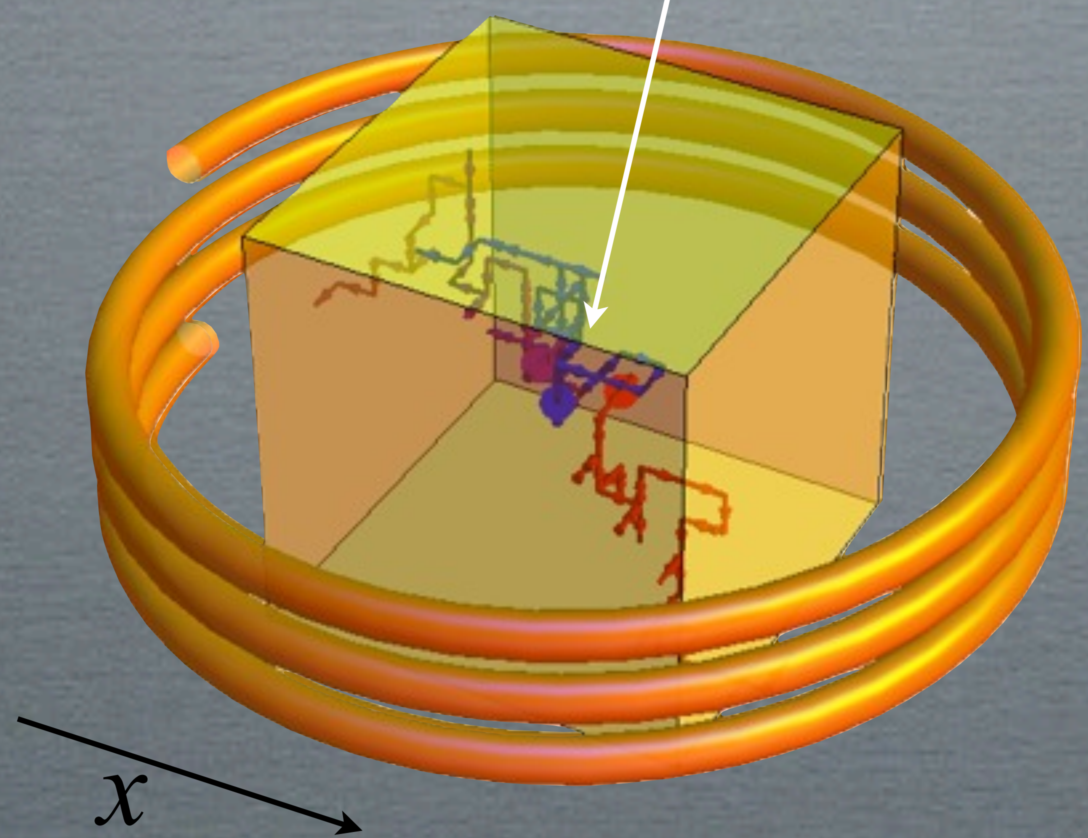
$$S(\varphi) = \int_\Omega dx\, \rho(x) e^{-i\varphi(x,t)}$$



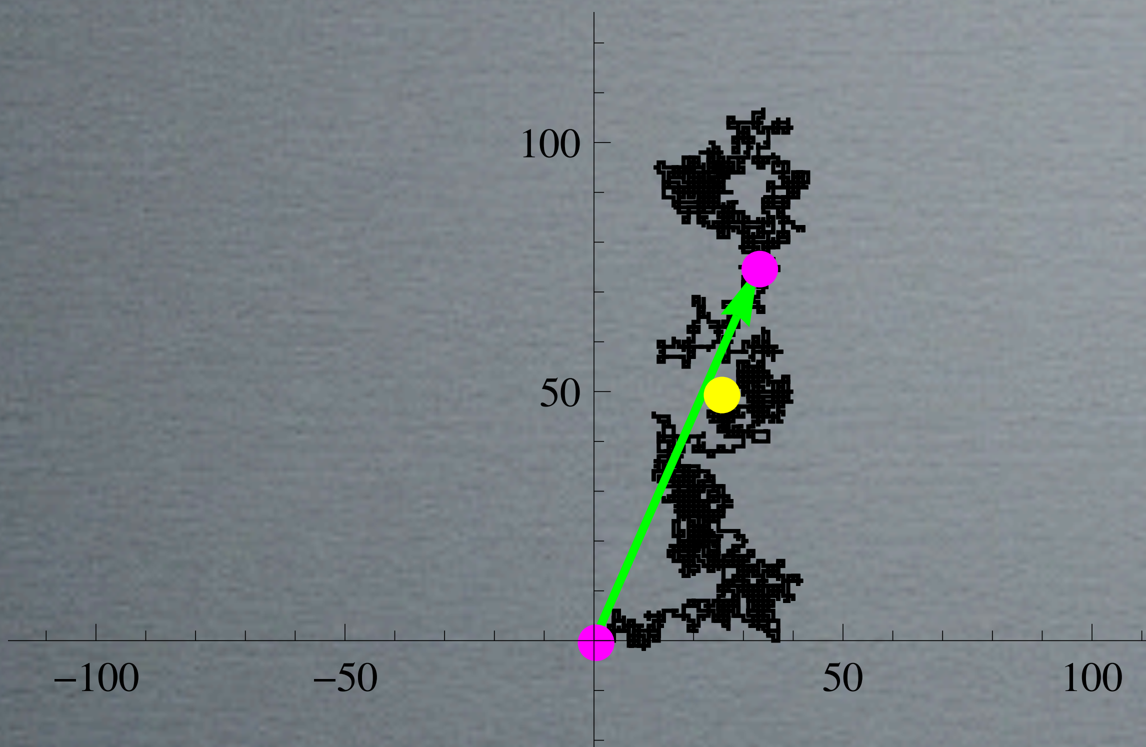
The MRI Signal

signal = Sum over all spins

$$S(\varphi) = \int_{\Omega} dx\, P(x,t)\, e^{-i\varphi(x,t)}$$



Diffusion phase in a Bipolar Pulse



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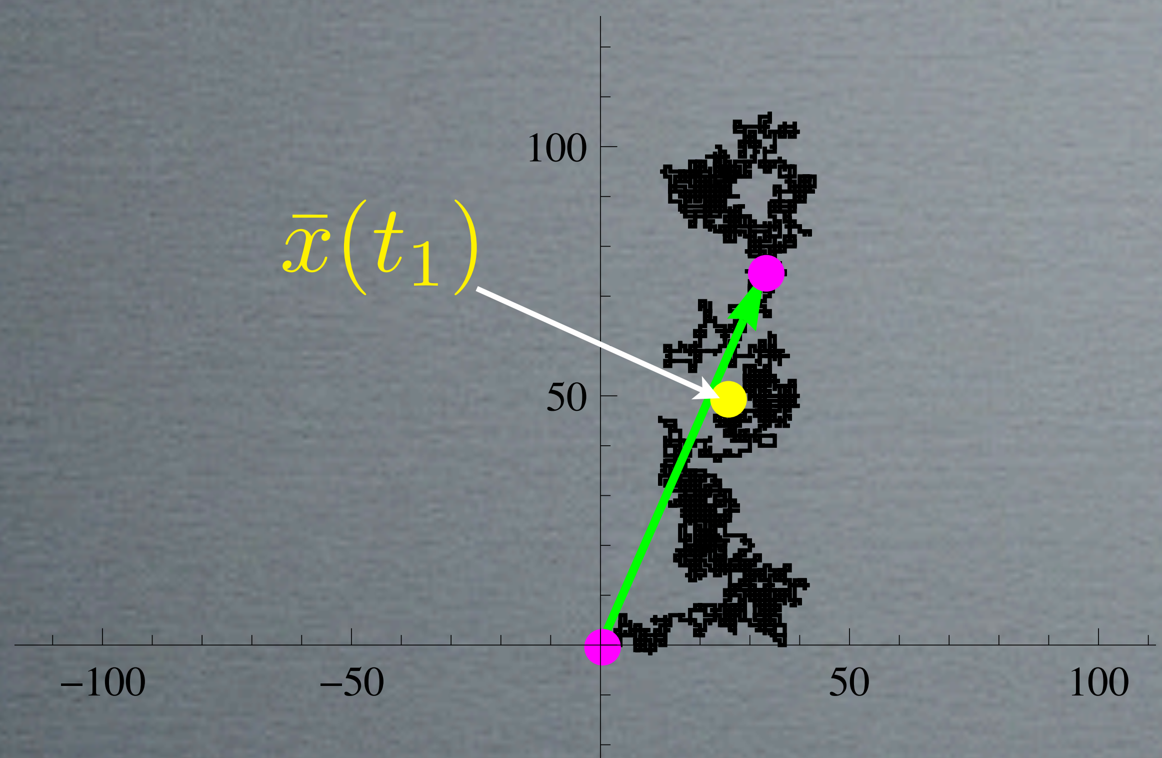
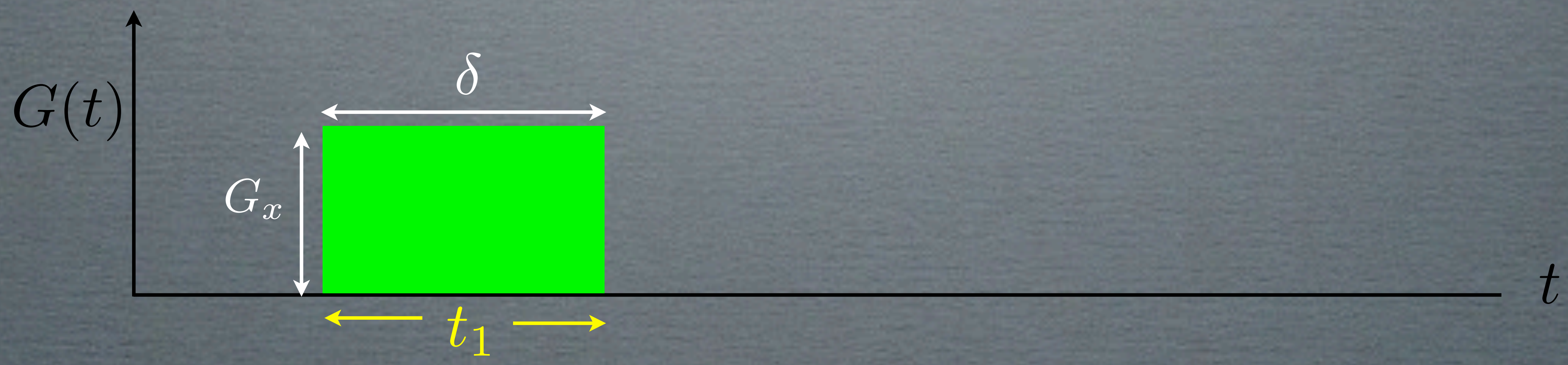
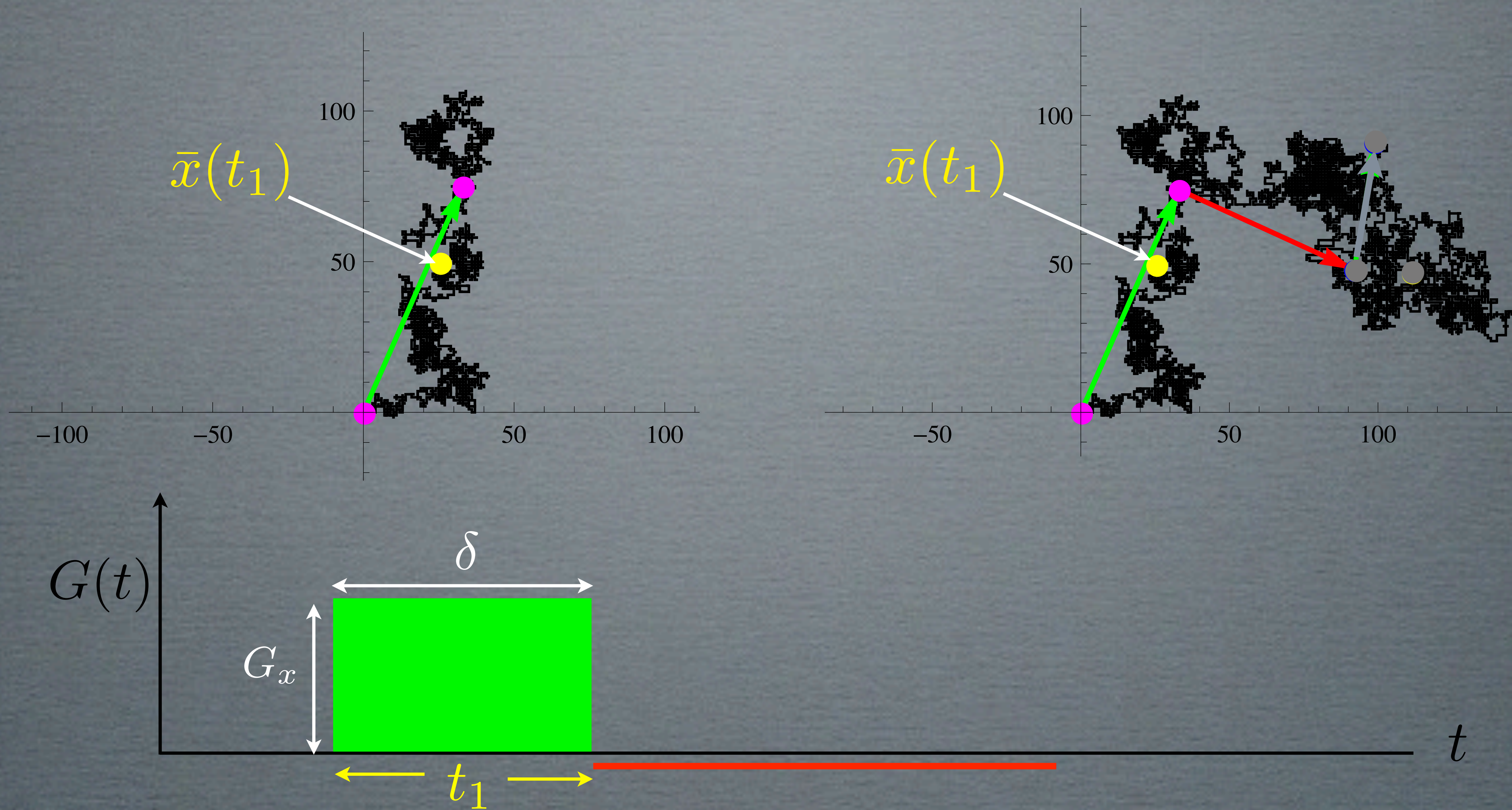
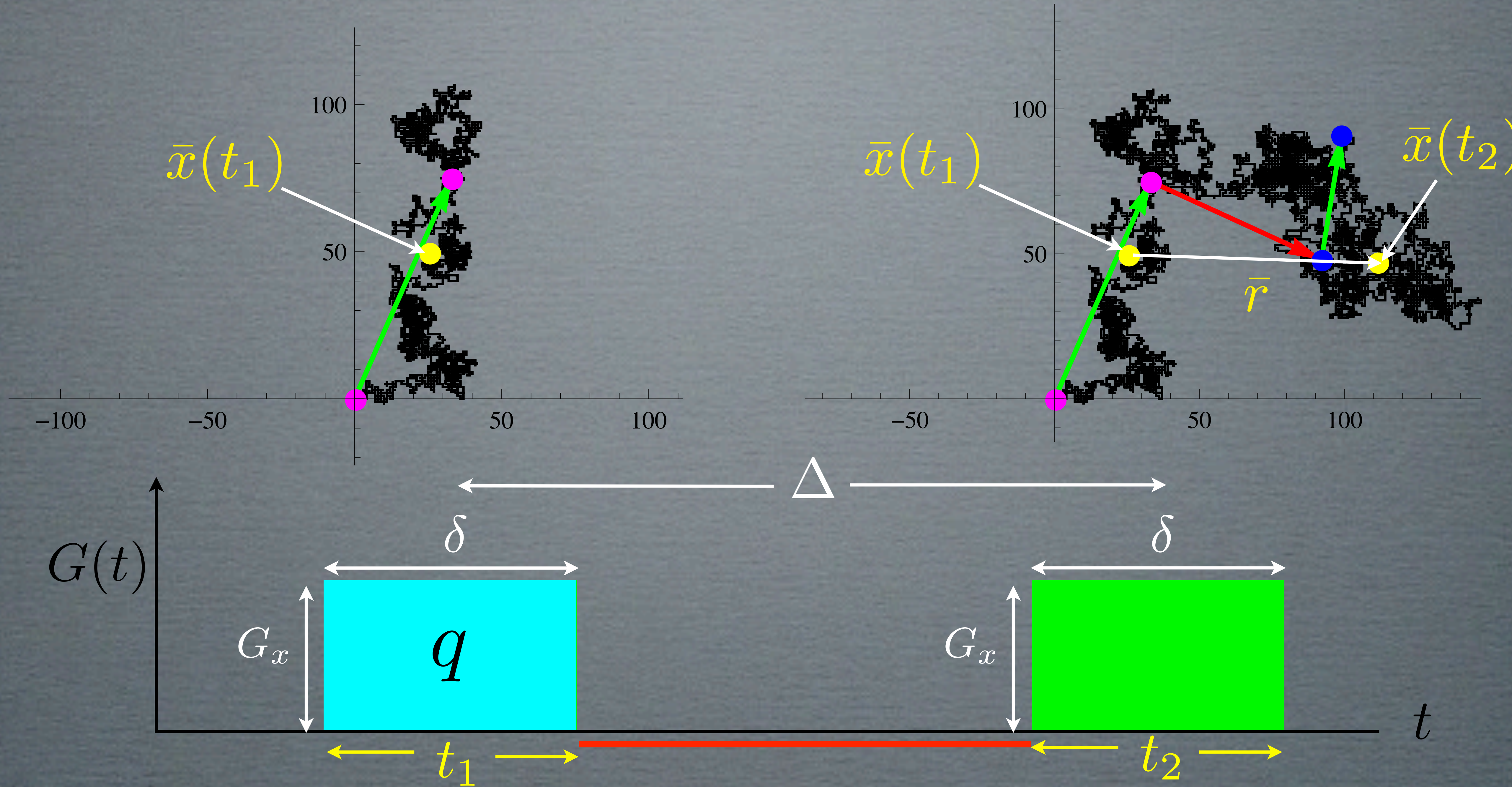




Diffusion phase in a Bipolar Pulse



Diffusion phase in a Bipolar Pulse



$$\varphi(x,t) = \underbrace{G\delta}_{q}\,\underbrace{[\bar{x}(t_2) - \bar{x}(t_1)]}_{\bar{r}} = q\bar{r}$$

The Diffusion Weighted Signal

The Diffusion Weighted Signal

Signal and Distribution are
Fourier Transform pairs

$$\mathfrak{s}(\boldsymbol{q}, \tau) = \int P(\bar{\boldsymbol{r}}, \tau) e^{-i\boldsymbol{q}\cdot\bar{\boldsymbol{r}}} d\bar{\boldsymbol{r}}$$

$$\Updownarrow$$

$$P(\bar{\boldsymbol{r}}, \tau) = \int \mathfrak{s}(\boldsymbol{q}, \tau) e^{i\boldsymbol{q}\cdot\bar{\boldsymbol{r}}} dq$$

So, in principal, you can measure *P(r,τ)* by collecting data throughout *q*-space, just like imaging.
In practice, *very* time consuming

The Estimation Problem for Gaussian Diffusion

The Estimation Problem for Gaussian Diffusion

$$S(\varphi) = \int_\Omega dx\, P(x,t)\, e^{-i\varphi(x,t)}$$

The Estimation Problem for Gaussian Diffusion

$$S(\varphi) = \int_{\Omega} d\bar{r}\, P(\bar{r}, t)\, e^{-iq\bar{r}}$$

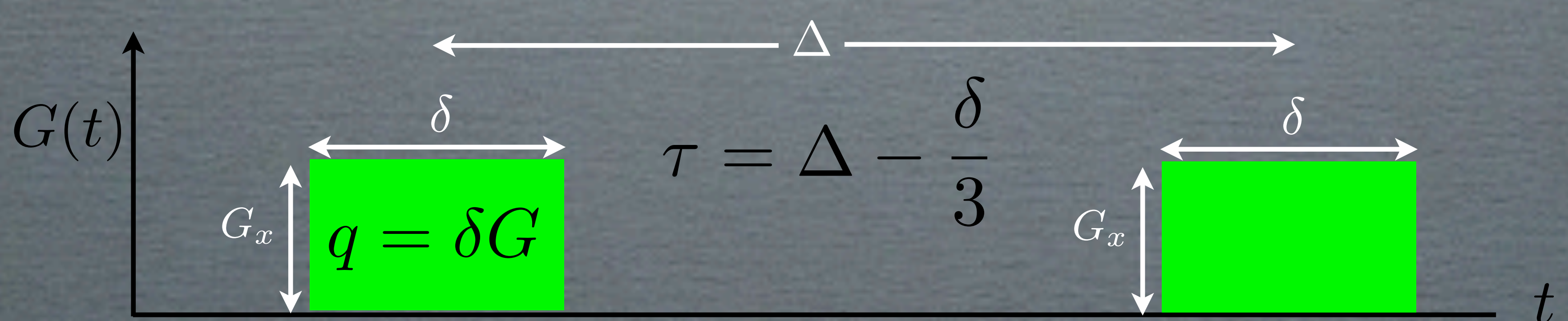
Gaussian

The Estimation Problem for Gaussian Diffusion

$$S(b) = S(0)\,e^{-bD} + \eta$$

- $S(b)$: measured signal
- $S(0)$: non-diffusion weighted signal (b=0)
- b: pulse sequence parameters
- D: object of our desire!
- η: noise

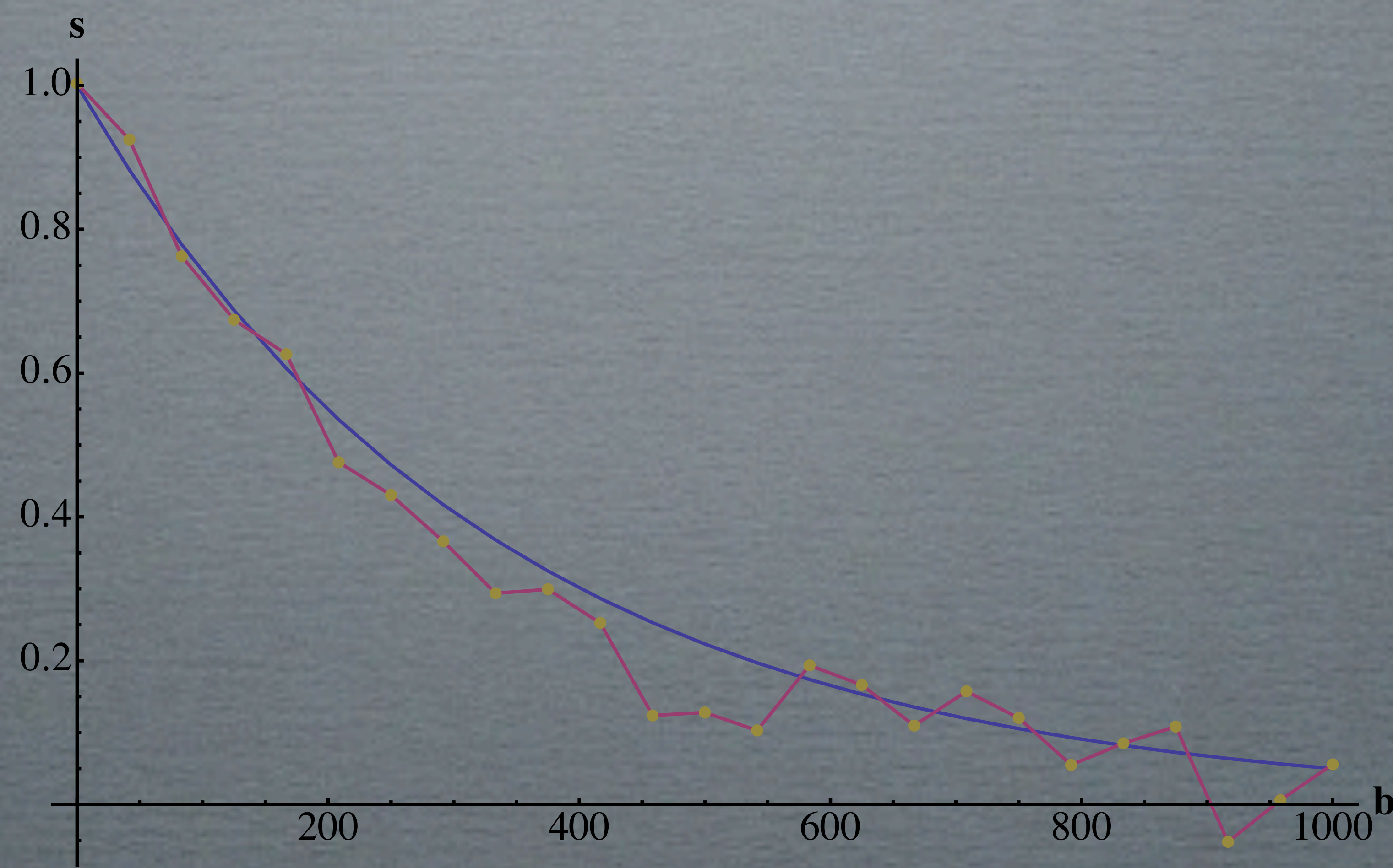
$$b = q^2\tau$$



$$\tau = \Delta - \frac{\delta}{3}$$

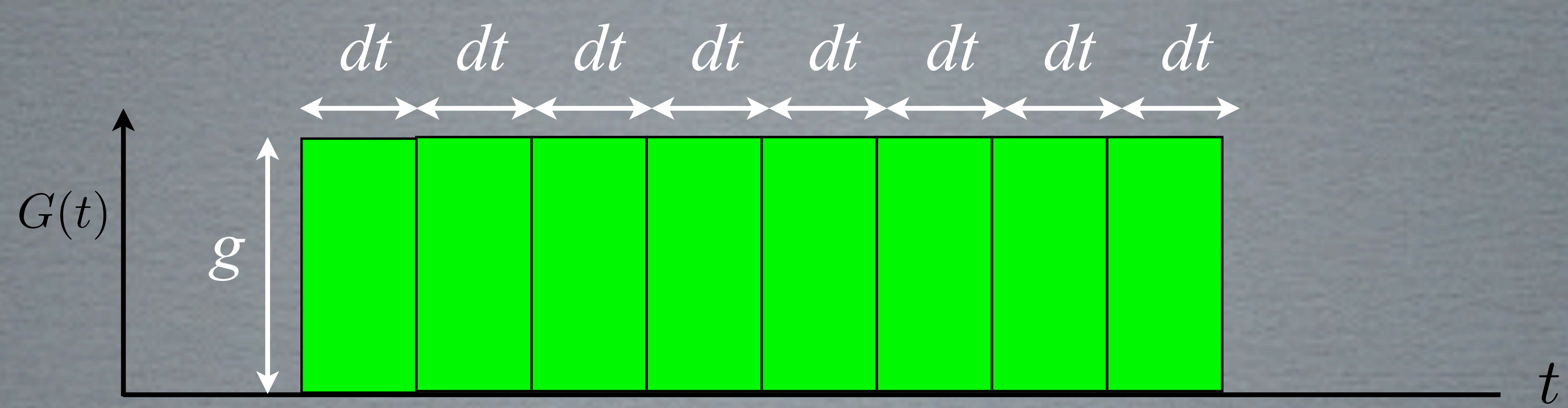
The signal from Gaussian Diffusion

$$s(b) = s(0)e^{-bD} + \eta(b)$$



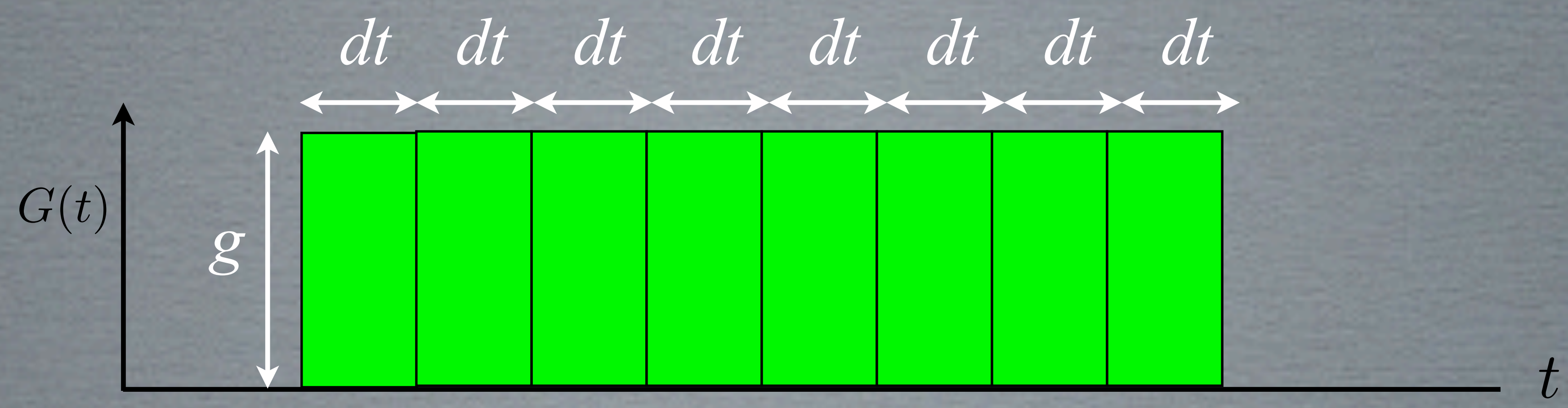
The b-value

The b-value



attenuation for each little time interval: $A_1 \; A_2 \; \cdots \; A_n$

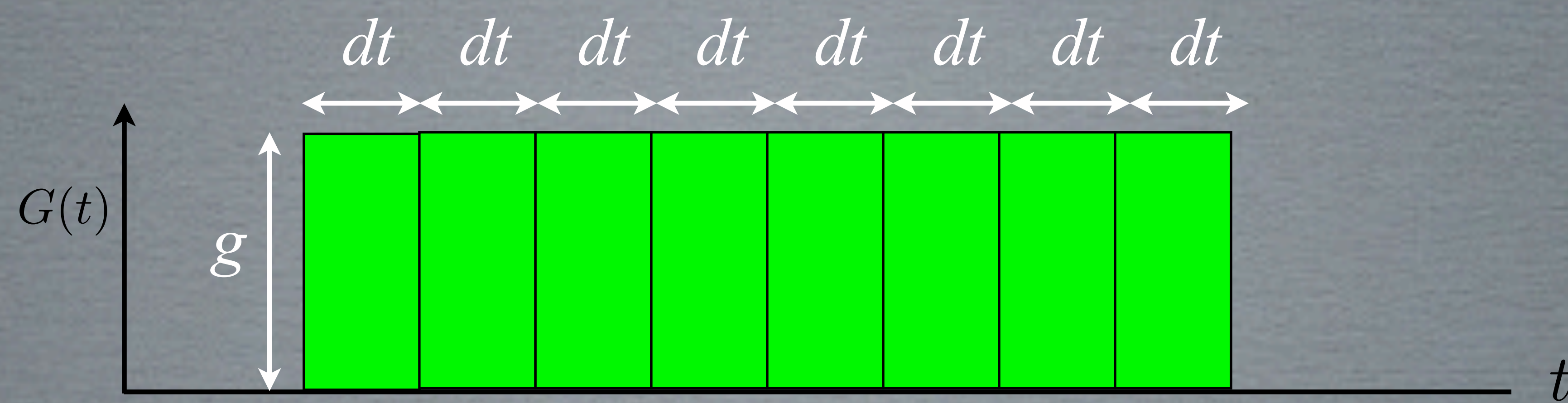
The b-value



attenuation for each little time interval: $A_1 \; A_2 \; \cdots \; A_n$

total attenuation $A_\tau = \prod_{i=1}^{n} A_i$ where $A_i = e^{-k^2 D\, dt}$

The b-value



attenuation for each little time interval: $A_1 \; A_2 \; \cdot \; \cdot \; \cdot \; A_n$

total attenuation $A_\tau = \prod_{i=1}^{n} A_i$ where $A_i = e^{-k^2 D\, dt}$

$$A_\tau = \prod_{i=1}^{n} e^{-k^2 D\, dt} = e^{-D \sum_{i=1}^{n} k^2\, dt} = e^{-D \int k^2\, dt}$$

$dt \to \epsilon$

b

$$A_\tau = e^{-D \int k^2 dt}$$

The b-value

The b-value

$$b = g^2\delta^2\left(\Delta - \frac{1}{3}\delta\right)$$

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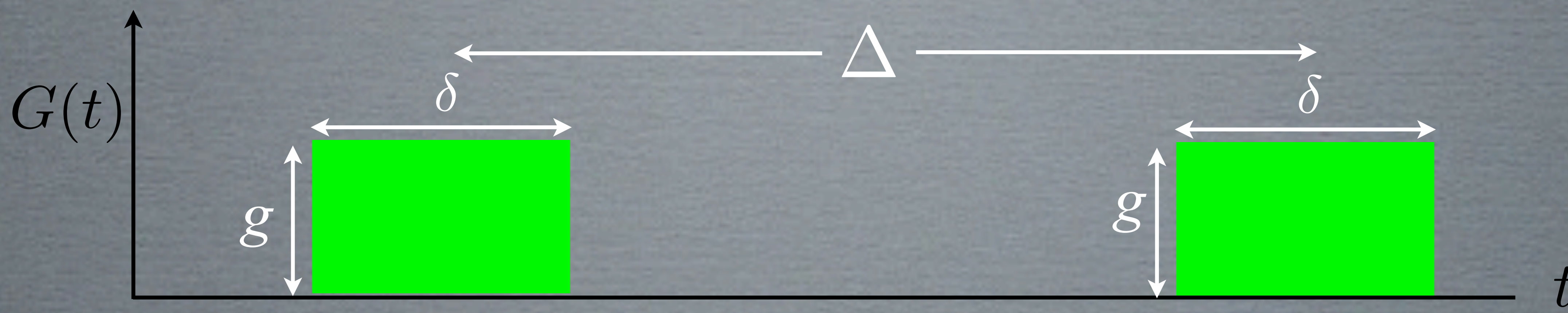
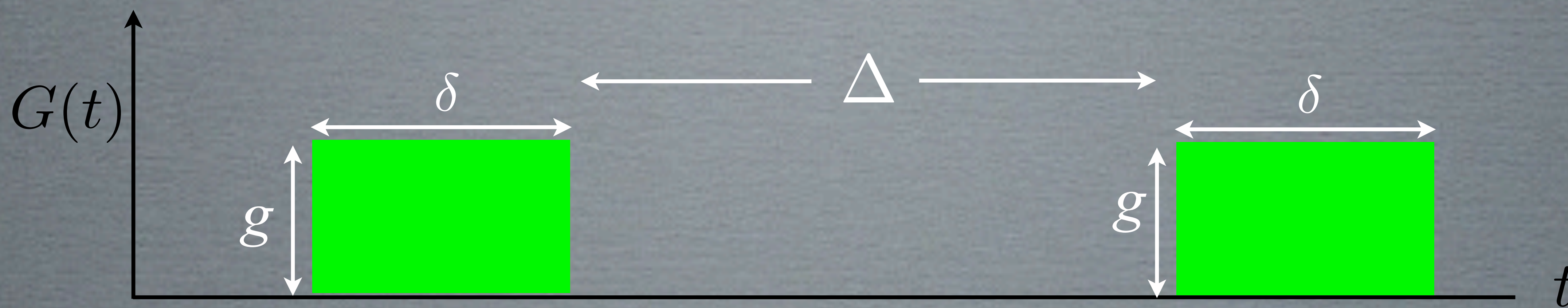


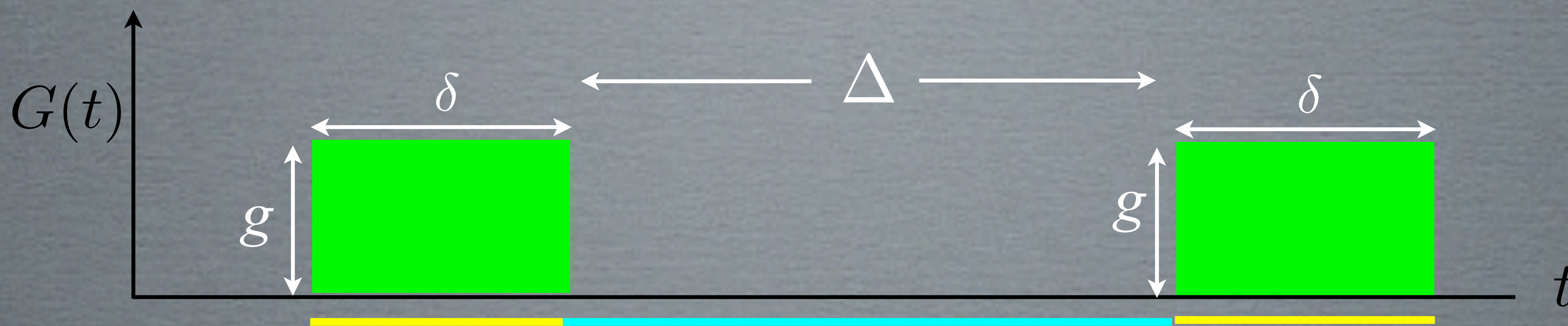
The b-value

$$b = g^2\delta^2(\Delta + \frac{2}{3}\delta)$$



The b-value

$$b = g^2\delta^2(\Delta + \frac{2}{3}\delta)$$



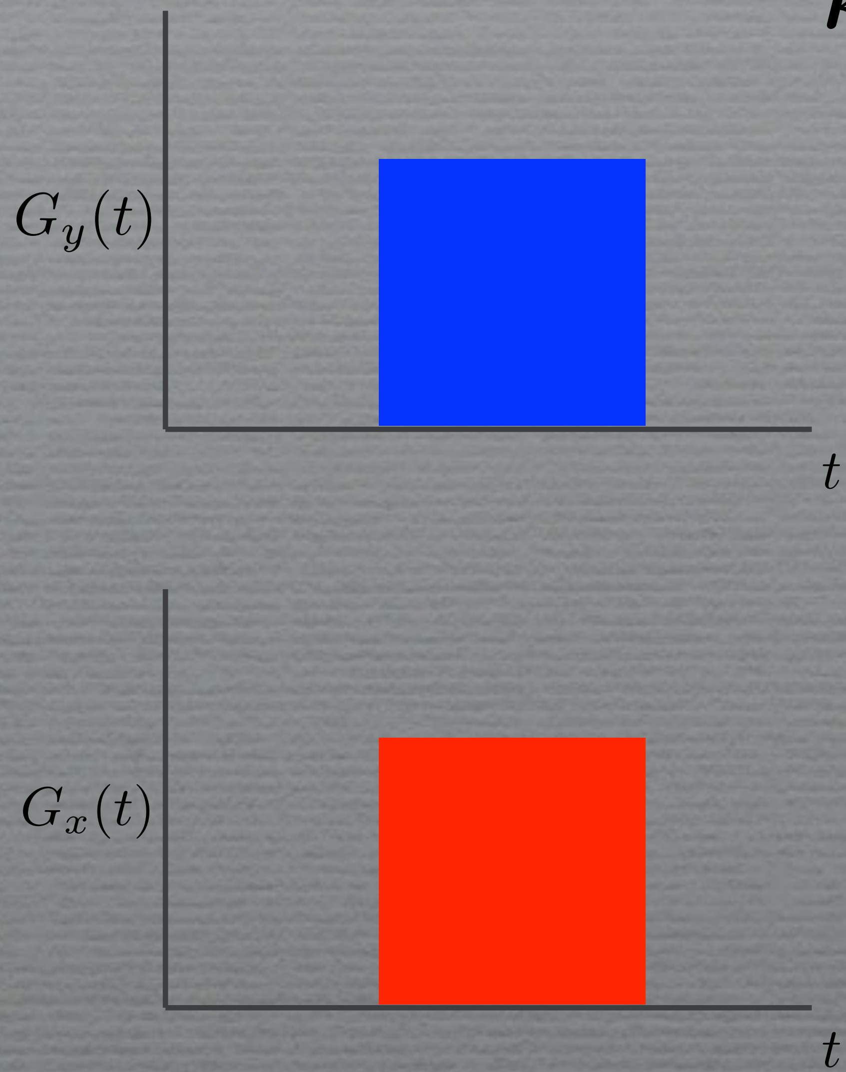
$$\int k^2\, dt = \quad g^2\int_0^{\delta} t^2\, dt \; + \; g^2\delta^2\int_0^{\Delta} dt \; + \; g^2\int_0^{\delta} t^2\, dt$$

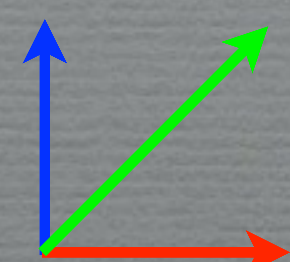
$$b = g^2\frac{\delta^3}{3} + g^2\delta^2\Delta + g^2\frac{\delta^3}{3}$$

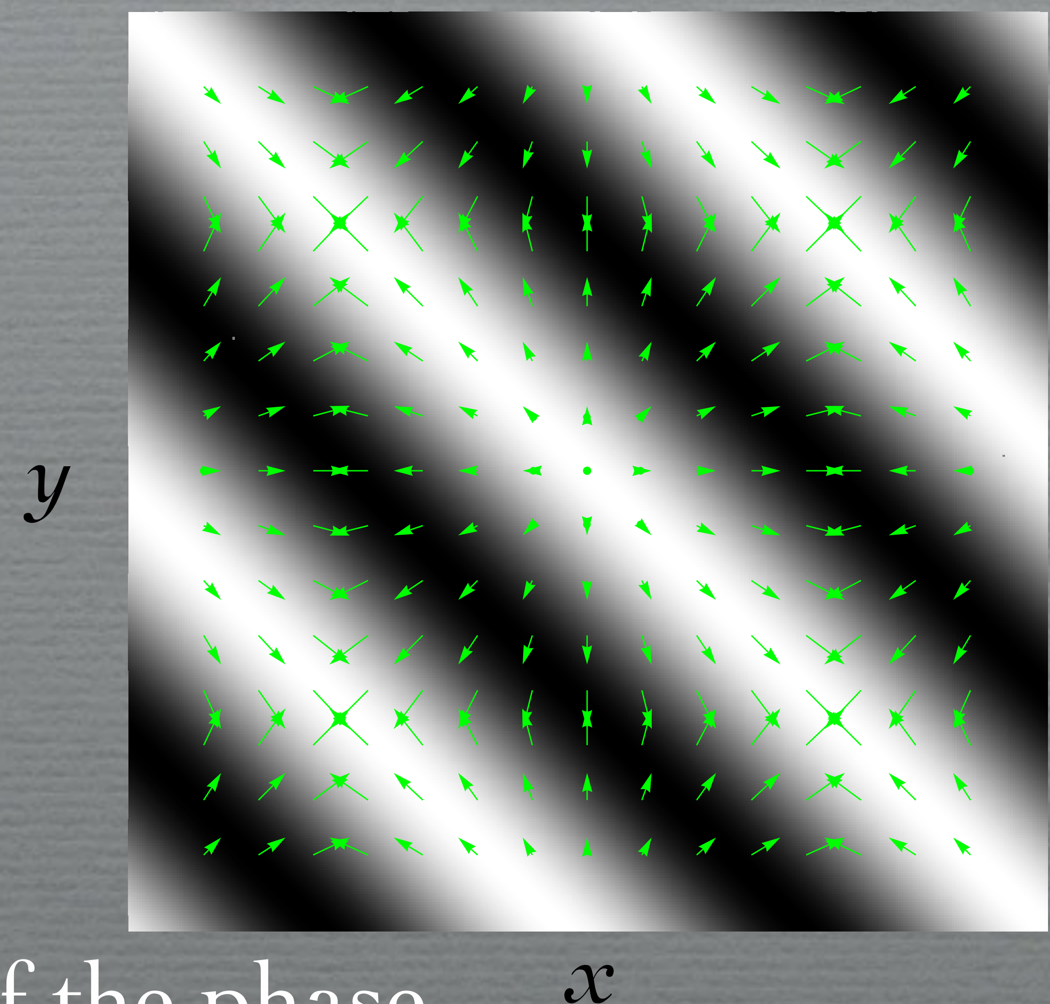
What gradients are doing to k-space

What gradients are doing to k-space

$$\boldsymbol{k}\cdot\boldsymbol{x} = k_x x + k_y y = \gamma G_x t x + \gamma G_y t y$$





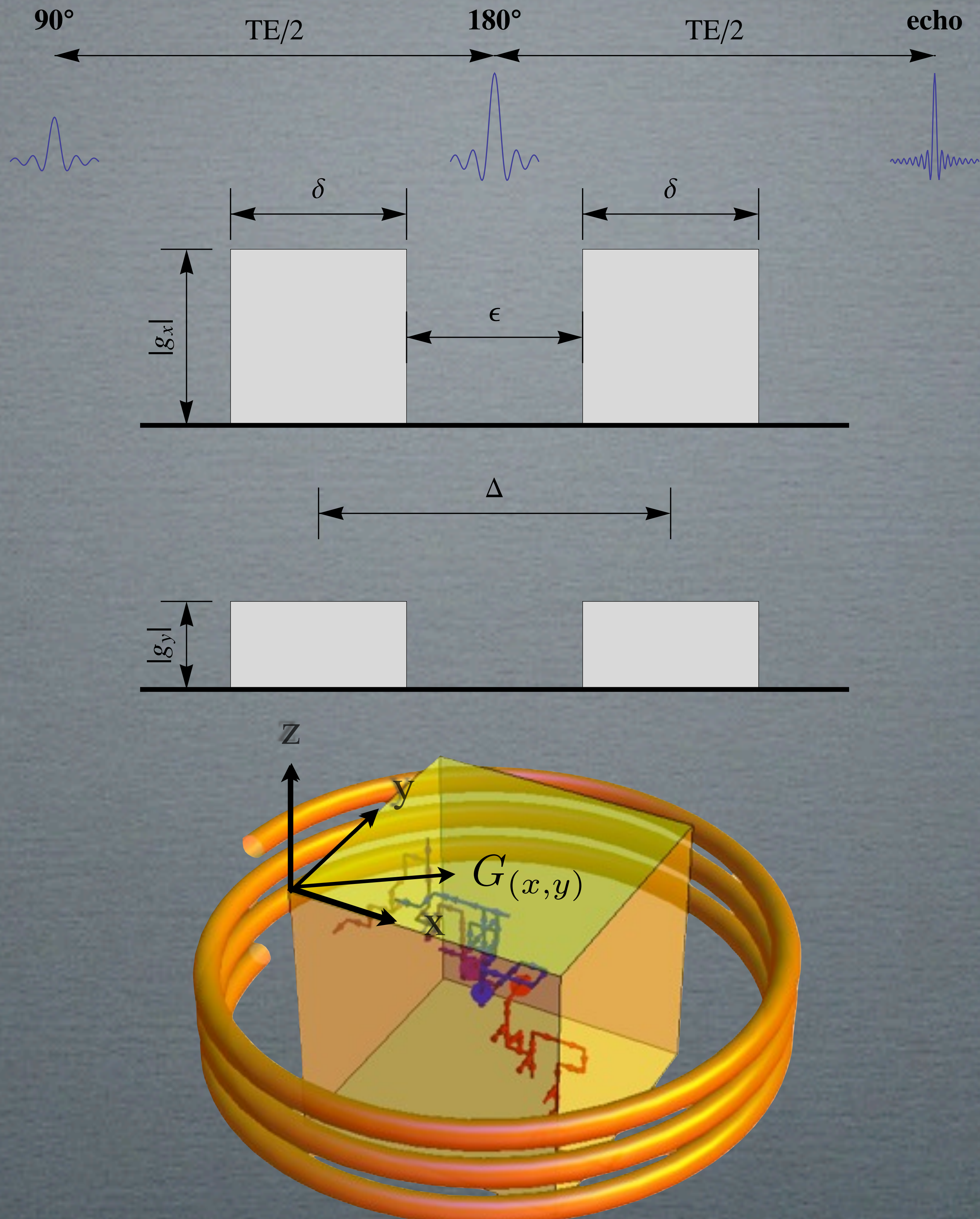


spatial modulation of the phase

gradients alter the k-space representation of the object

Directional Diffusion Encoding

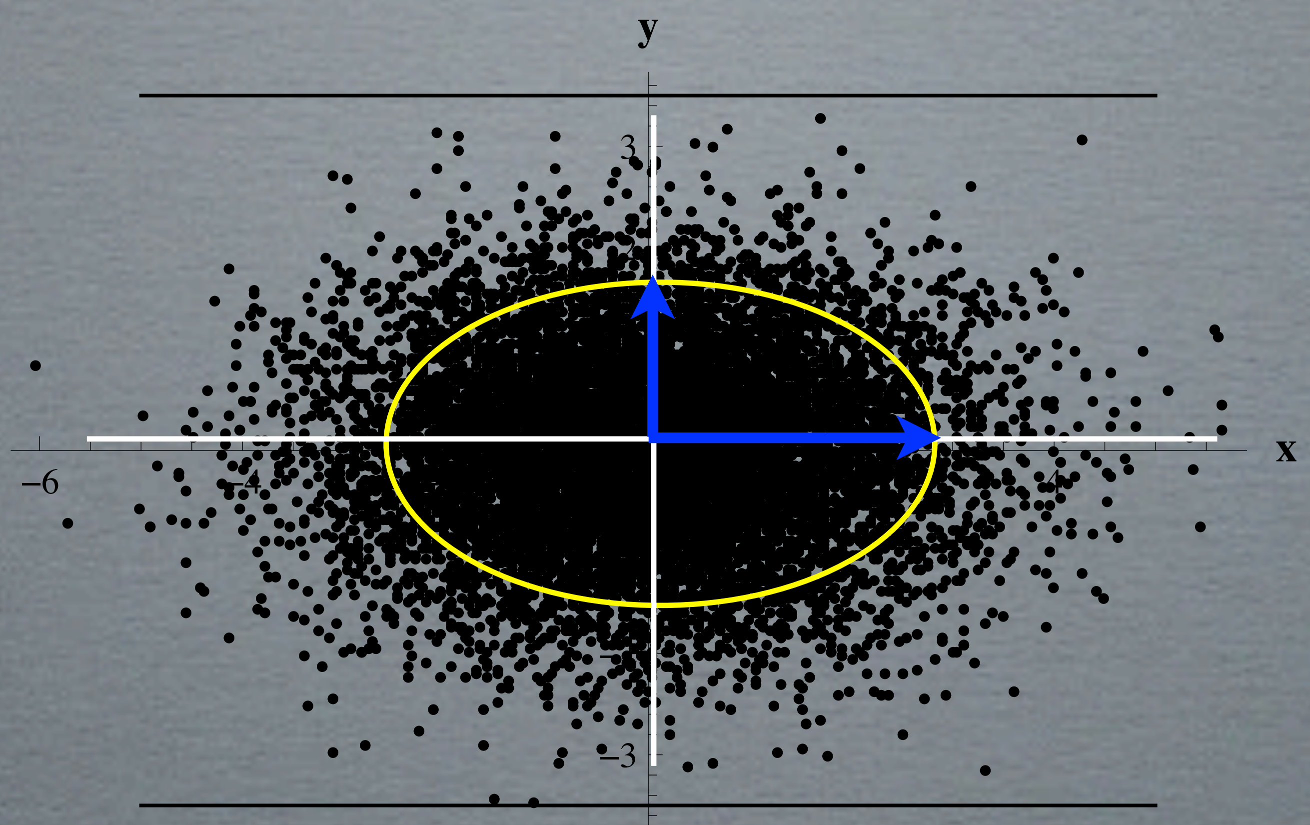
Directional Diffusion Encoding



Anisotropic diffusion in 2D

Anisotropic diffusion in 2D

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$

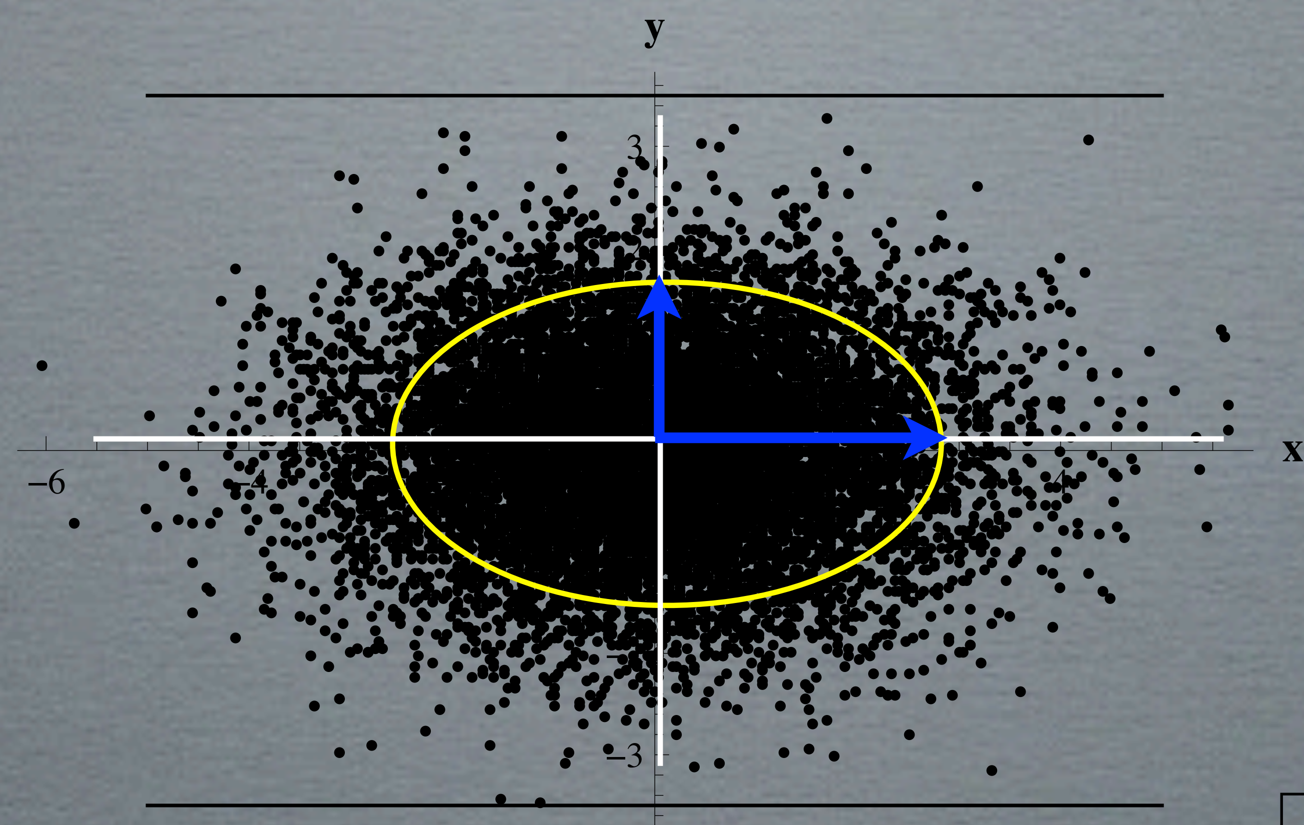


Covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}$$

Anisotropic diffusion in 2D

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$



Diffusion Tensor

Covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} = 2\tau \begin{pmatrix} D_x & 0 \\ 0 & D_y \end{pmatrix}$$

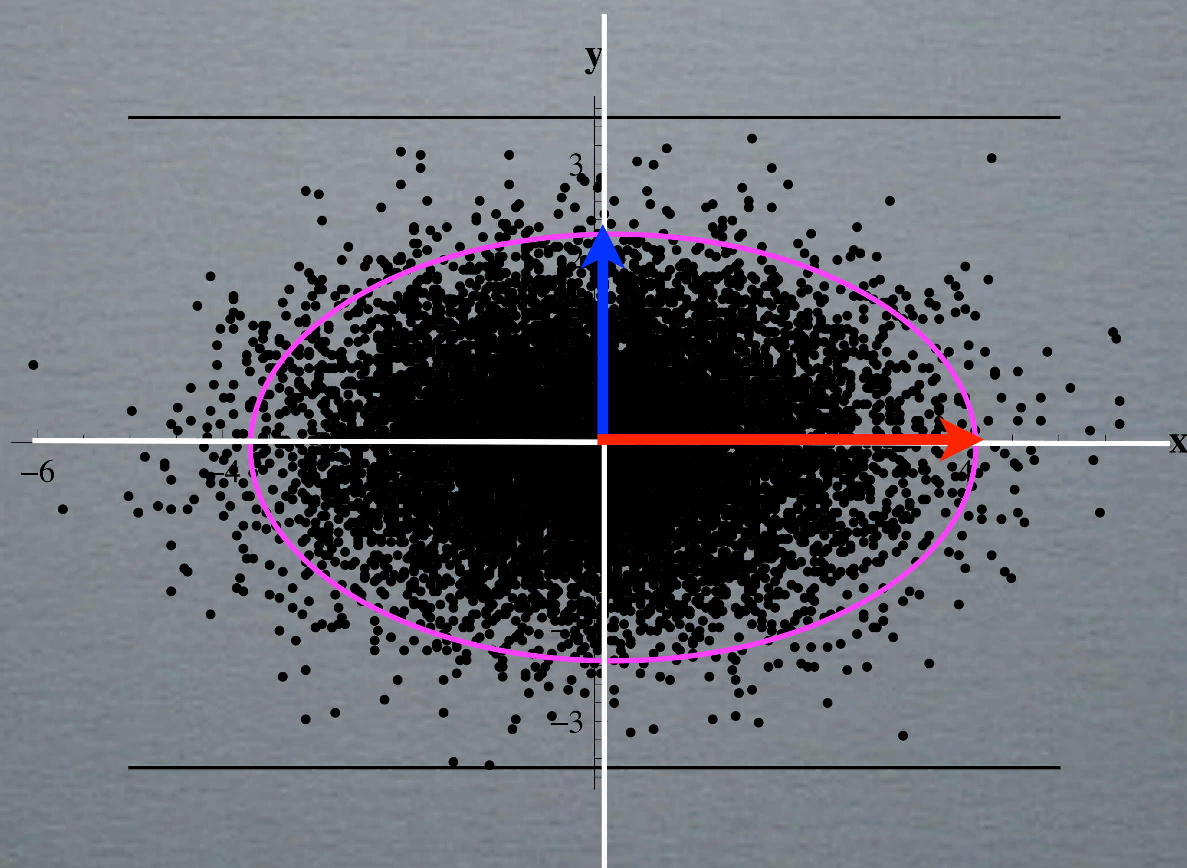
$\{D_x, D_y\}$ are the *principal diffusivities*

Measuring the Diffusion Tensor

Measuring the Diffusion Tensor

$$S(b, \hat{r}) = S(0)e^{-b\tilde{D}} + \eta$$

$$\boldsymbol{D} = \begin{pmatrix} D_x & 0 \\ 0 & D_y \end{pmatrix}$$



Measuring the Diffusion Tensor

$$S(b, \hat{r}) = S(0)e^{-b\tilde{D}} + \eta$$

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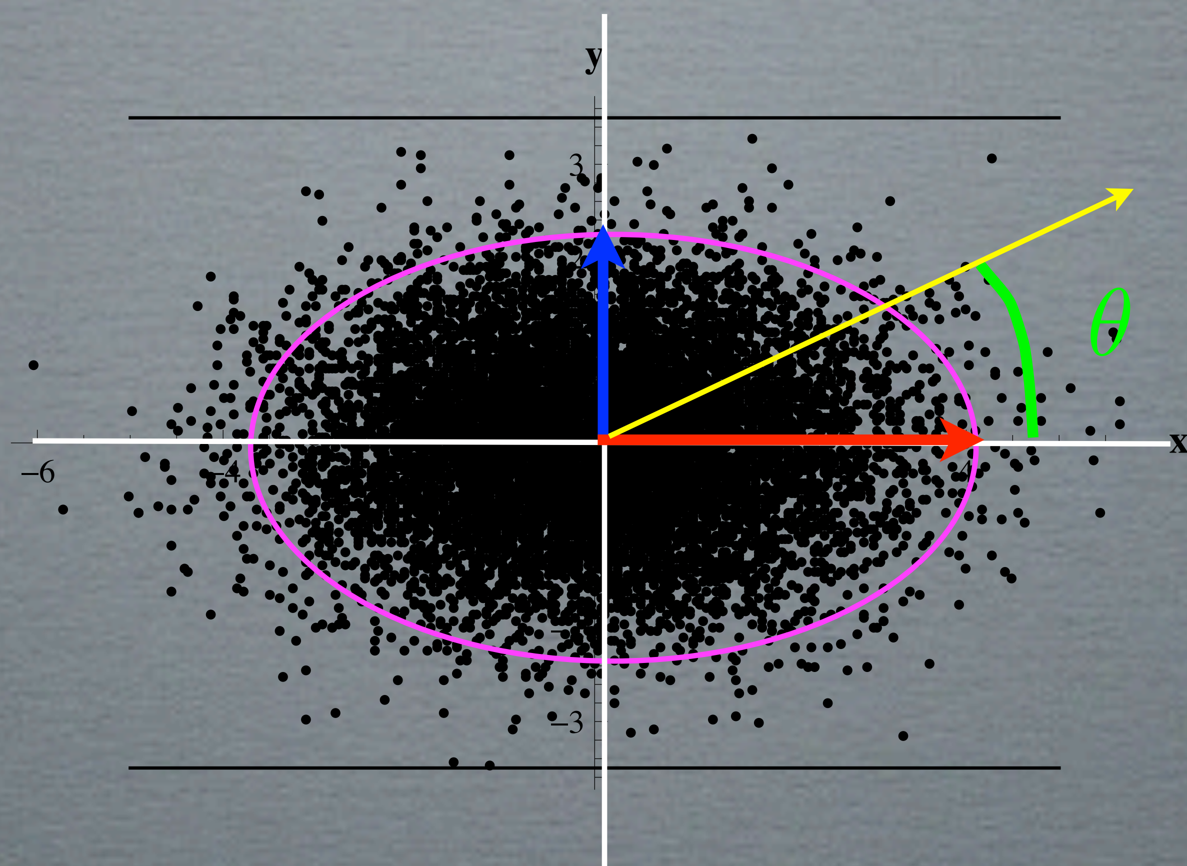


$$\hat{r} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

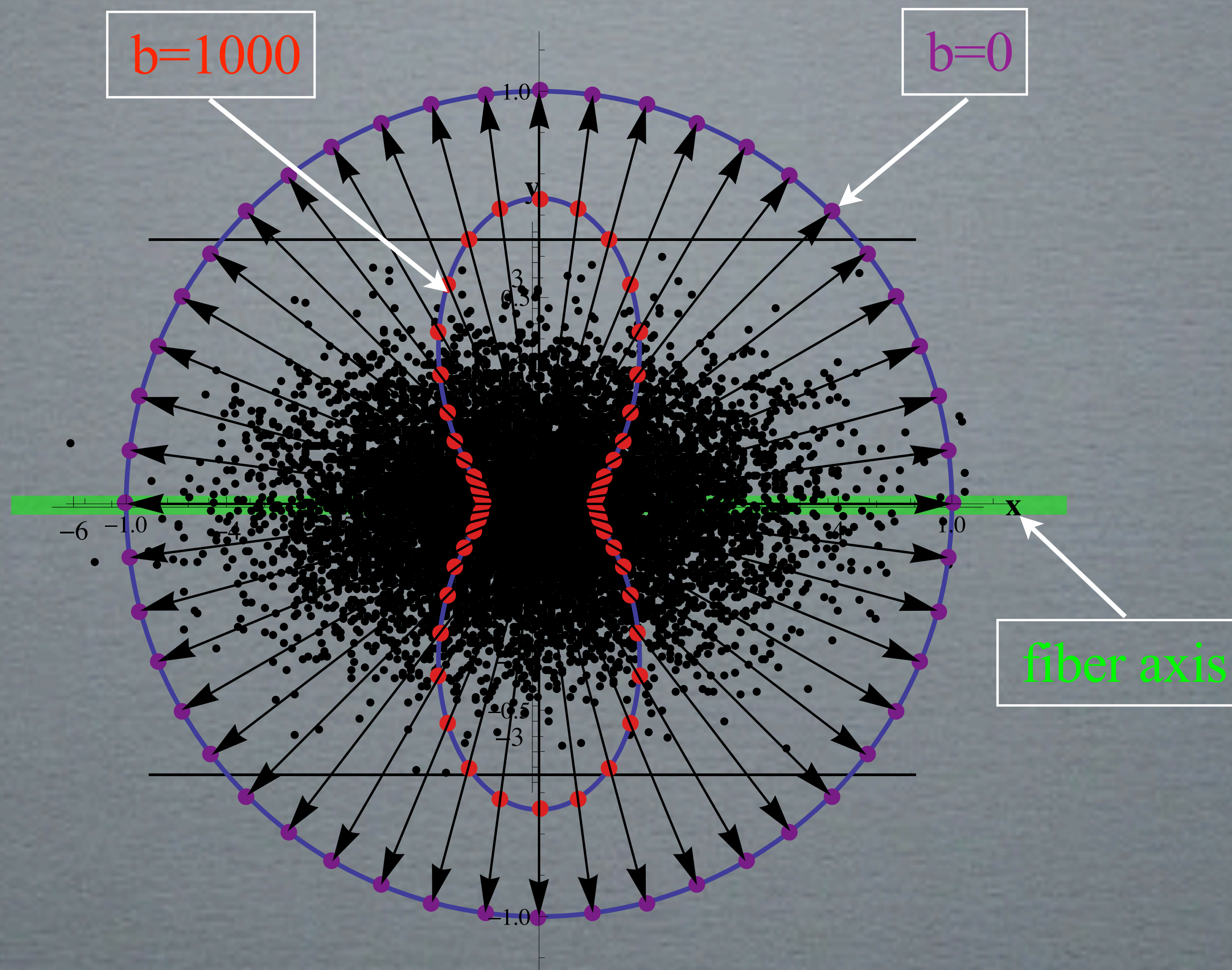
measurement direction

$$\tilde{D} = \hat{r}^t D \hat{r} = D_x \cos^2\theta + D_y \sin^2\theta$$

projection of an ellipsoid!
not like projection of a vector

Measuring the Diffusion Tensor

Measuring the Diffusion Tensor

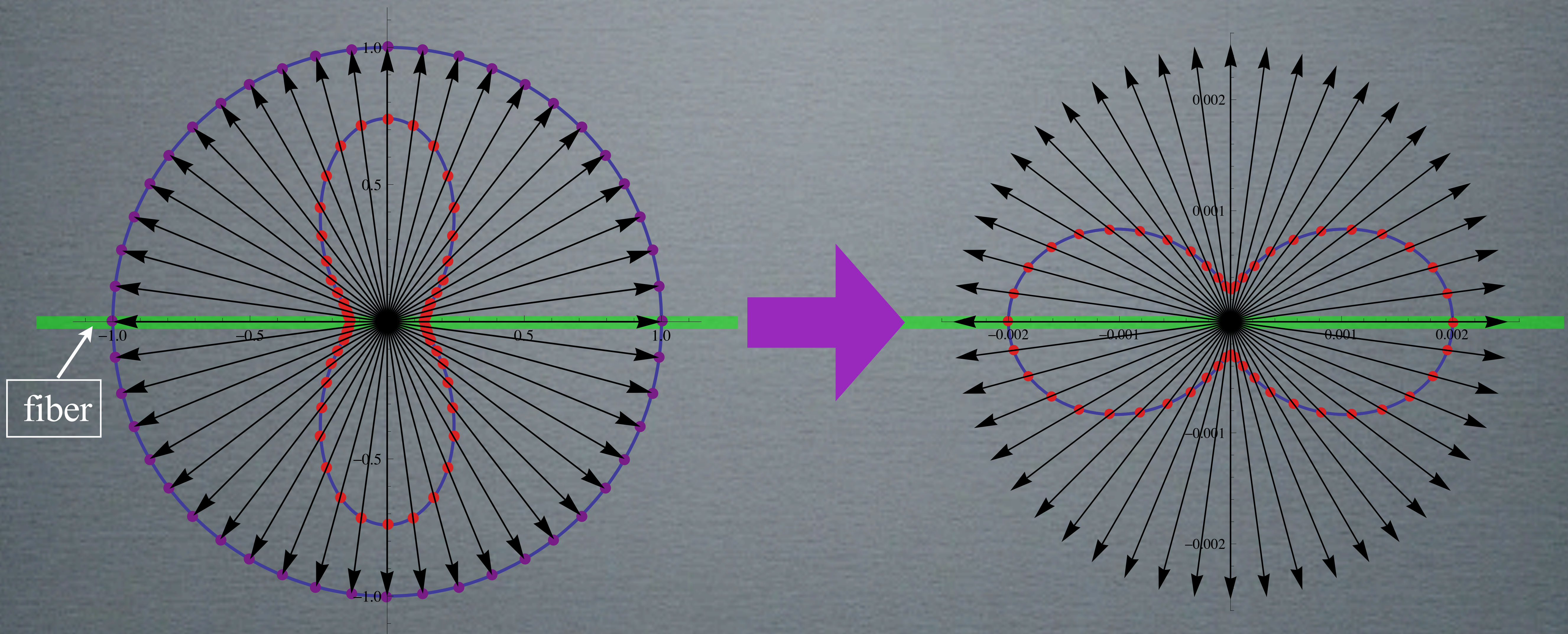


$$S(b,\theta) = S(0)e^{-bD(\theta)} + \eta$$

$$D(\theta) = \lambda_x \cos^2\theta + \lambda_y \sin^2\theta$$

The Shape of Diffusion

The Shape of Diffusion

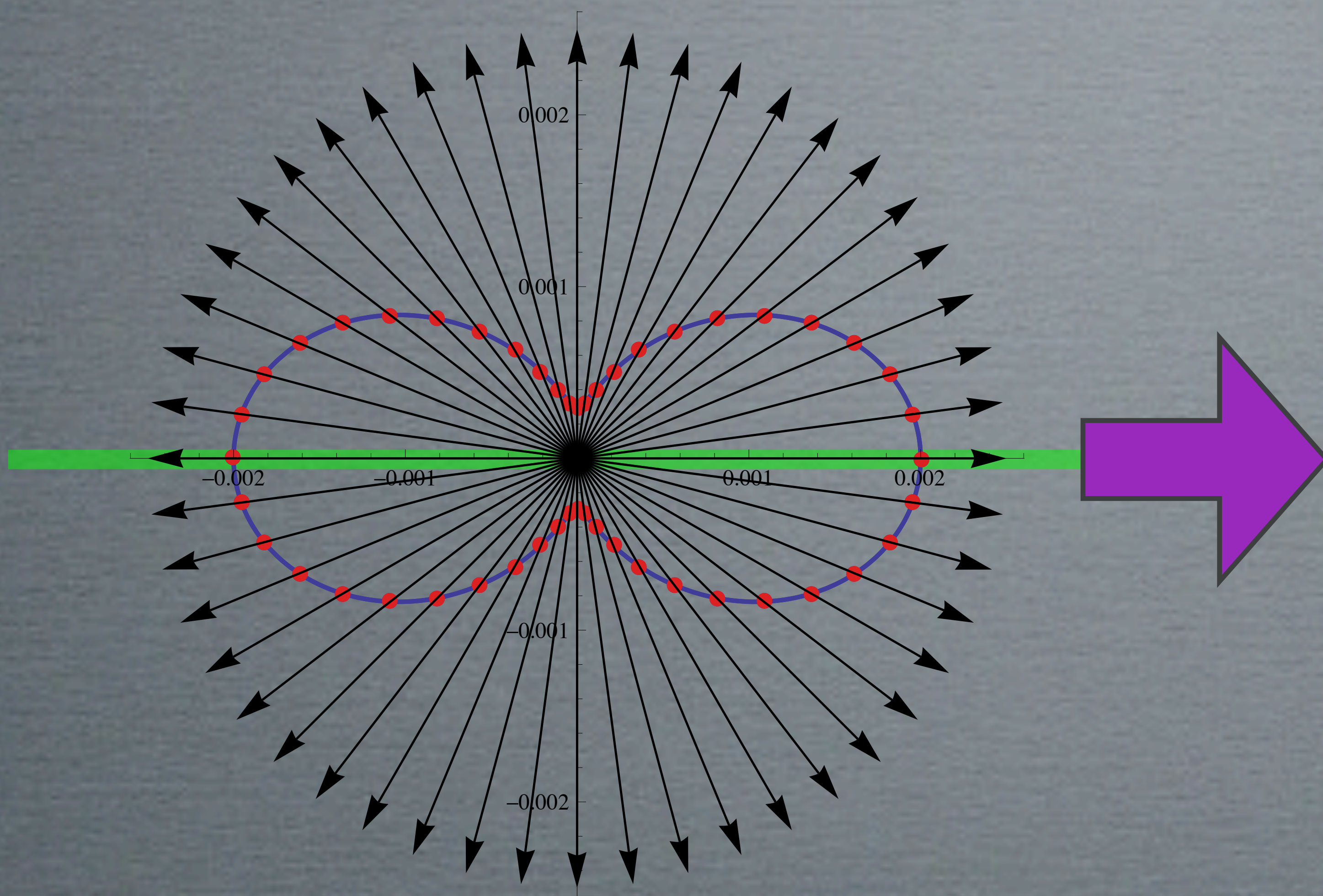


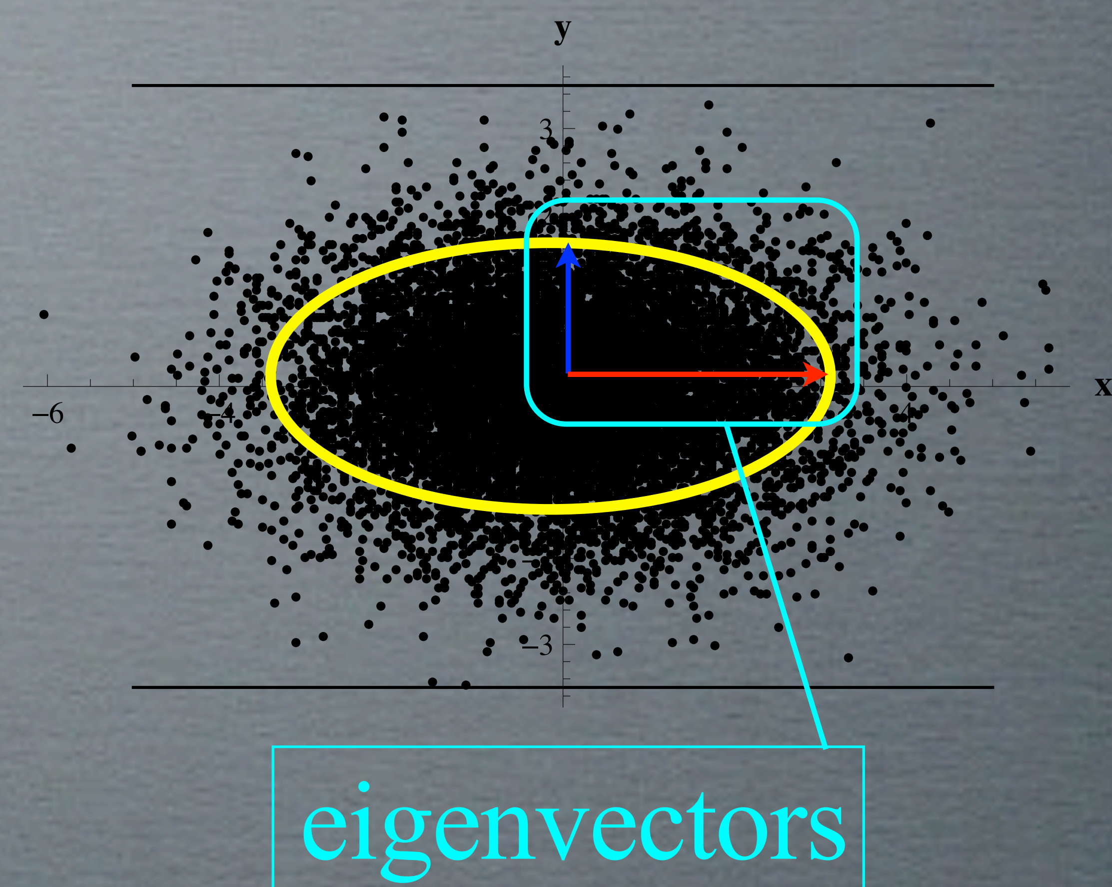
signal $S_b(\theta)$

$$D_{app}(\theta) = -\frac{1}{b}\log\left(\frac{S_b}{S_0}\right)$$

The Estimation of Diffusion

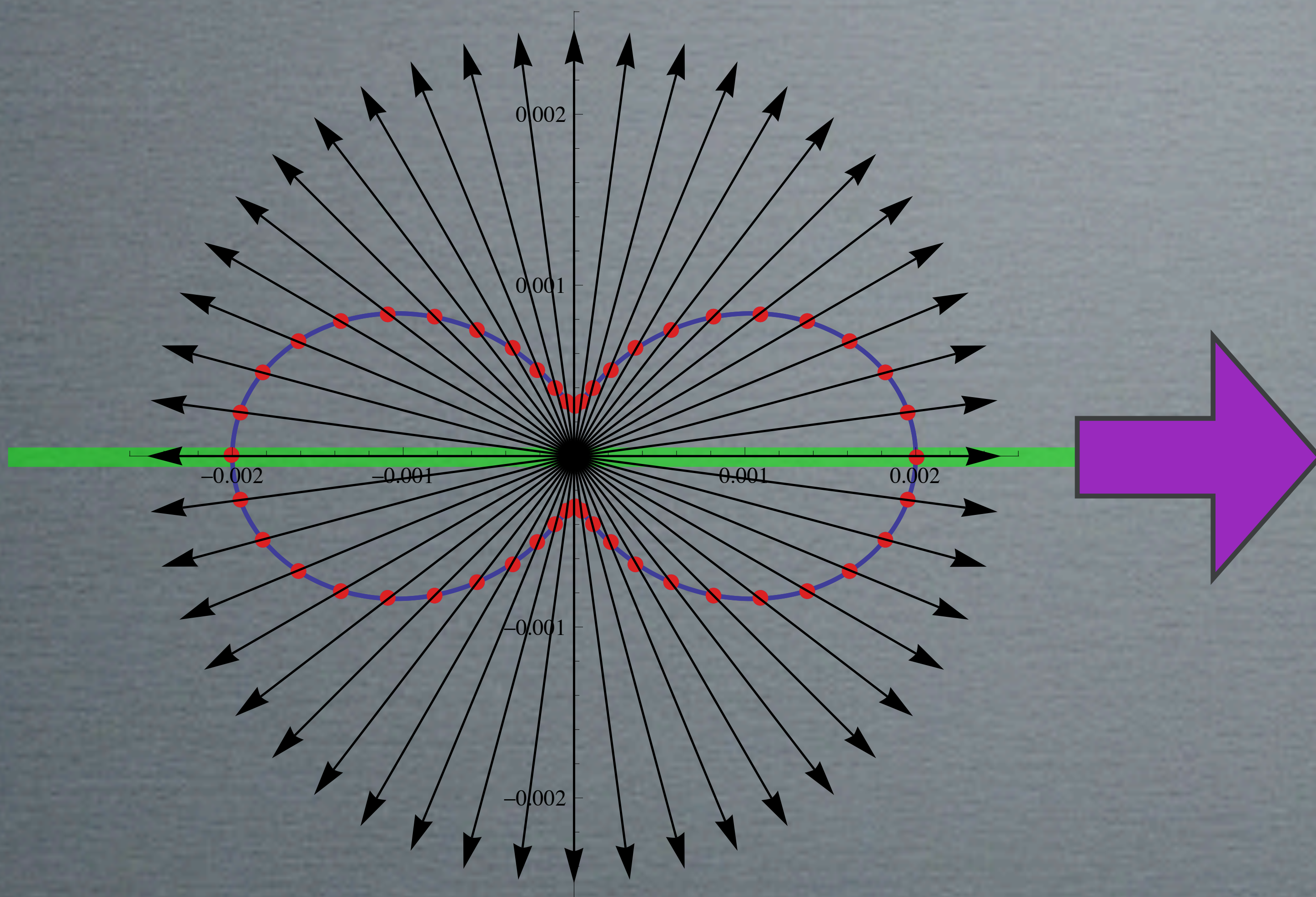
The Estimation of Diffusion

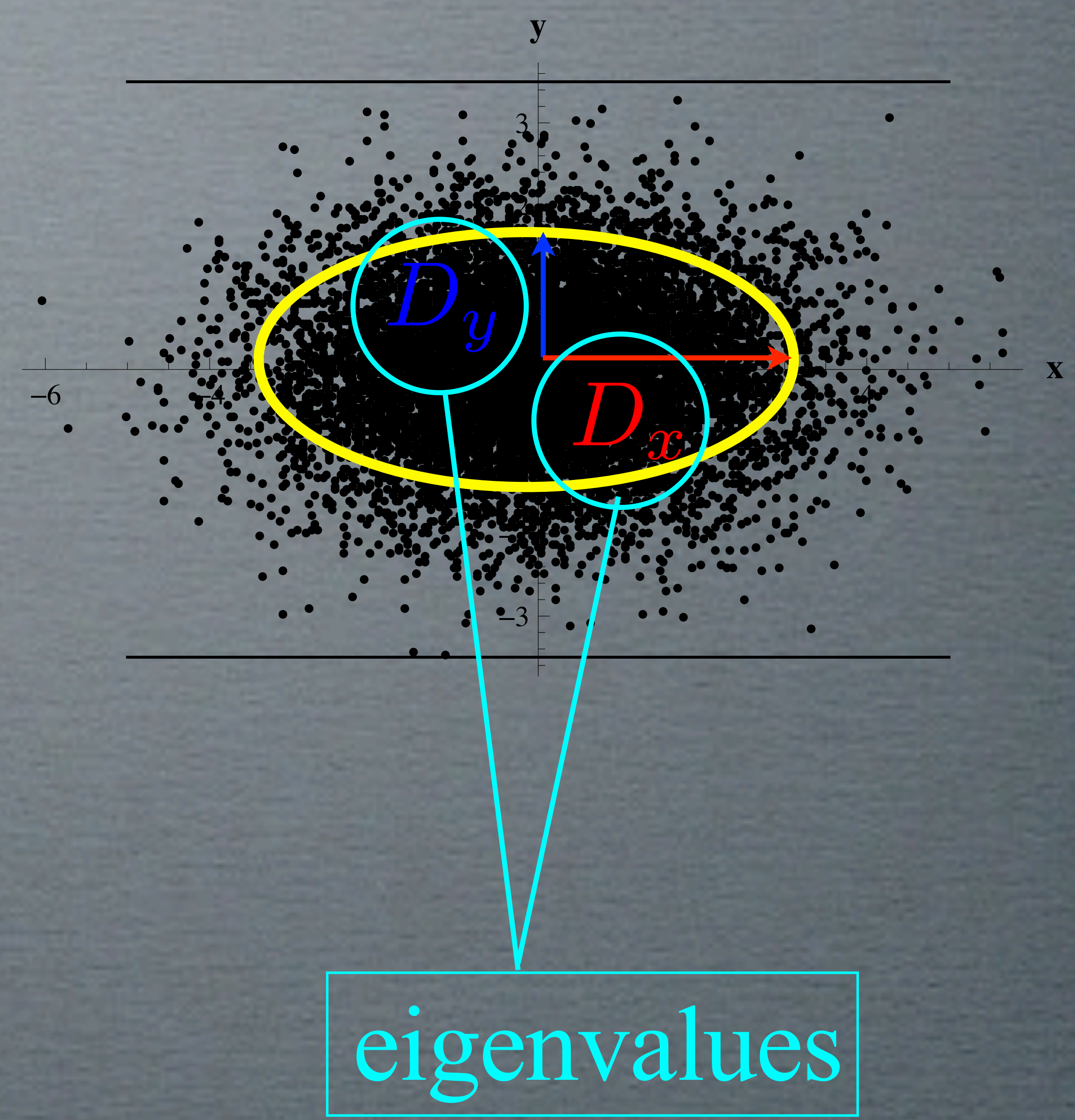




$$D_{app}(\theta) = -\frac{1}{b}\log\left(\frac{S_b}{S_0}\right)$$

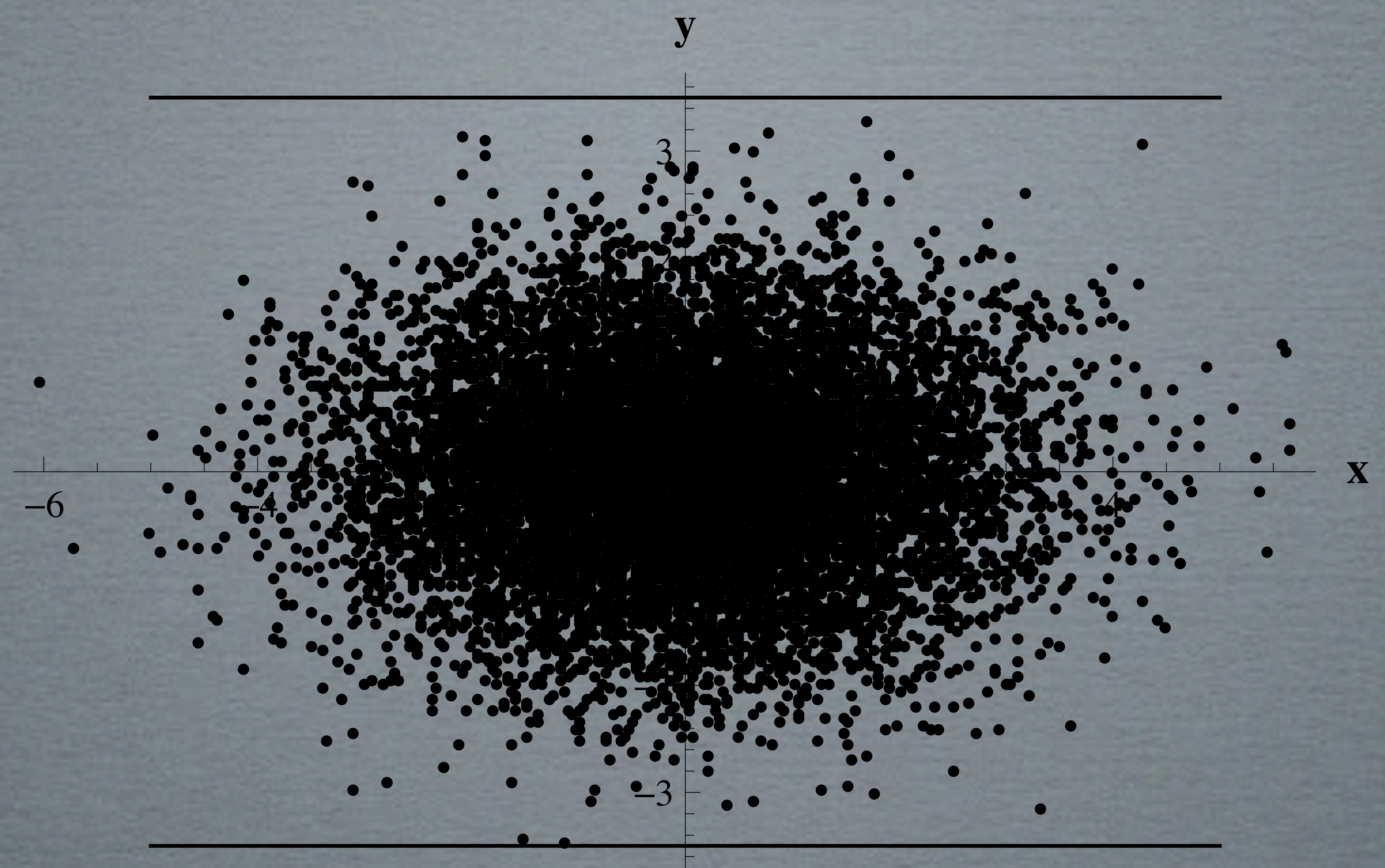
The Estimation of Diffusion



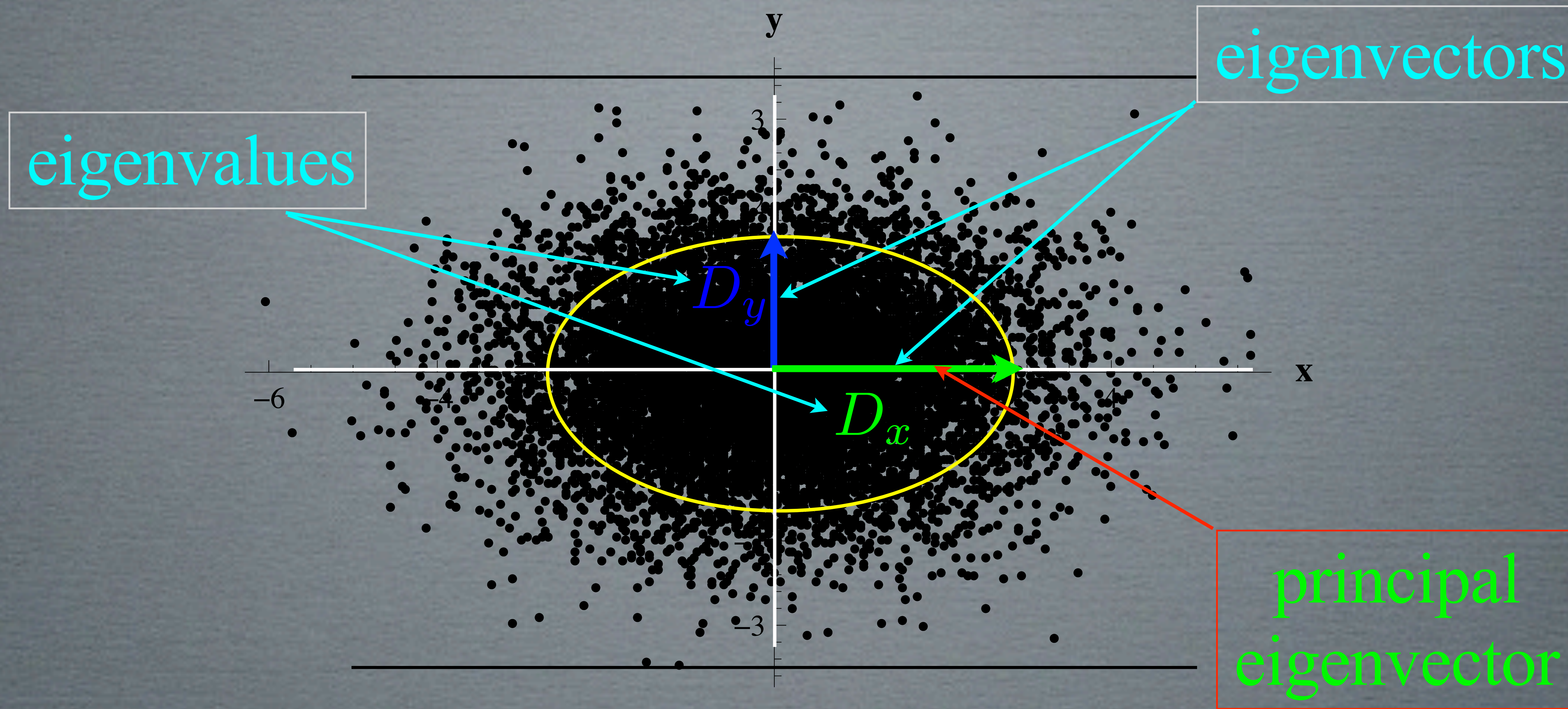


$$D_{app}(\theta) = -\frac{1}{b}\log\left(\frac{S_b}{S_0}\right)$$

Anisotropic Gaussian Diffusion



Anisotropic Gaussian Diffusion

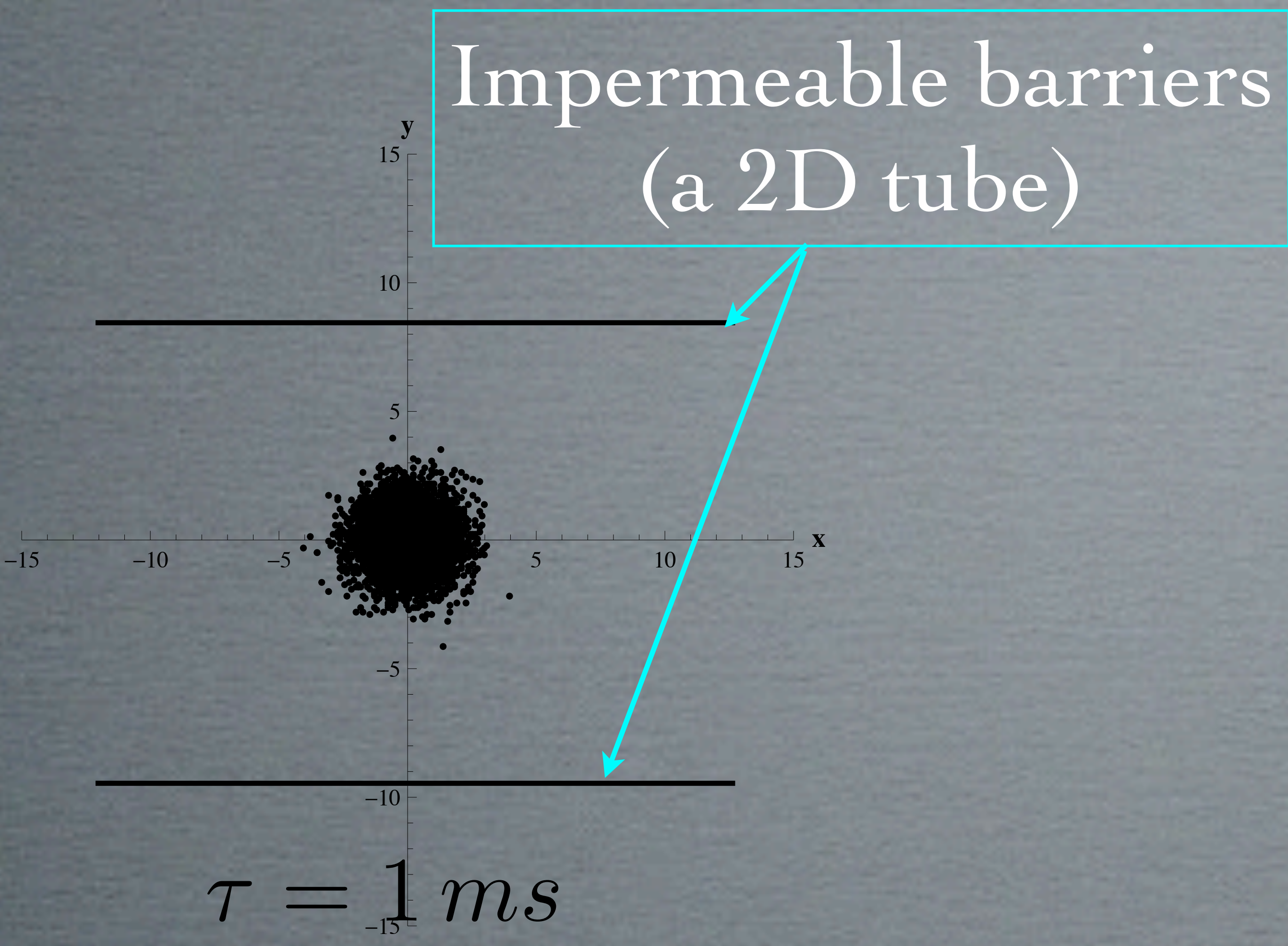


1. The relative dimensions of the contours tells us about local structure

2. The orientation of the eigenvectors is related to the orientation of the structure

Anisotropic diffusion in 2D

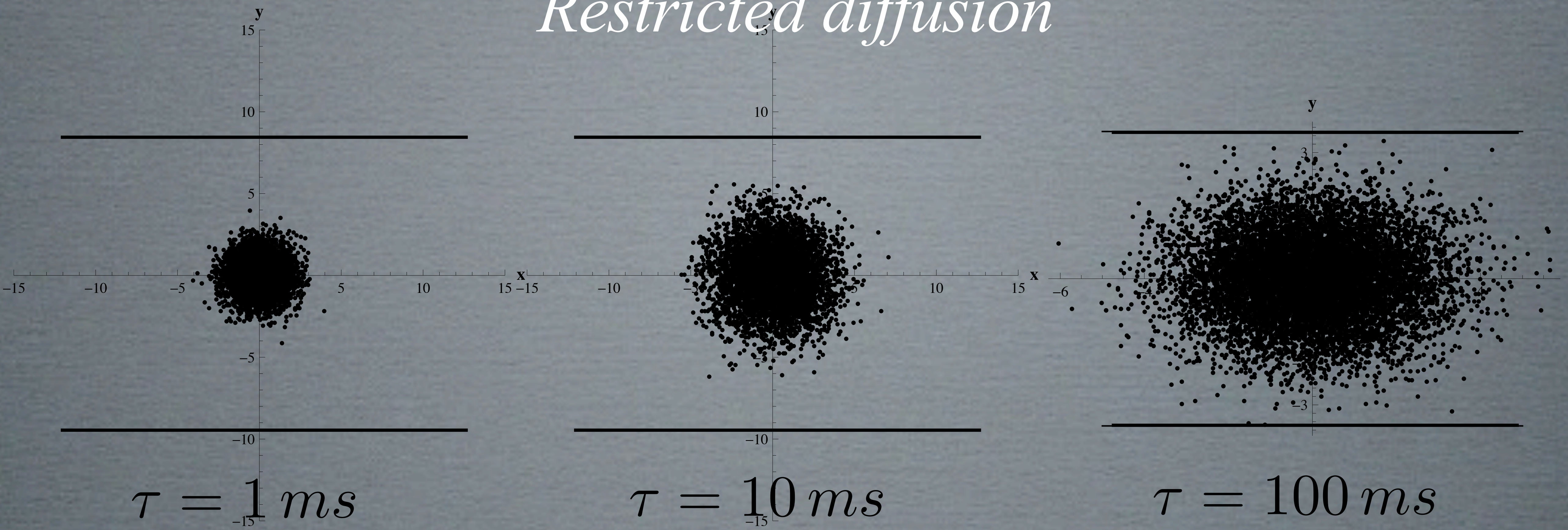
Anisotropic diffusion in 2D



$\tau = 1\,ms$

Anisotropic diffusion in 2D

Restricted diffusion

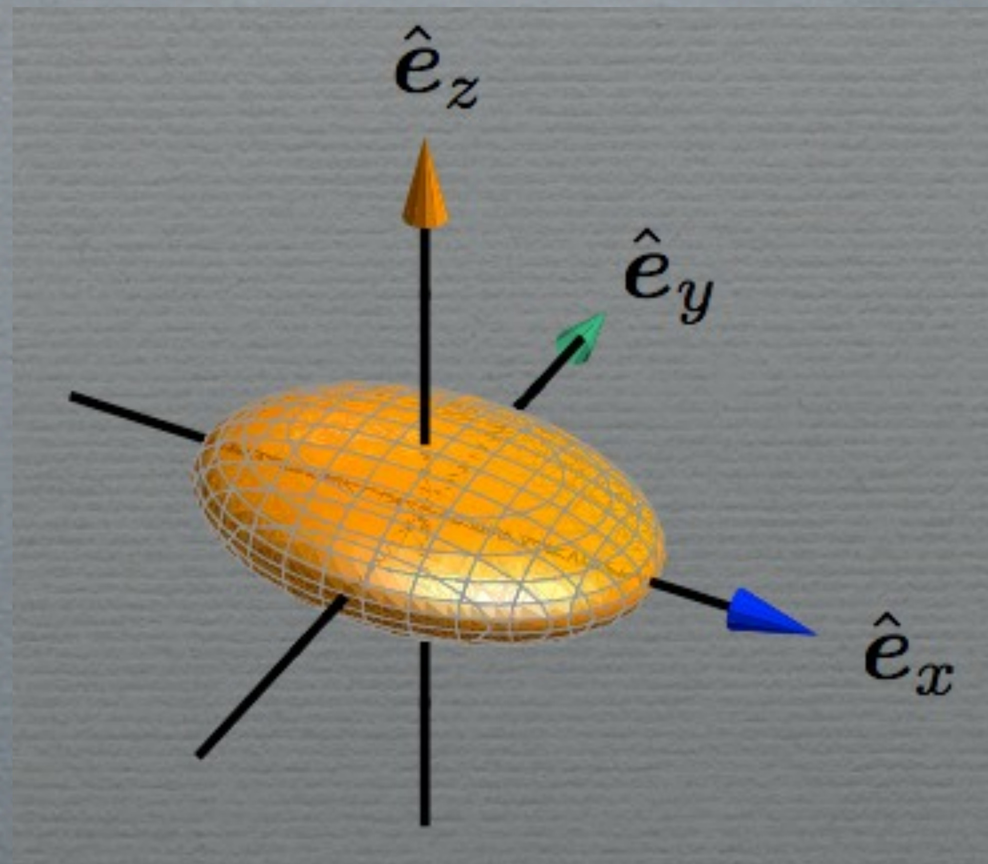


1. Anisotropy induced by local geometry
2. Sensitivity to geometry depends upon diffusion time τ
3. While the D of the liquid may be a constant, there is an *apparent diffusion coefficient* (ADC) that varies with direction

The 3D Gaussian Distribution:

The 3D Gaussian Distribution:

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$



Covariance matrix

Diffusion Tensor

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{xx}^2 & 0 & \\ 0 & \sigma_{yy}^2 & 0 \\ 0 & 0 & \sigma_{zz}^2 \end{pmatrix} = 6\tau \begin{pmatrix} D_x & 0 & \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{pmatrix}$$

The Diffusion Tensor

The Diffusion Tensor

The three eigenvectors of ***D***

$$\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$$

are the three unique directions along which the molecular displacements are uncorrelated

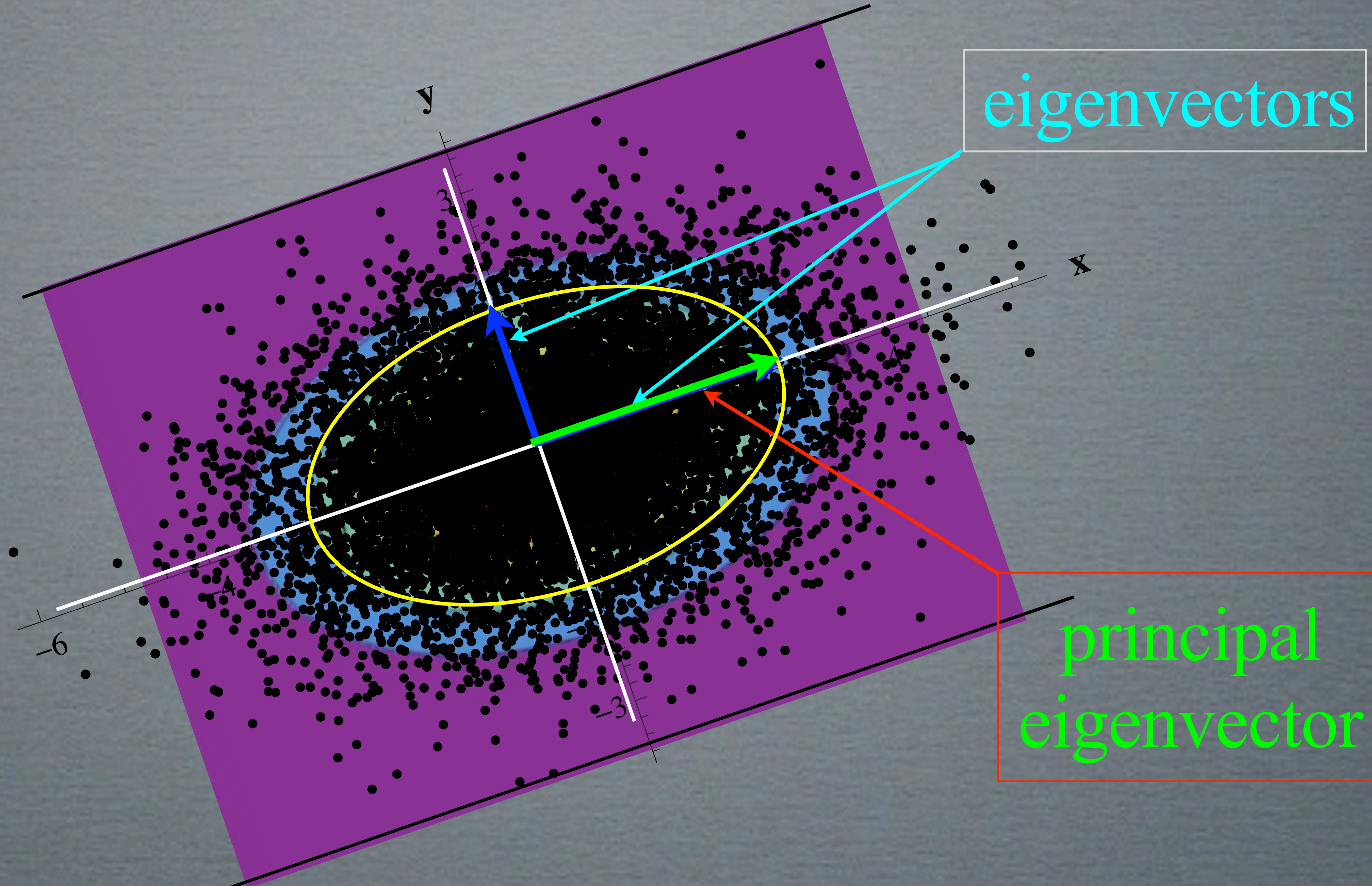
The three eigenvalues of ***D***

$$\{D_x, D_y, D_z\}$$

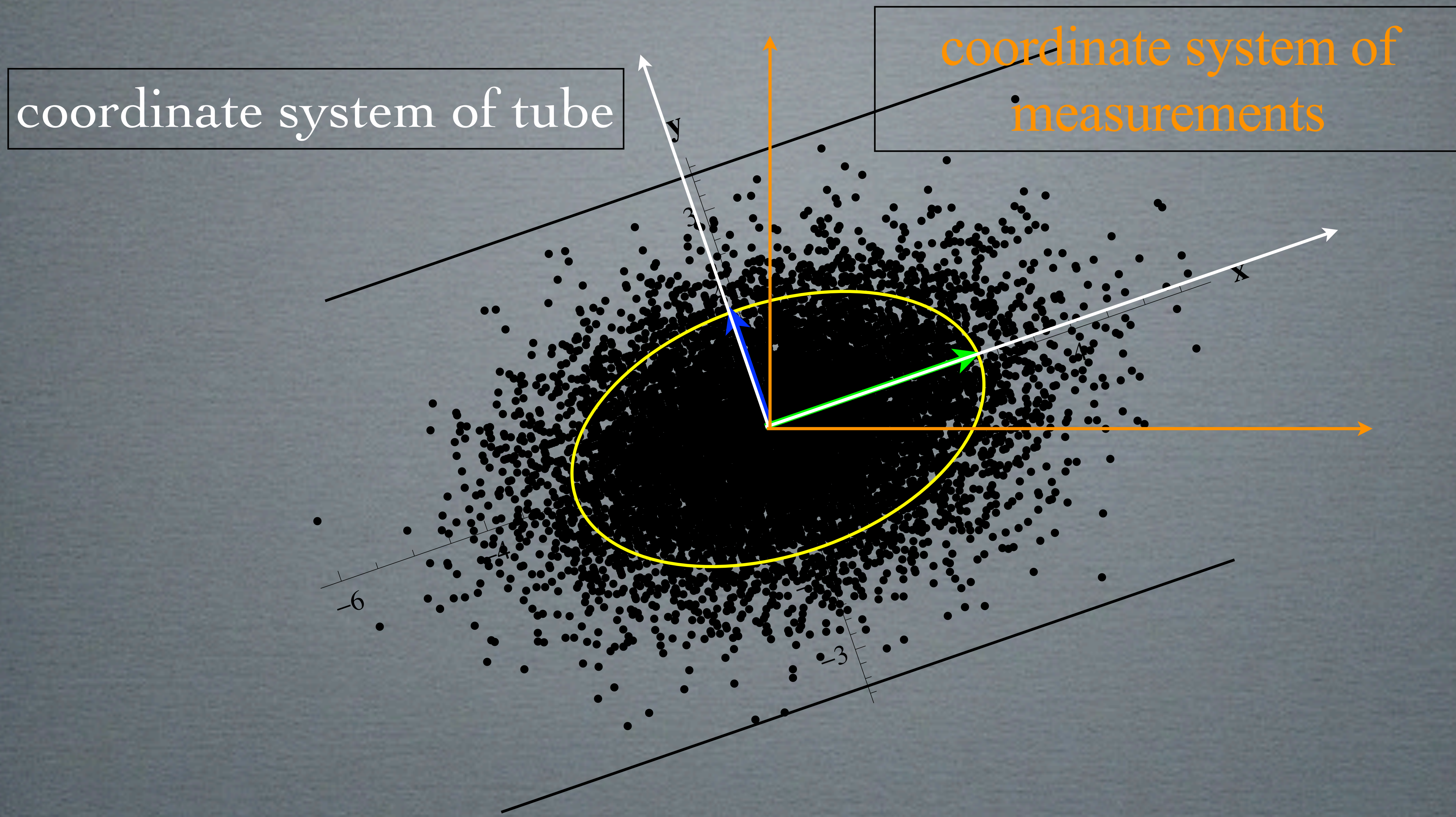
are the principle diffusivities

Rotated tube

Rotated tube



Rotated tube



If the tube is not aligned with the coordinate system of the measurements, the diffusion along the measurement axes appears correlated

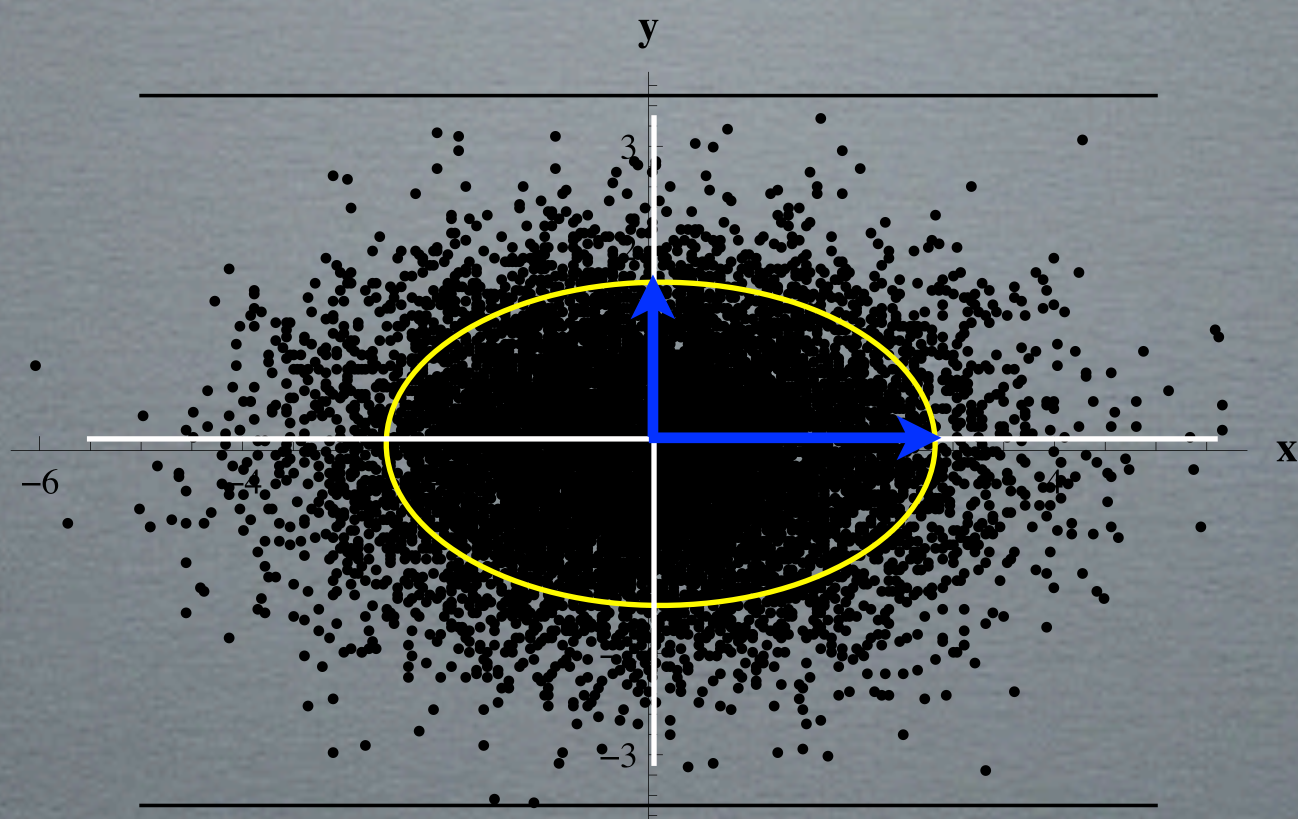
The 2D Gaussian Distribution:

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$

The 2D Gaussian Distribution:

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$

$$\boldsymbol{r} = \{x, y\}$$



Covariance matrix

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The 2D Gaussian Distribution:

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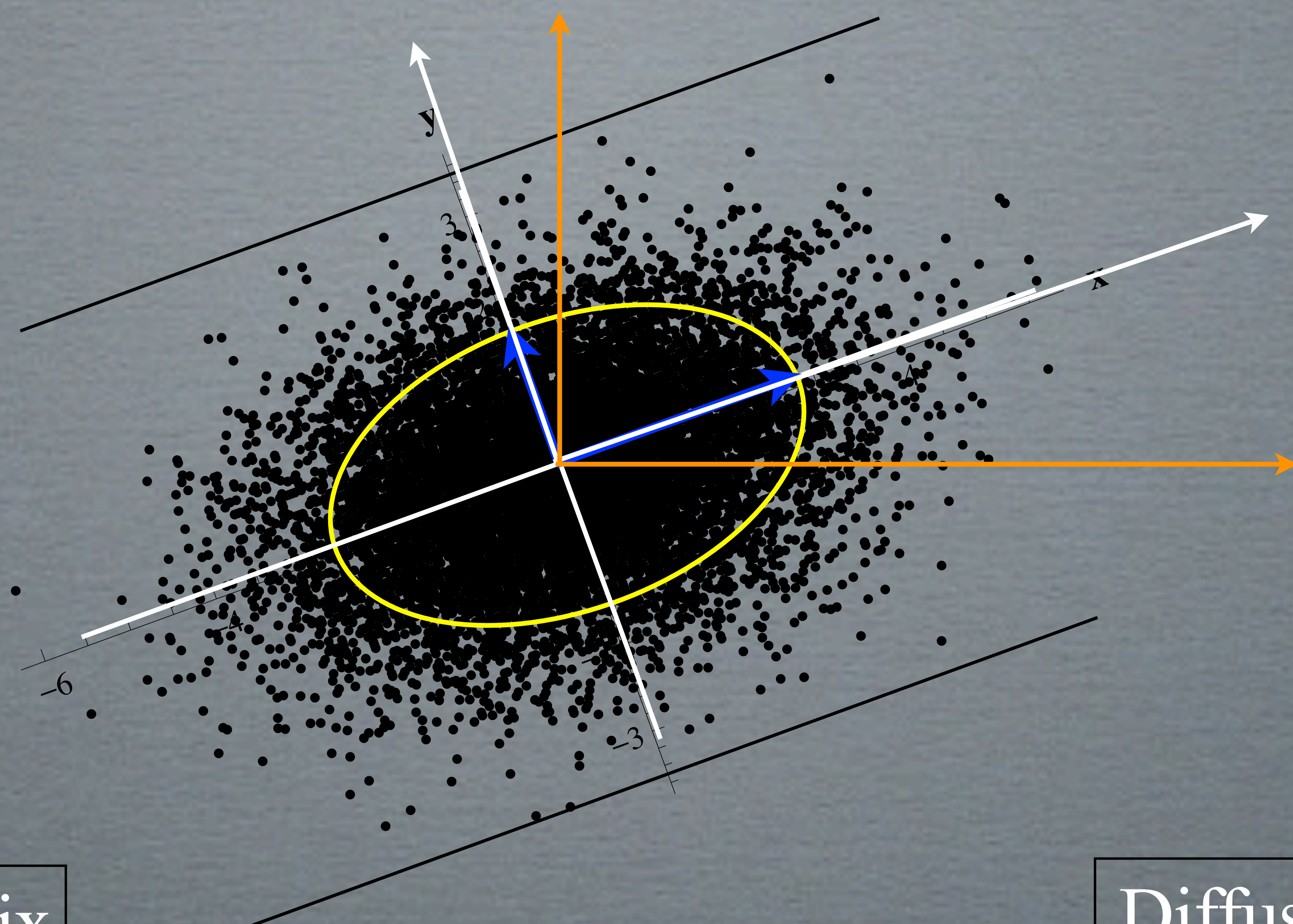


Covariance matrix

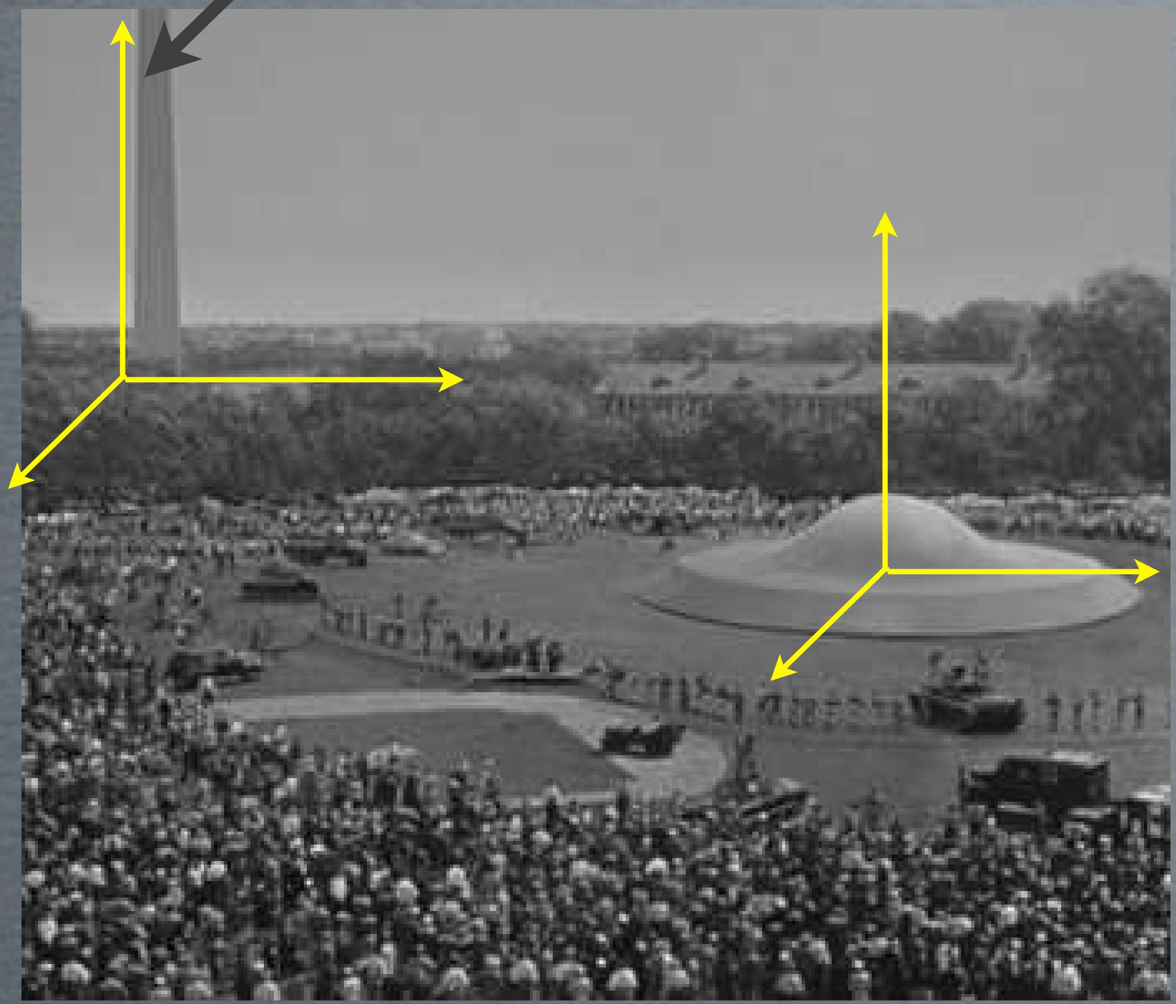
Diffusion Tensor

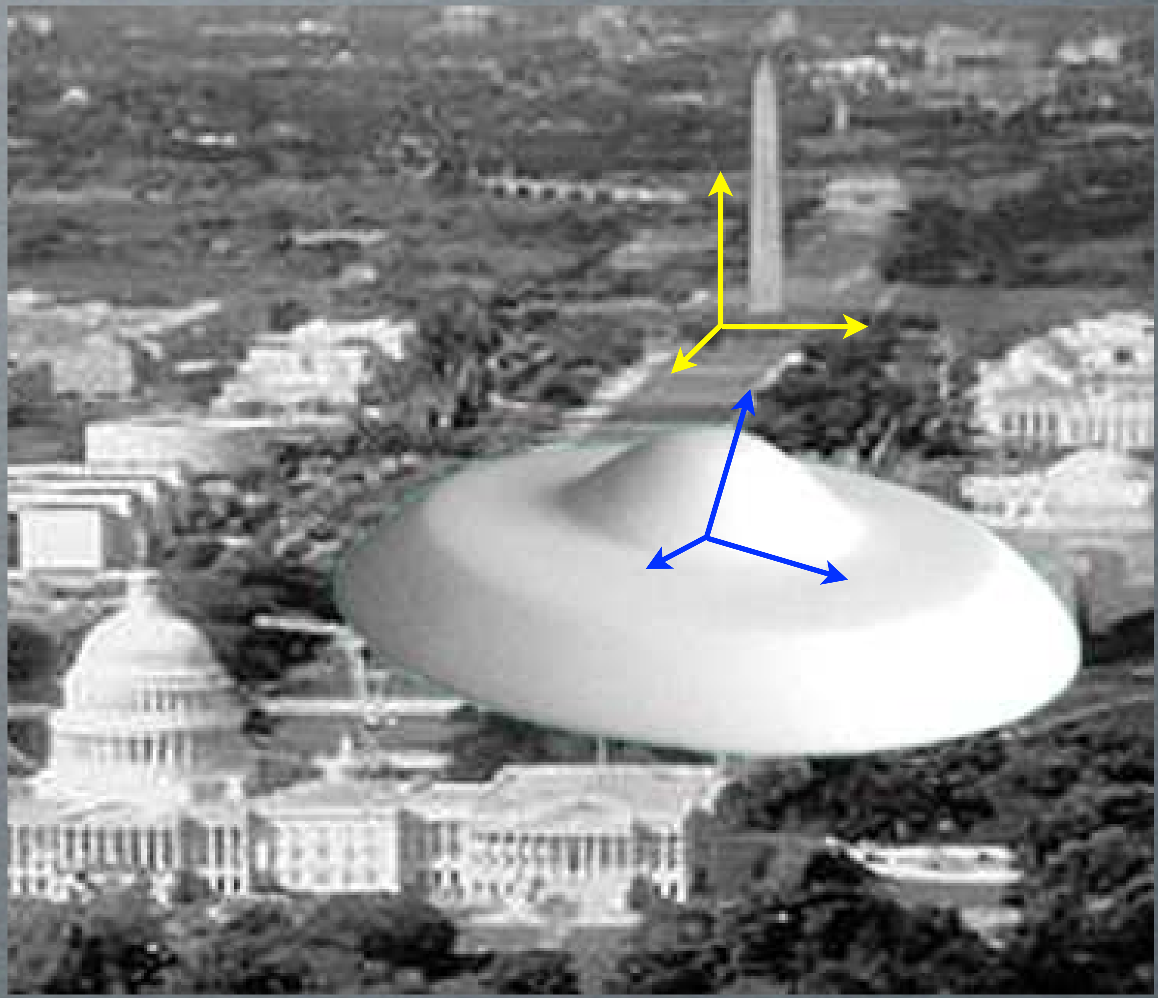
$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 \end{pmatrix} = 4\tau \begin{pmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{pmatrix}$$

Generally fibers are not aligned along magnet coordinates!

Generally fibers are not aligned along magnet coordinates!

laboratory
coordinate system





same orientation as laboratory coordinate system

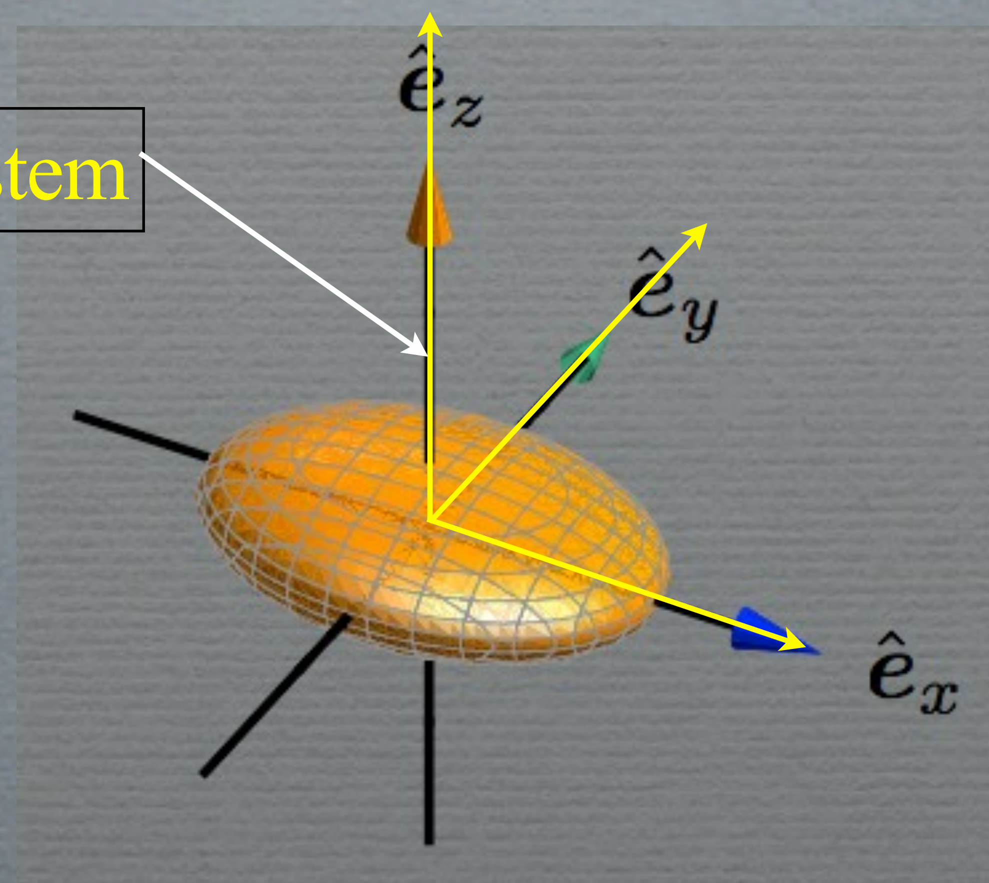
rotated relative to laboratory coordinate system

The 3D Gaussian Distribution:

The 3D Gaussian Distribution:

$$P(\boldsymbol{r}|\boldsymbol{r}_0,\tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$

scanner coordinate system



Covariance matrix

Diffusion Tensor

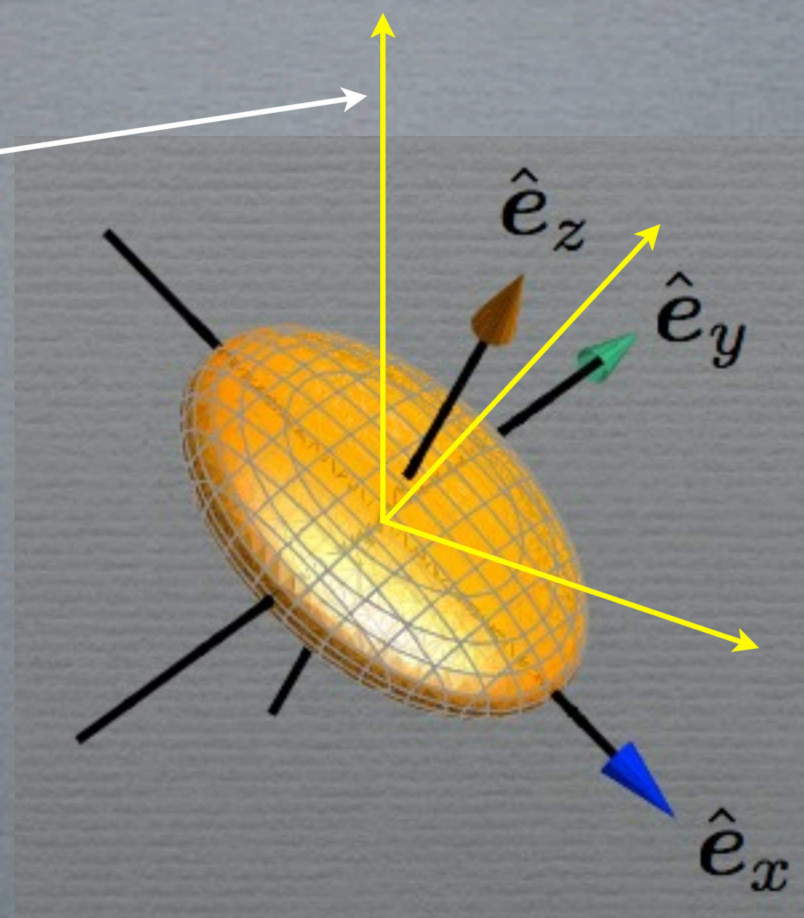
$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma^2_{xx} & 0 & \\ 0 & \sigma^2_{yy} & 0 \\ 0 & 0 & \sigma^2_{zz} \end{pmatrix} = 6\tau \begin{pmatrix} D_x & 0 & \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{pmatrix}$$

The 3D Gaussian Distribution:

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$

The 3D Gaussian Distribution:

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$



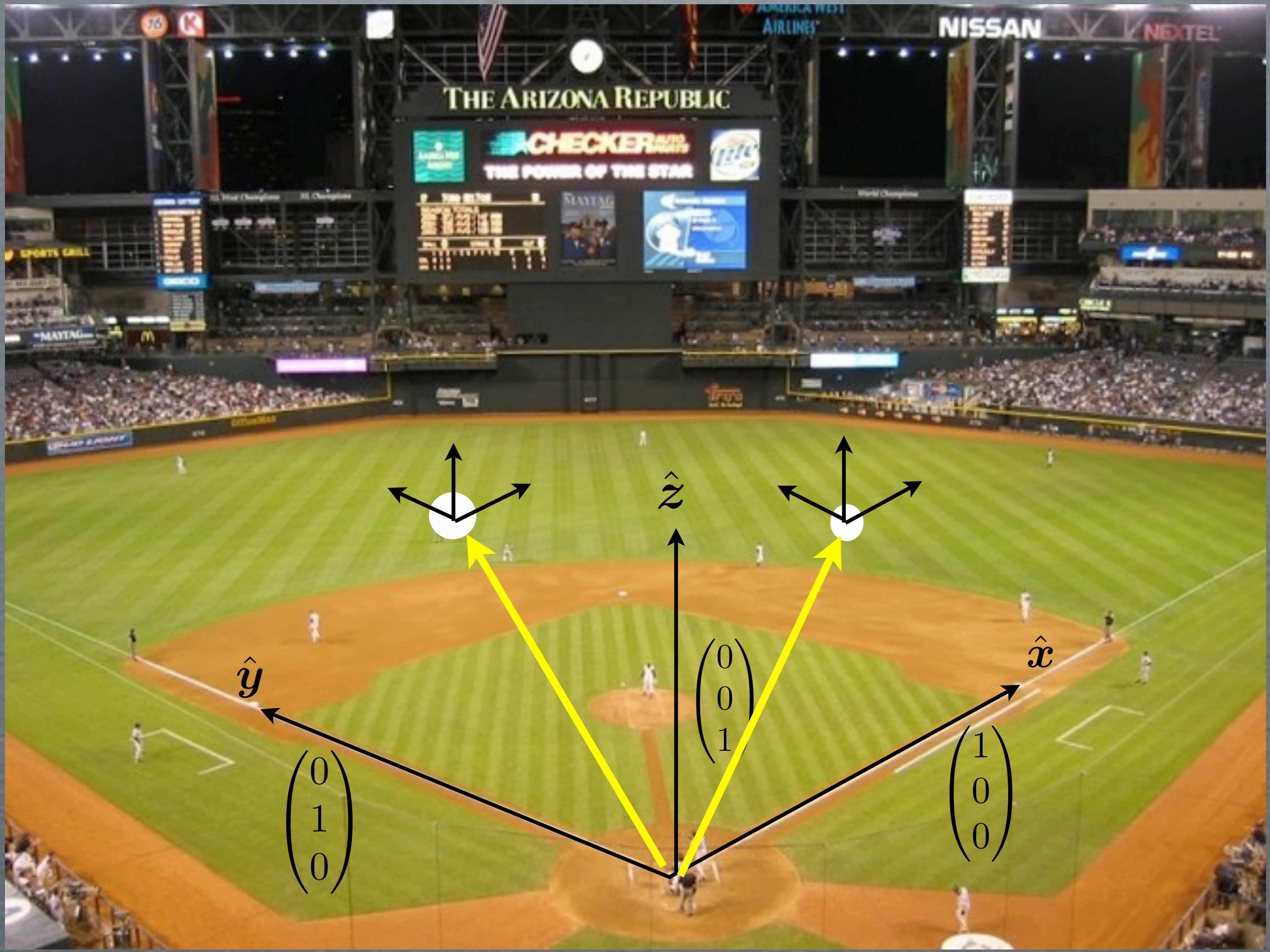
Covariance matrix

Diffusion Tensor

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{xz}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{yz}^2 \\ \sigma_{zx}^2 & \sigma_{zy}^2 & \sigma_{zz}^2 \end{pmatrix} = 6\tau \underbrace{\begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}}_{\boldsymbol{D}}$$

Tensor Rotations

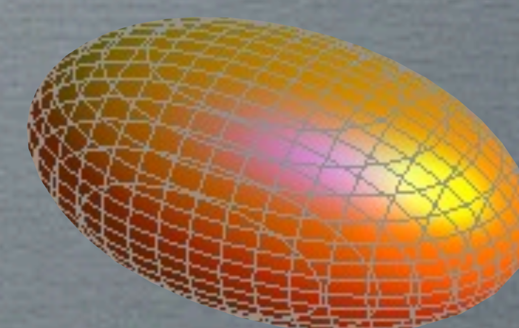
Tensor Rotations



A baseball is spherical - it has no sense of orientation

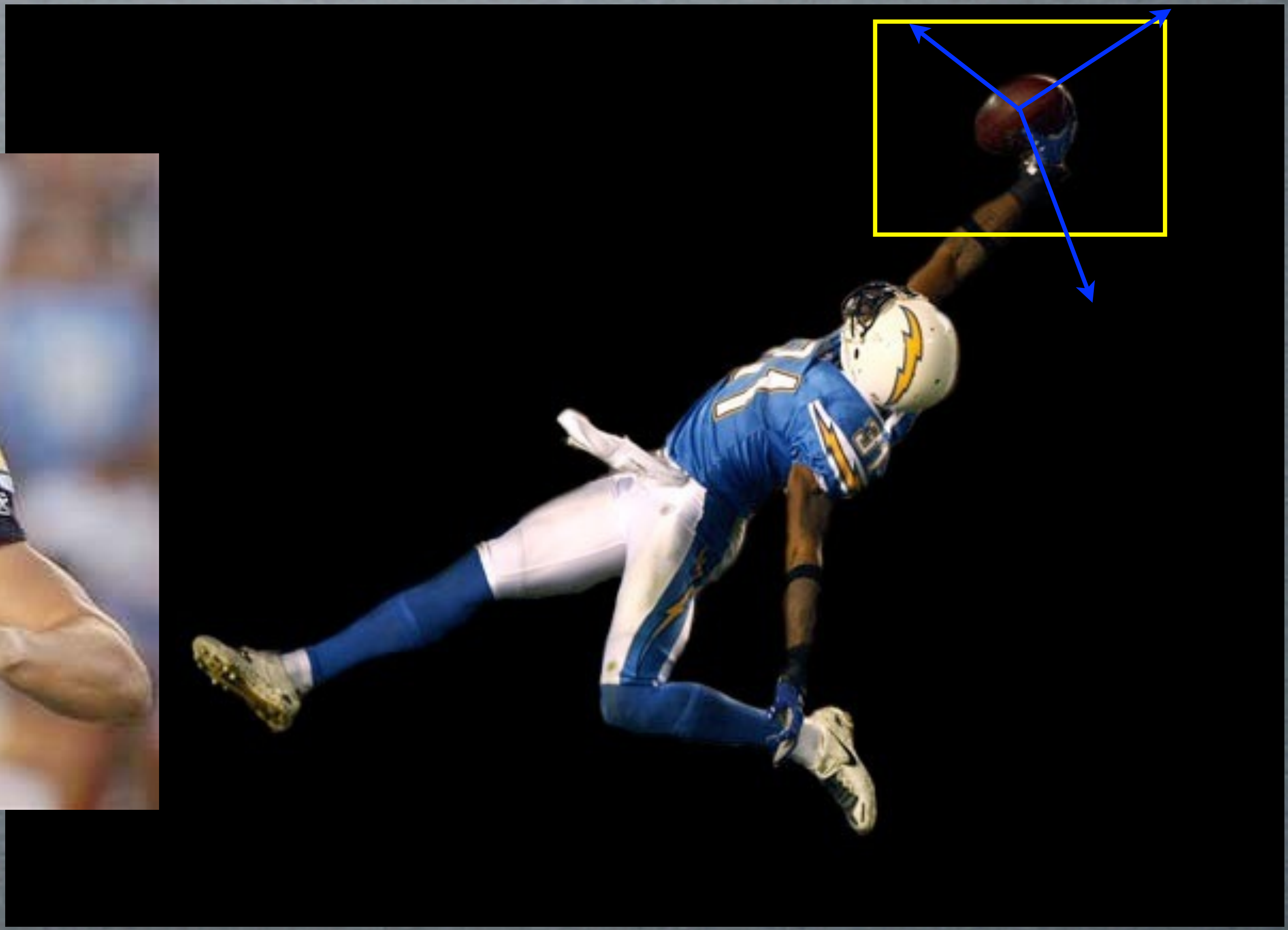
Tensor Rotations

$\approx$

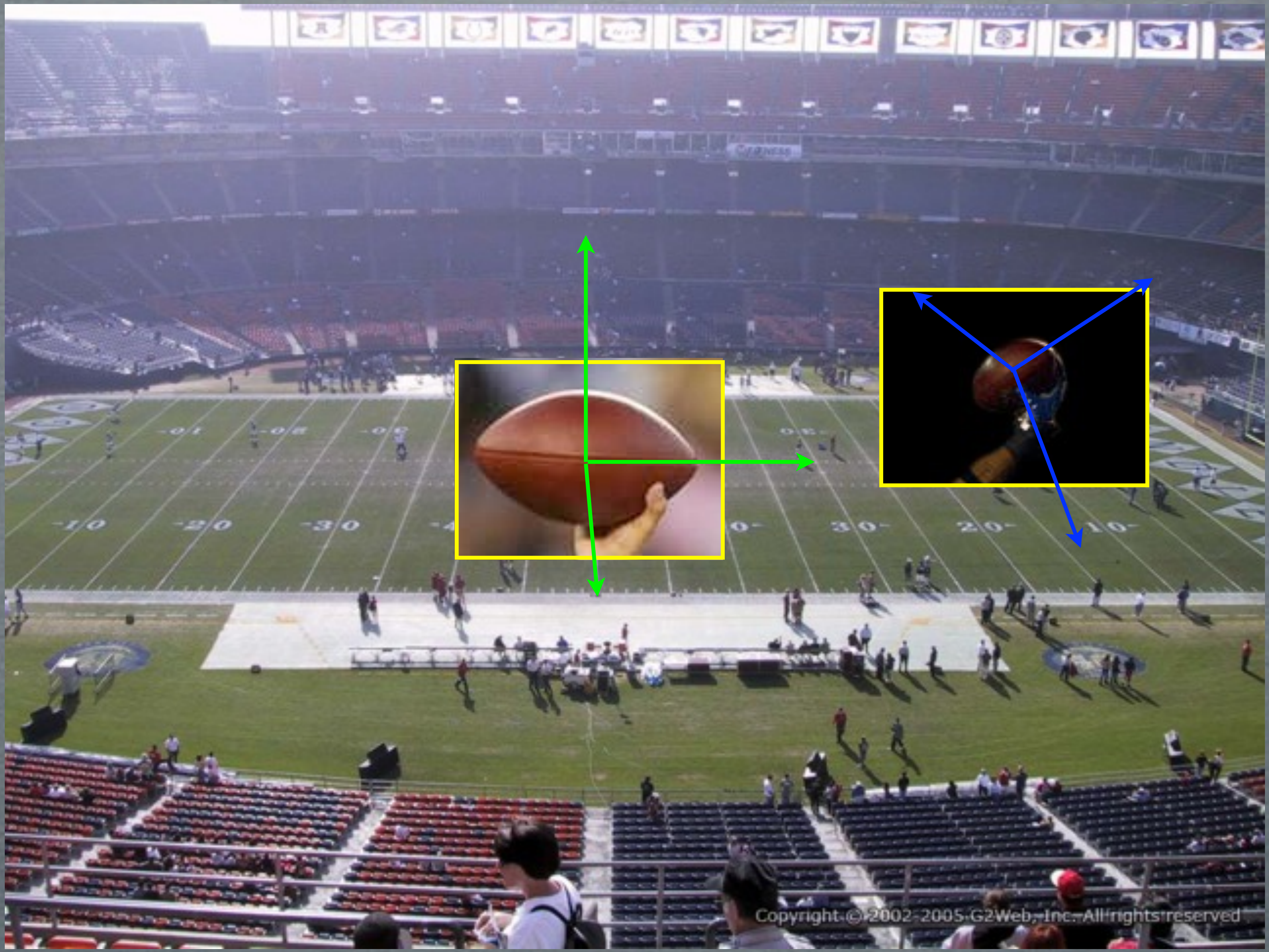


... but a football is ellipsoidal, and does!

Tensor Rotations

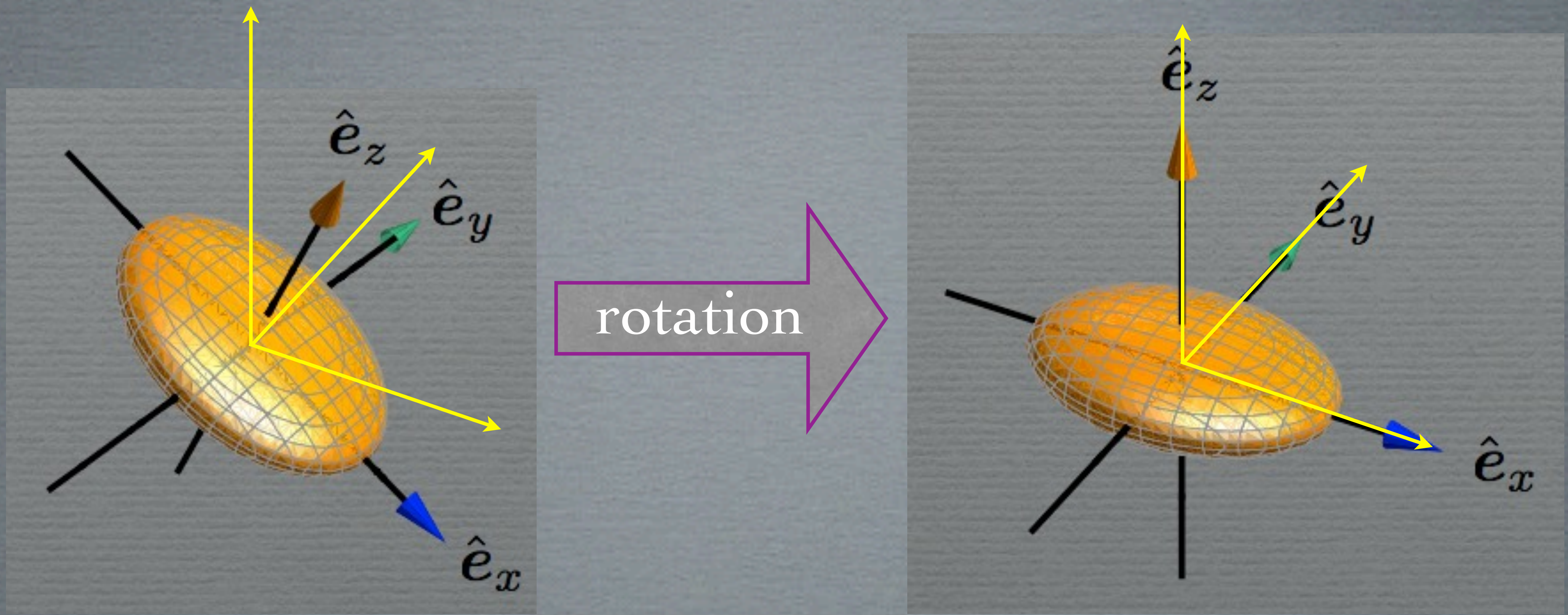


Tensor Rotations



What we want

What we want



$$\boldsymbol{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix} \xrightarrow{\text{rotation}} \boldsymbol{D} = \begin{pmatrix} D_x & 0 & \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{pmatrix}$$

This is what eigenvector routines do!

The Estimation of Diffusion
CAUTION

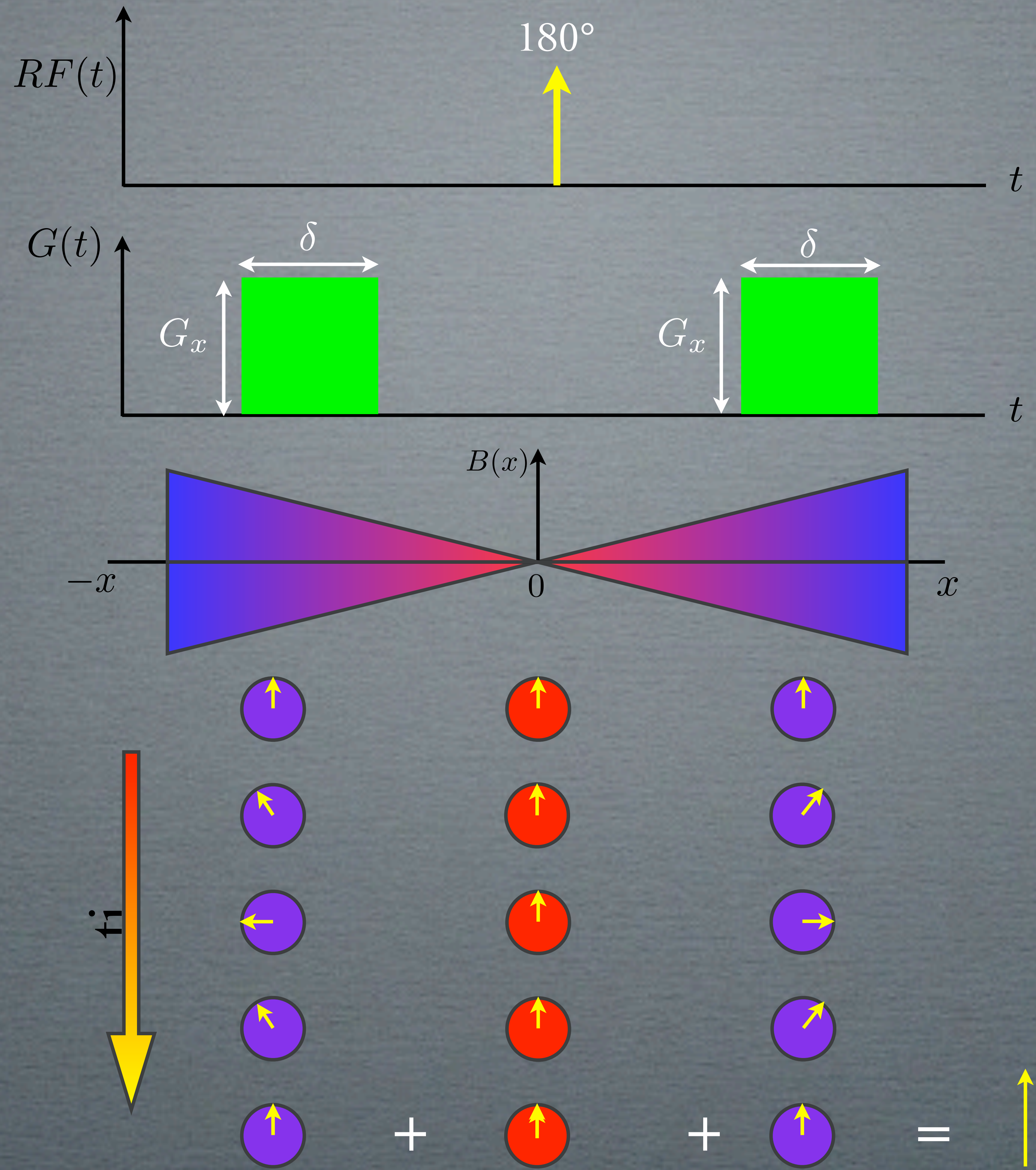
The Estimation of Diffusion
caution

$$S(b, \hat{r}) = S(0)e^{-bD(\hat{r})} + \eta$$

$$D(\hat{r}) = -\frac{1}{b} \log\left(\frac{S(b, \hat{r})}{S(0)} - \eta\right)$$

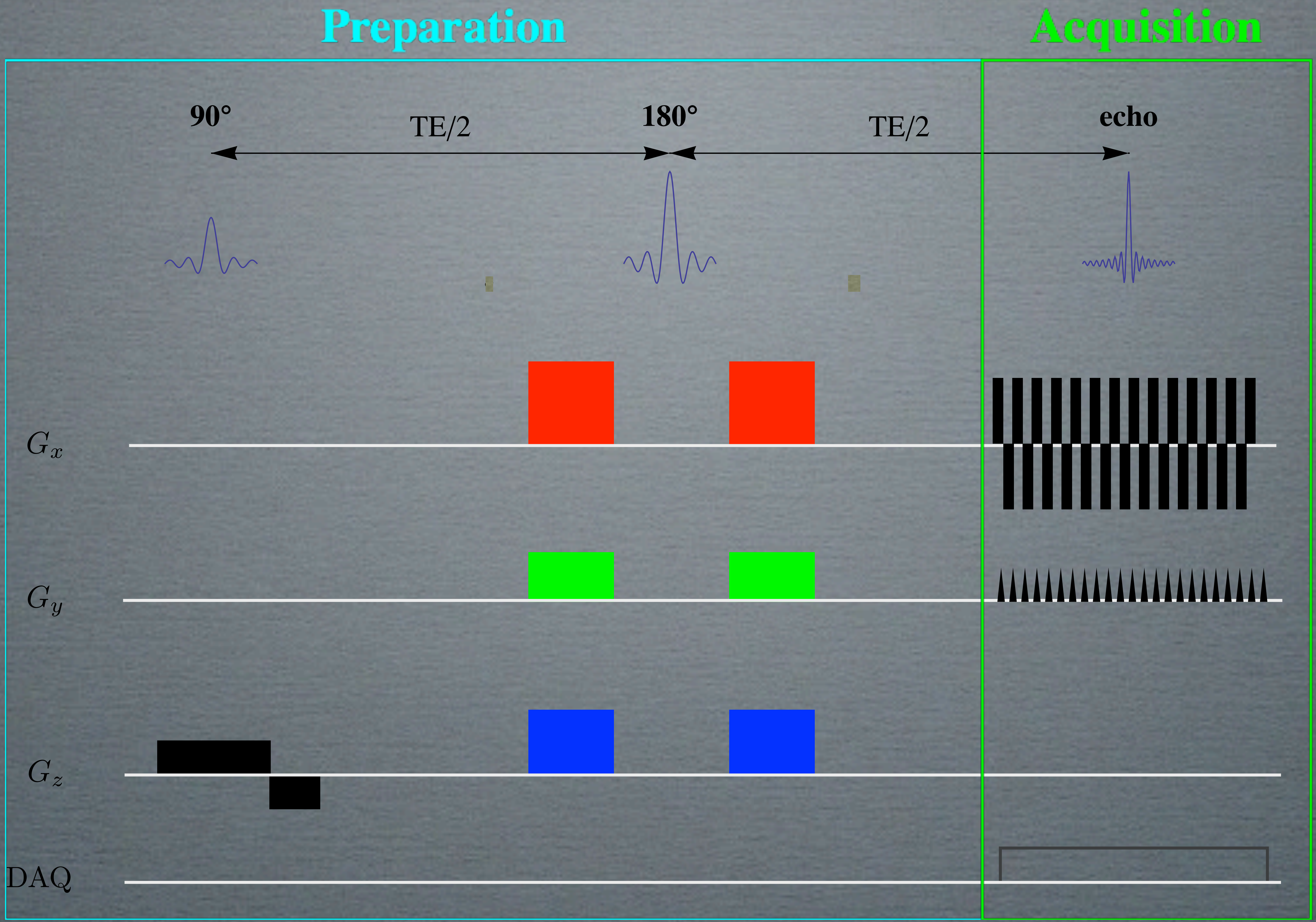
Not additive noise anymore!

The Bipolar Gradient Pulse (spin echo)



Extension to Imaging

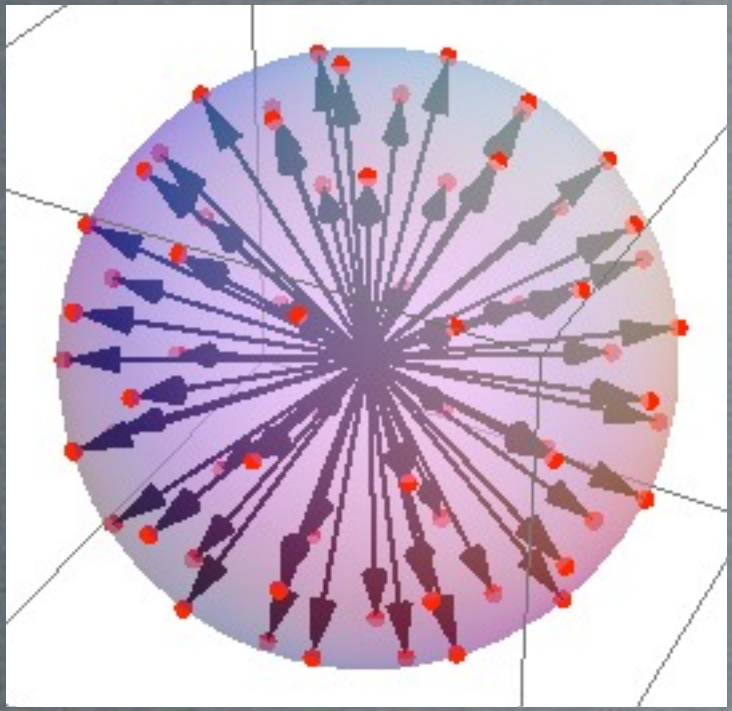
Extension to Imaging

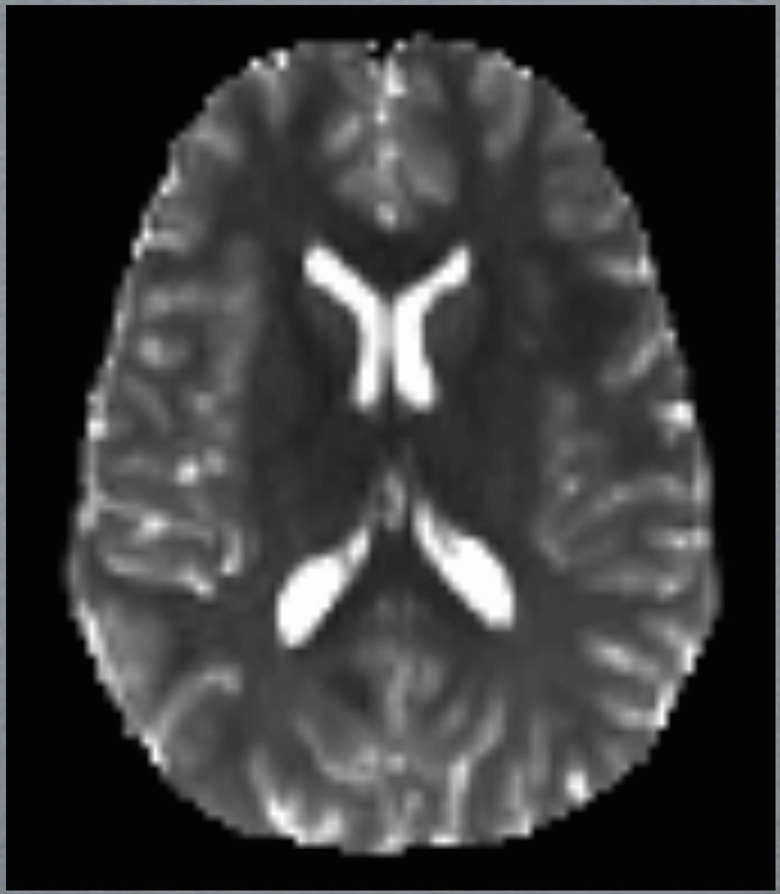


Because the diffusion weighting does not interfere with the stationary tissue signal, we can "insert" it into a standard imaging procedure

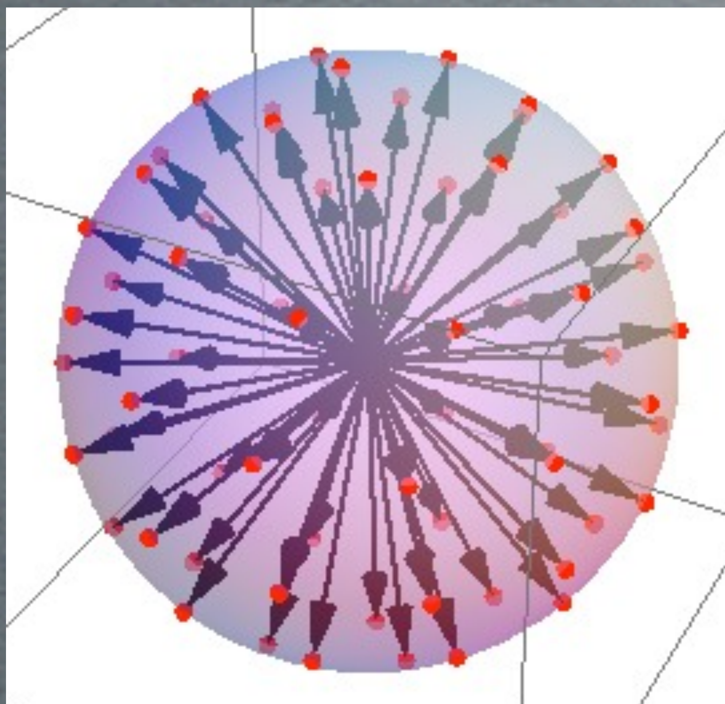
DTI

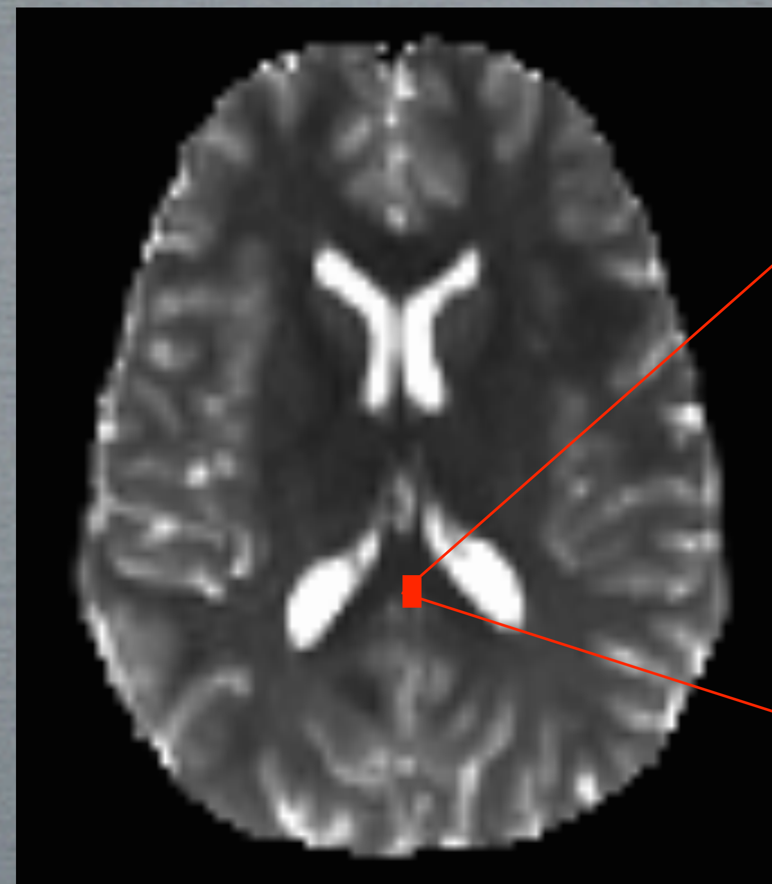
DTI



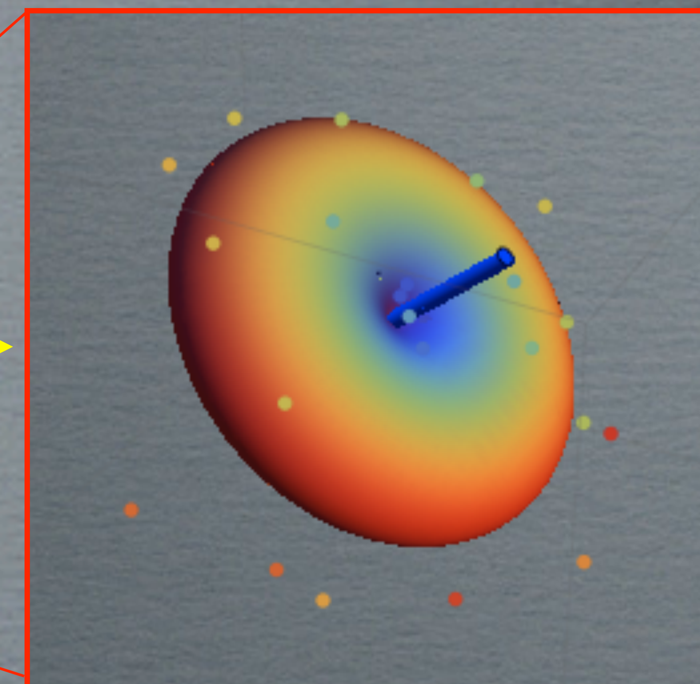


DTI

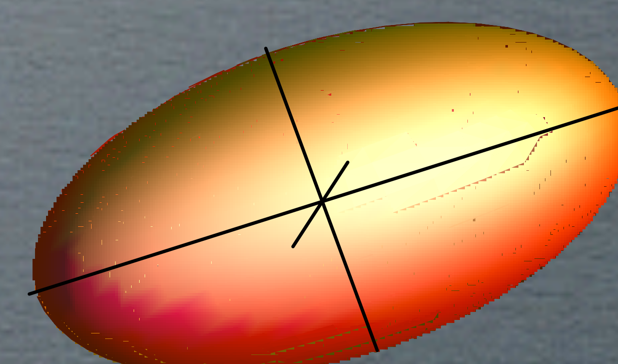




voxel signal
from multiple images at
different directions

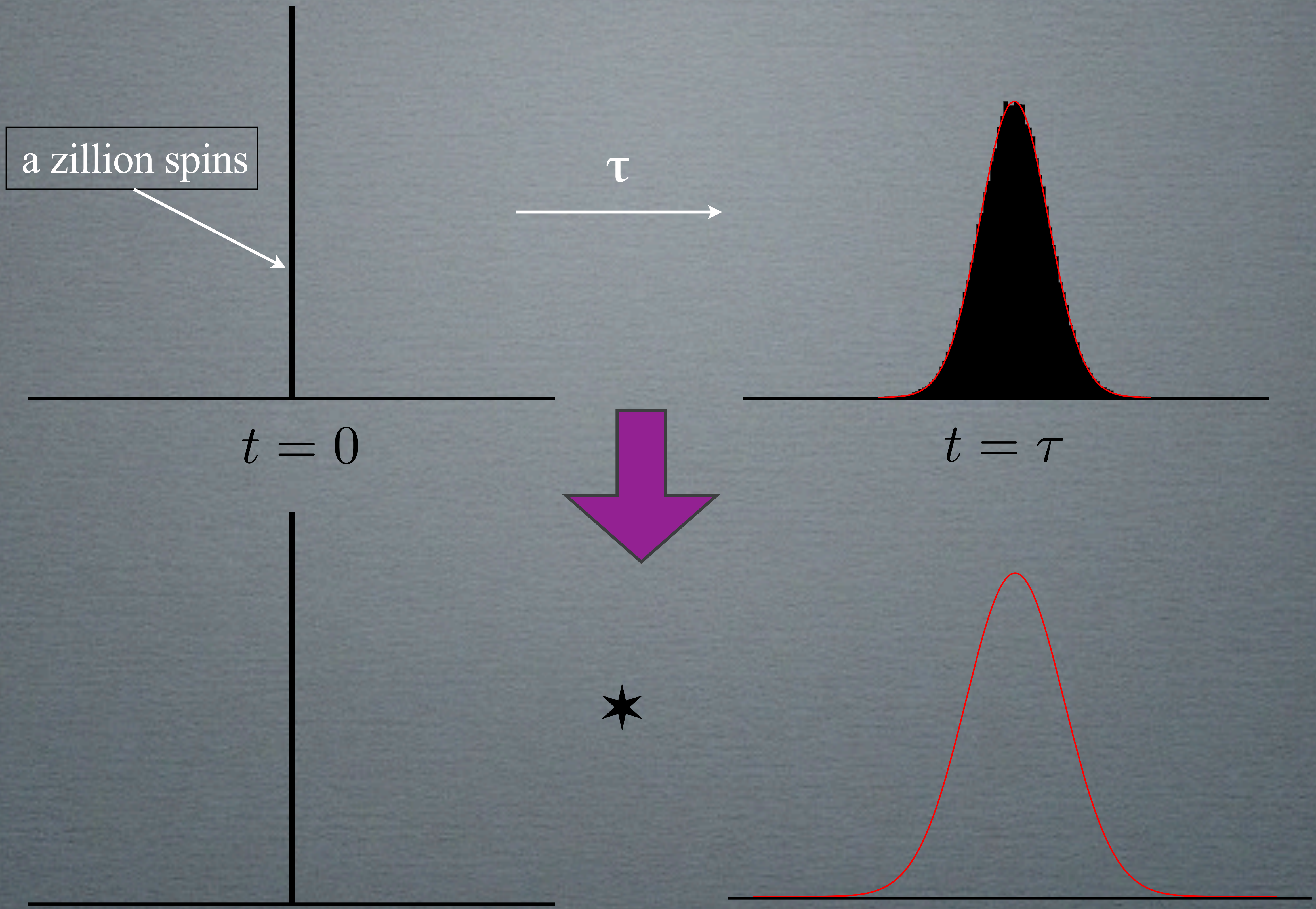


reconstruct **D**
(diffusion ellipsoid)

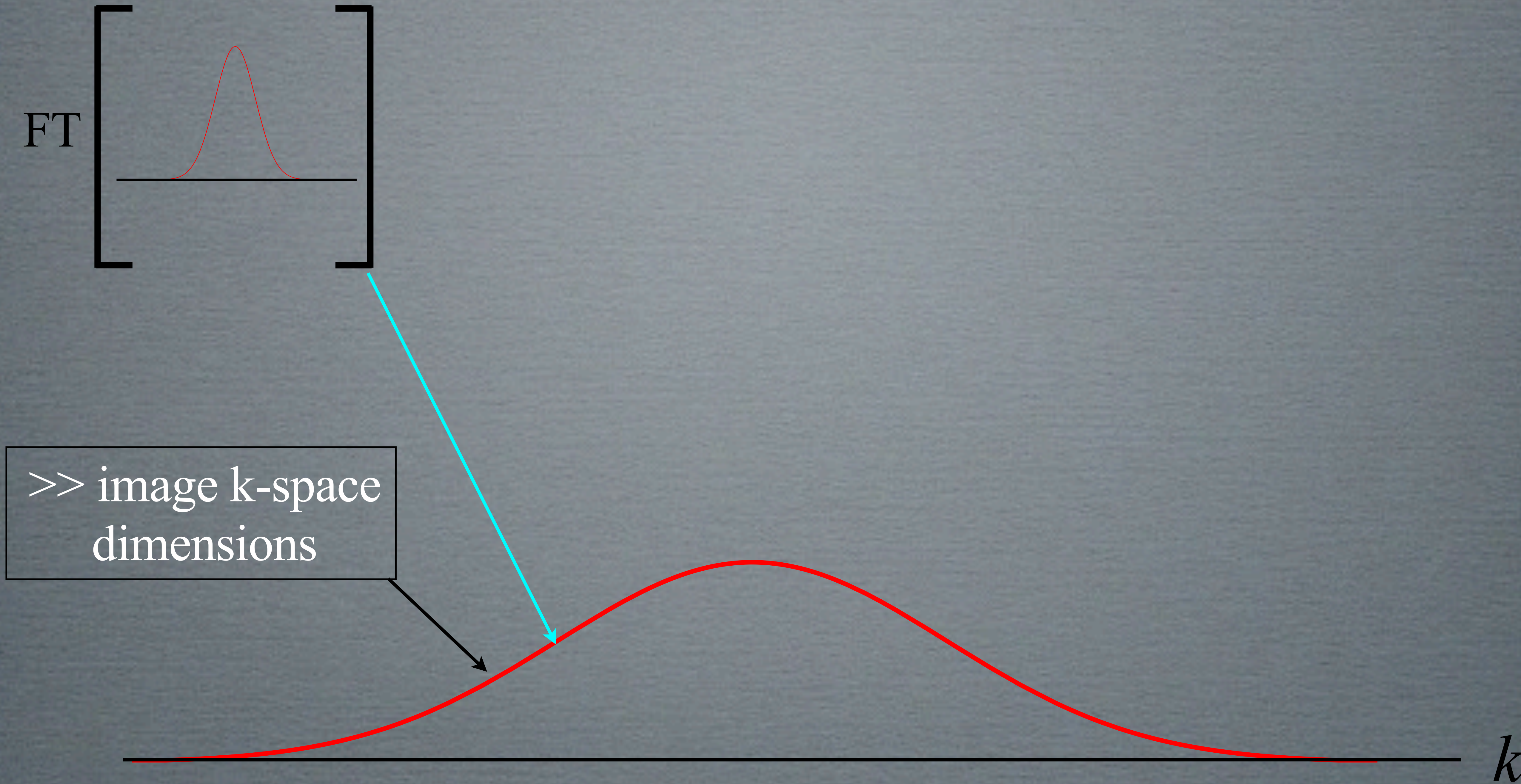


Diffusion acts as a convolution in the image domain

Diffusion acts as a convolution in the image domain

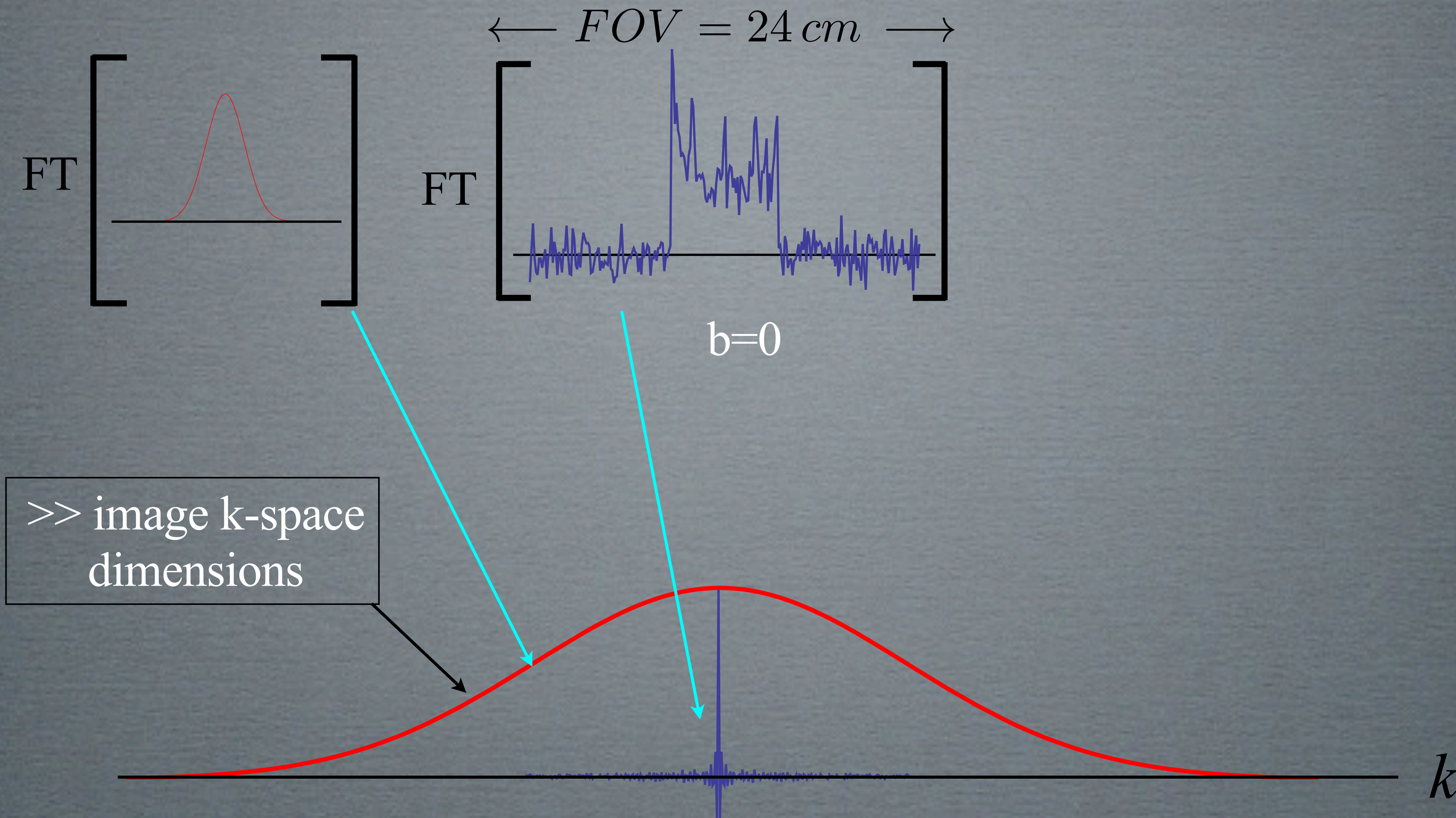


$\sigma \approx 15 \mu m \ll$ voxel dimensions

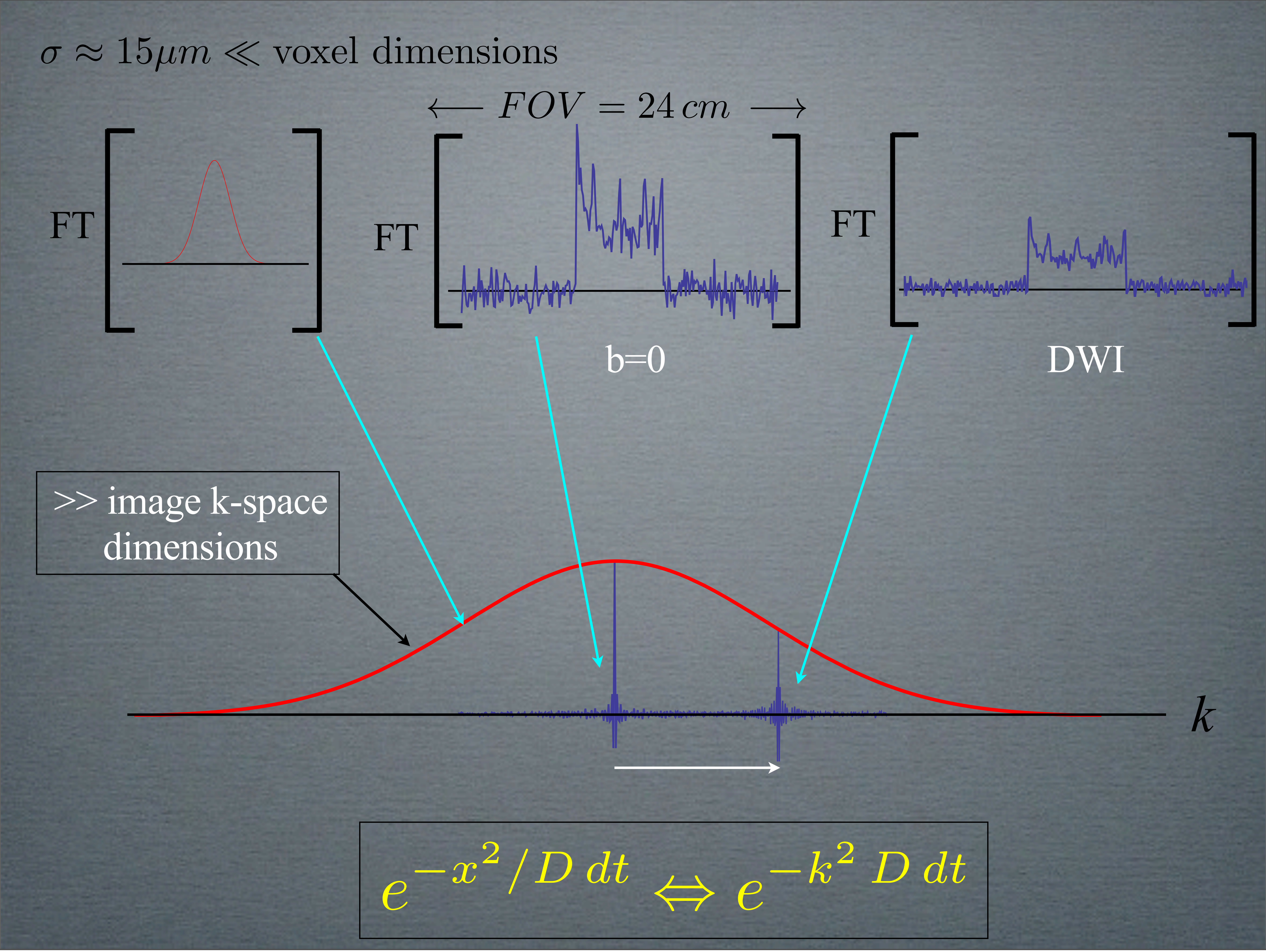


$$e^{-x^2/D\,dt} \Leftrightarrow e^{-k^2\,D\,dt}$$

$\sigma \approx 15\mu m \ll$ voxel dimensions



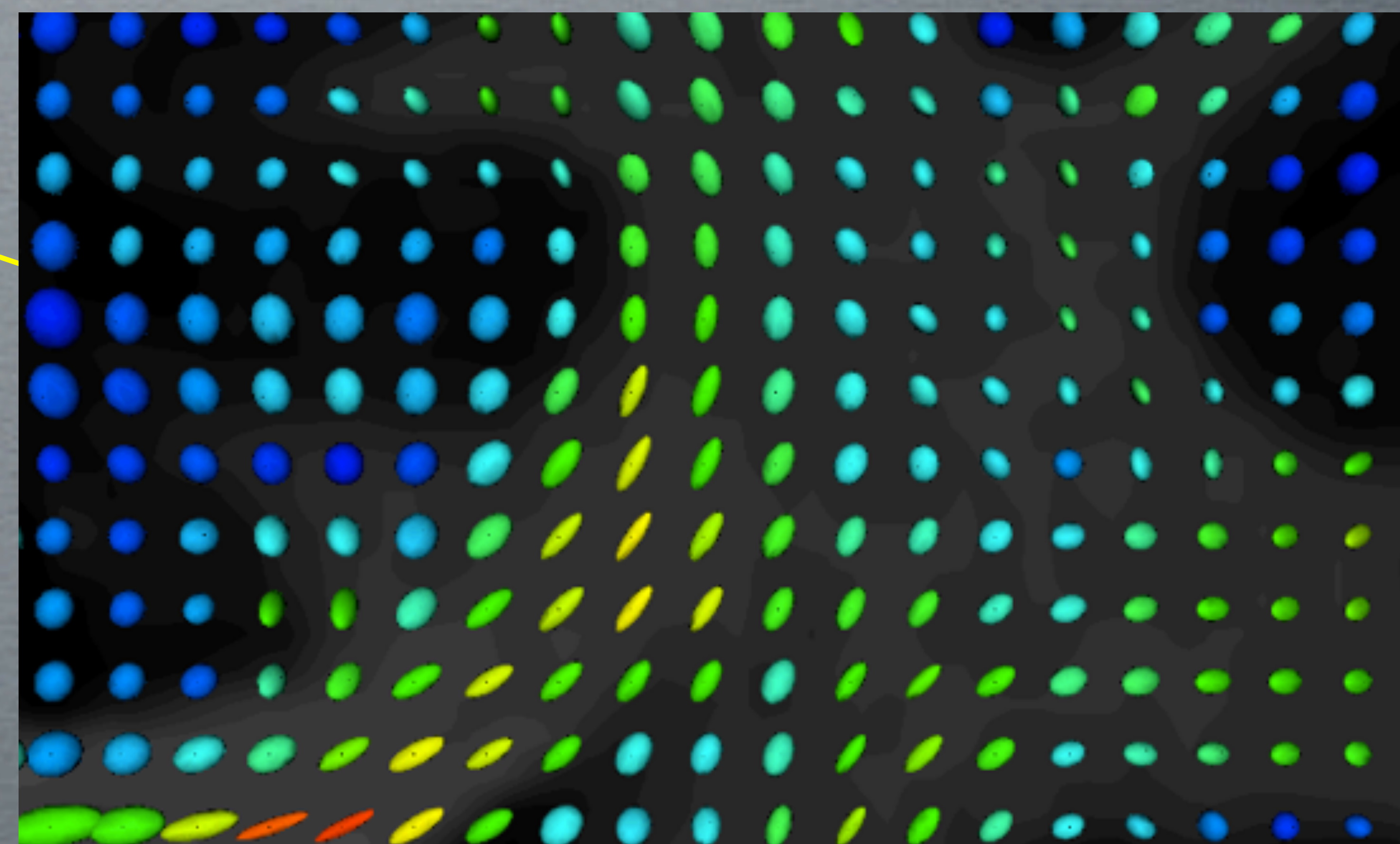
$$e^{-x^2/D\,dt} \Leftrightarrow e^{-k^2\,D\,dt}$$



$\sigma \approx 15\mu m \ll$ voxel dimensions
$\longleftarrow FOV = 24\,cm \longrightarrow$
FT
FT
FT
b=0
DWI
>> image k-space dimensions
k
$e^{-x^2/D\,dt} \Leftrightarrow e^{-k^2\,D\,dt}$

Diffusion Ellipsoid

Diffusion Ellipsoid



diffusion ellipsoids

Average Diffusion in a voxel

Three eigenvalues of D are the three principle mean-squared displacements along its three principal directions

$$D = \begin{pmatrix} \lambda_x & 0 & 0 \\ 0 & \lambda_y & 0 \\ 0 & 0 & \lambda_z \end{pmatrix}$$

$\langle D \rangle = (\lambda_1 + \lambda_2 + \lambda_3)/3 = \langle \lambda \rangle$

$$= \mathrm{Tr}(D)$$

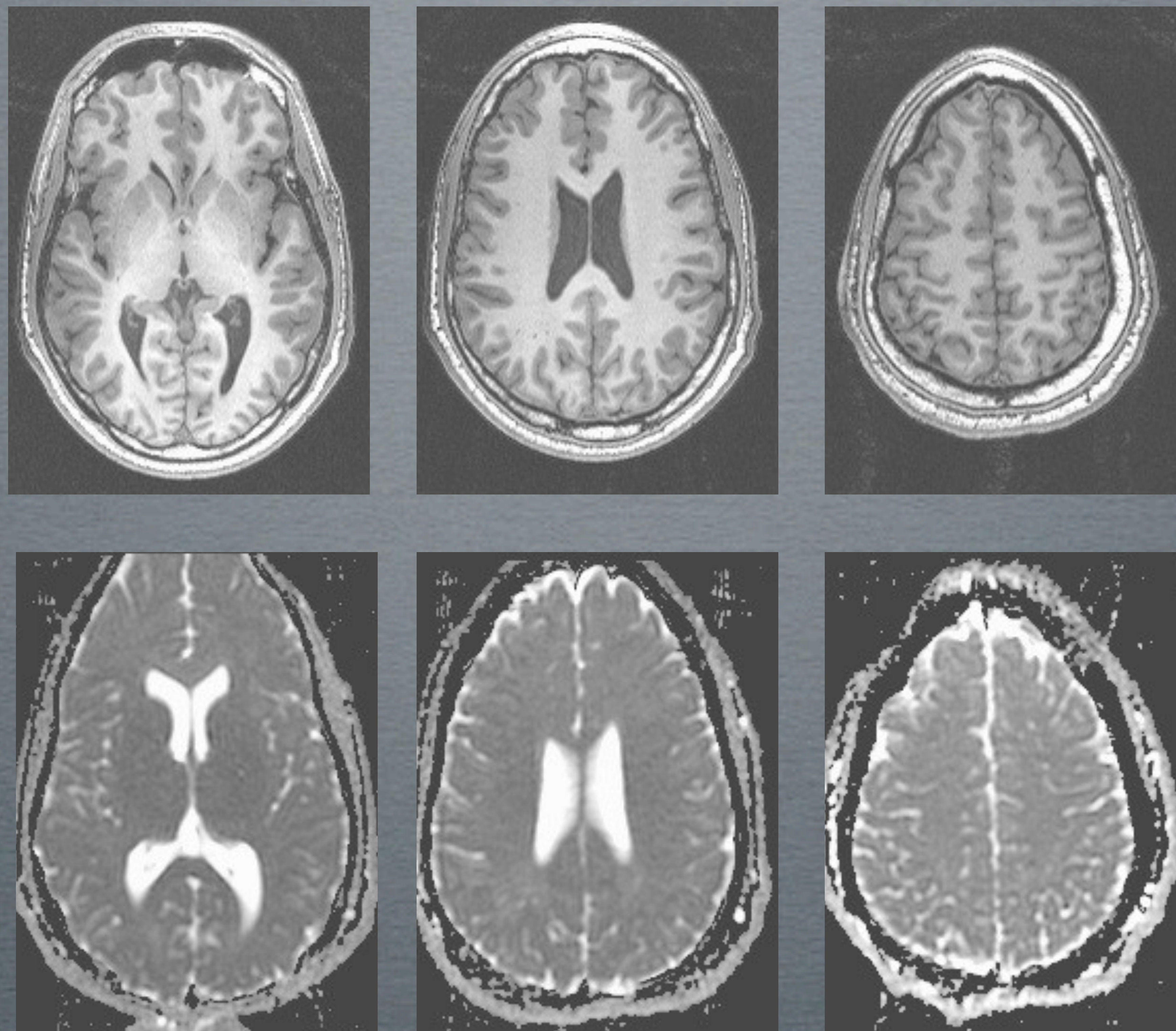
Tr = Trace = sum of diagonal elements

Average Diffusion in a voxel

Average Diffusion in a voxel

anatomical

mean D



Diffusion Anisotropy in a voxel

One measure of diffusion anisotropy is the variance of the eigenvalues, normalized to the mean-squared eigenvalue

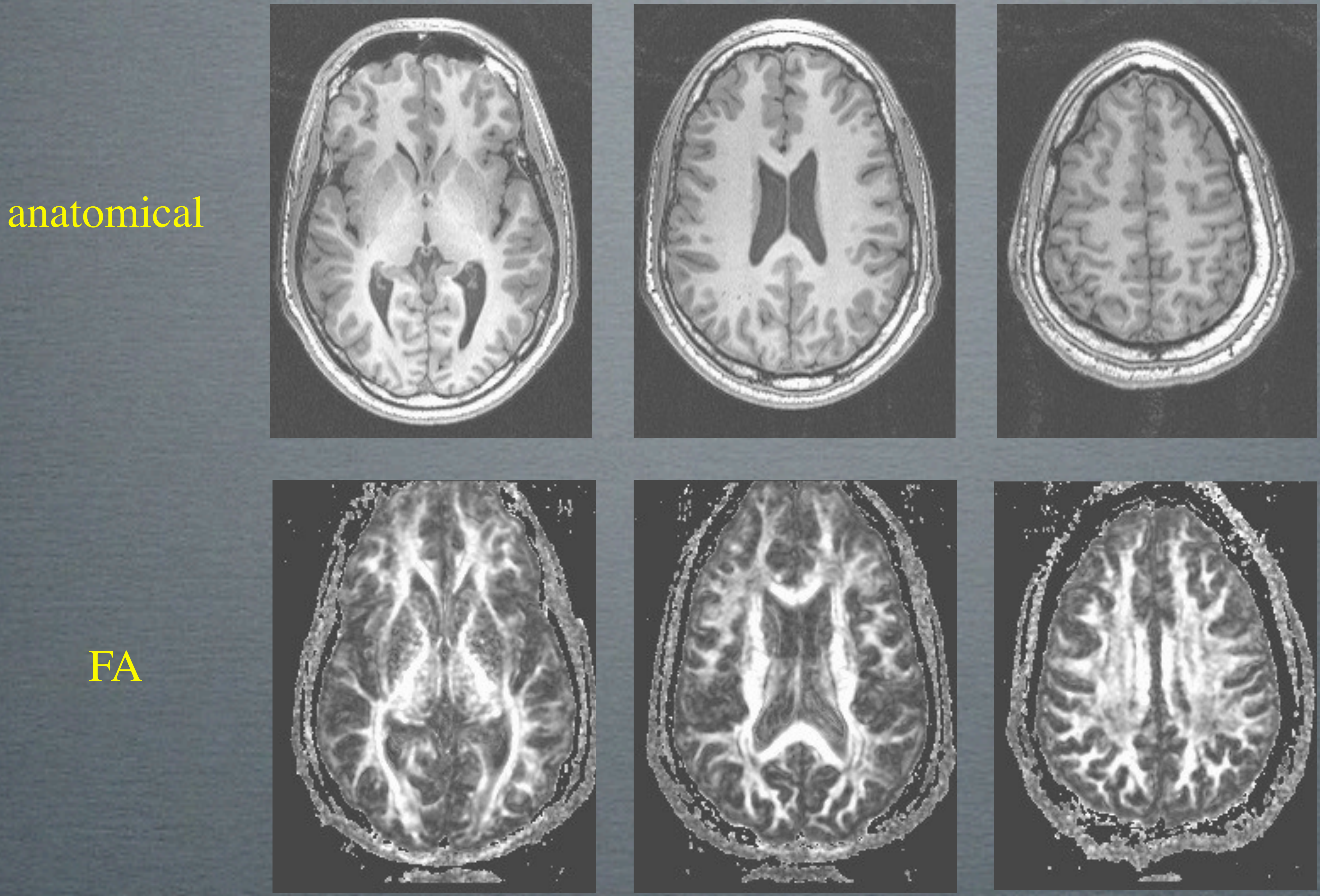
$$\text{anisotropy} \propto \frac{(\lambda_x - \bar{\lambda})^2 + (\lambda_y - \bar{\lambda})^2 + (\lambda_z - \bar{\lambda})^2}{\bar{\lambda}^2}$$

Fractional Anisotropy

$$\mathrm{FA} \equiv \sqrt{\frac{3}{2}\frac{\sigma_\lambda^2}{\bar{\lambda}^2}}$$

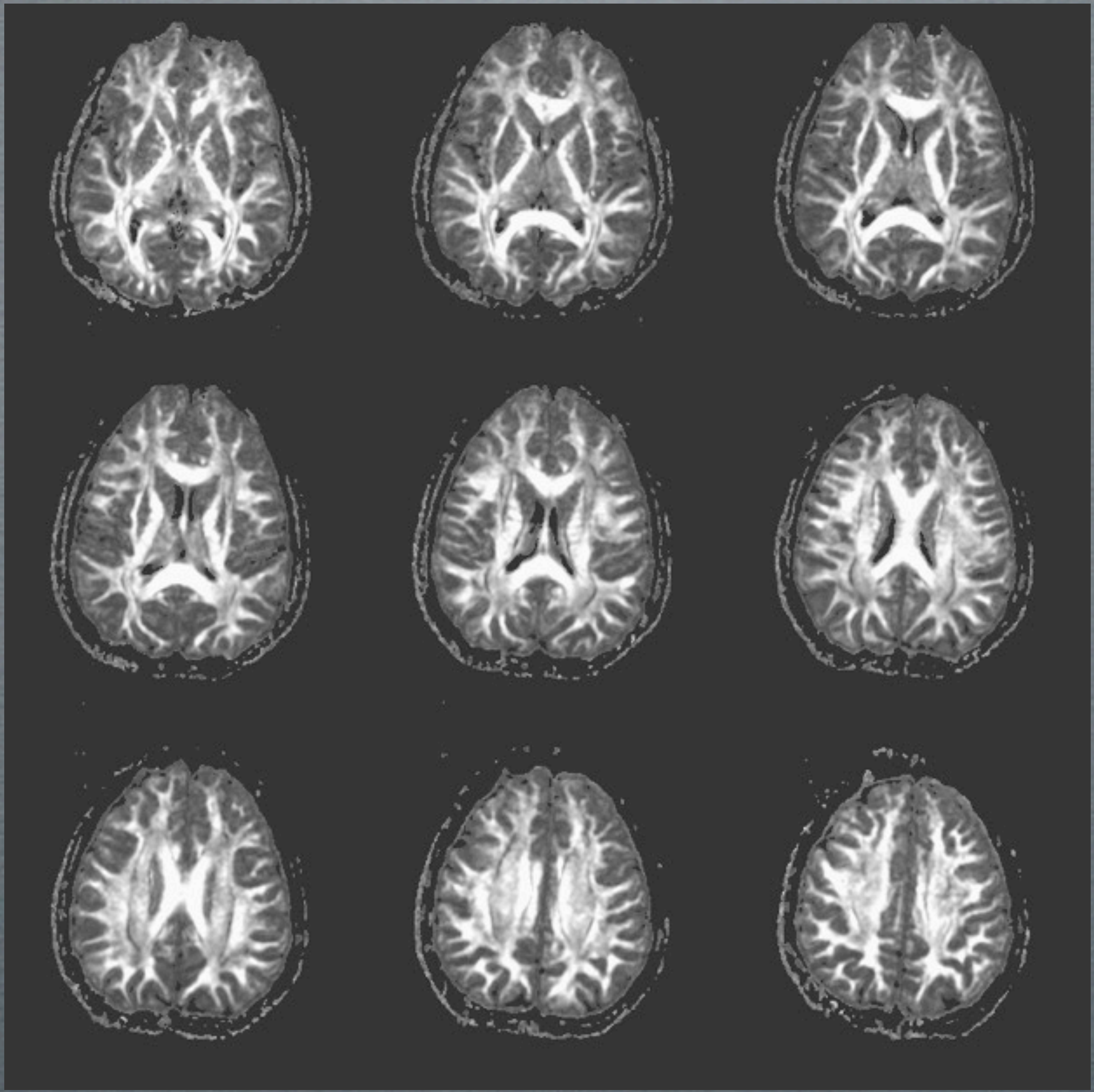
Diffusion Anisotropy in a voxel

Diffusion Anisotropy in a voxel

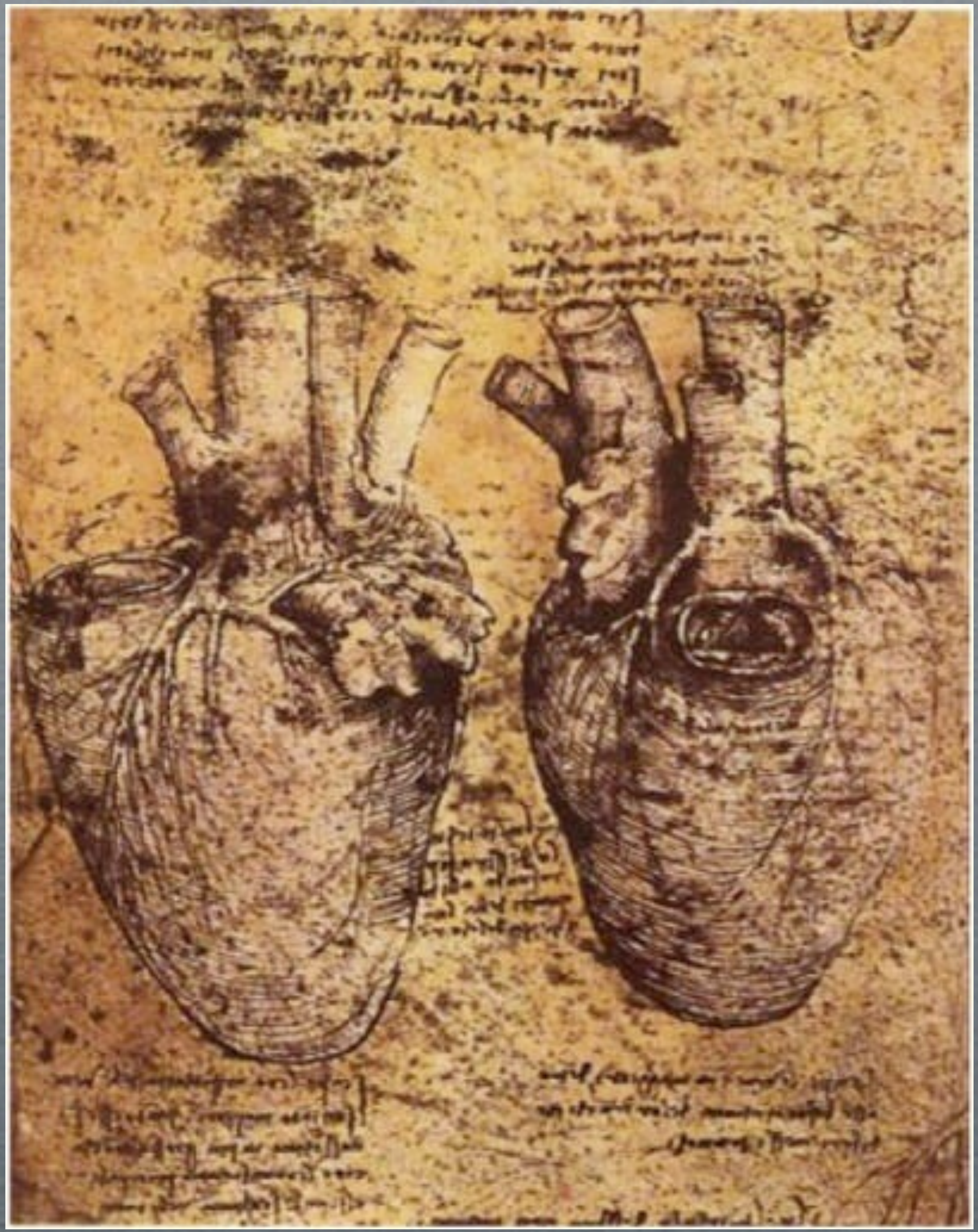


Diffusion Anisotropy

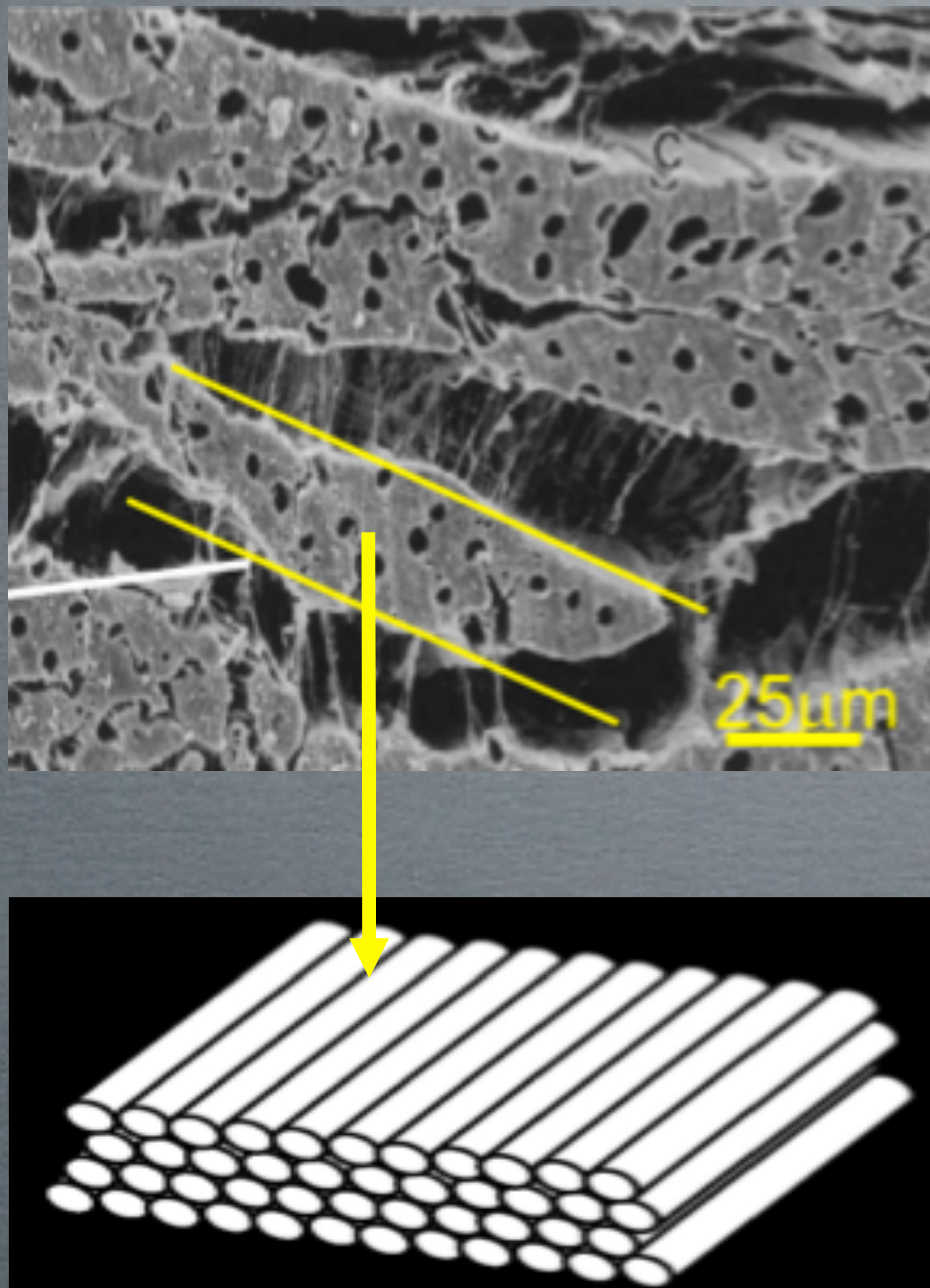
Diffusion Anisotropy



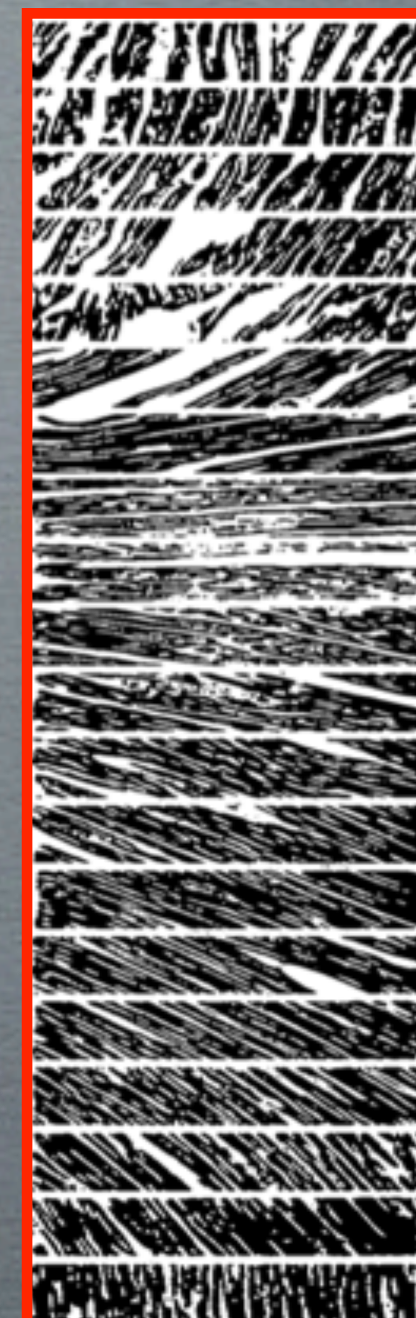
The uses of anisotropy: Cardiac Mechanics



The uses of anisotropy: Cardiac Mechanics

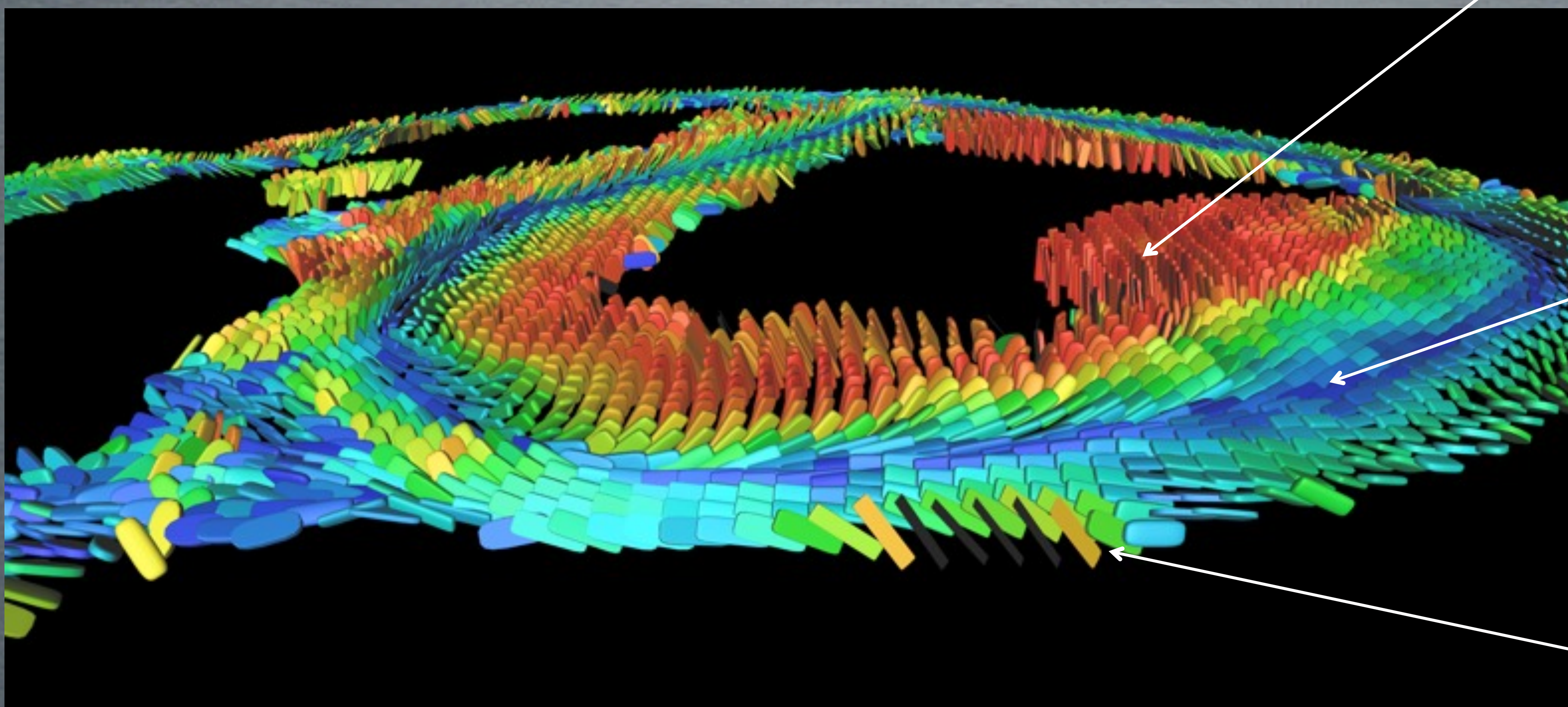


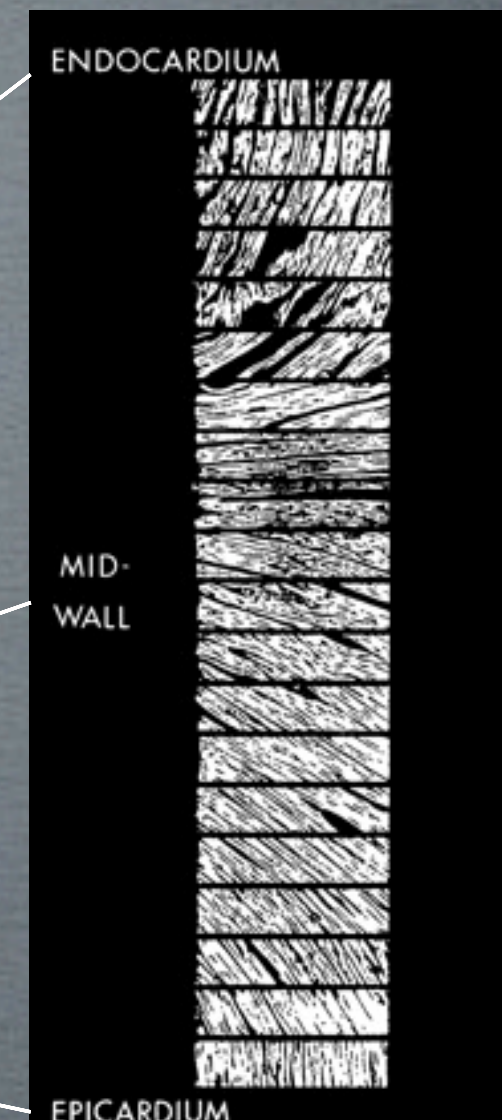
Legrice et al. 1994



Streeter et al. 1969

The uses of anisotropy: Cardiac Mechanics

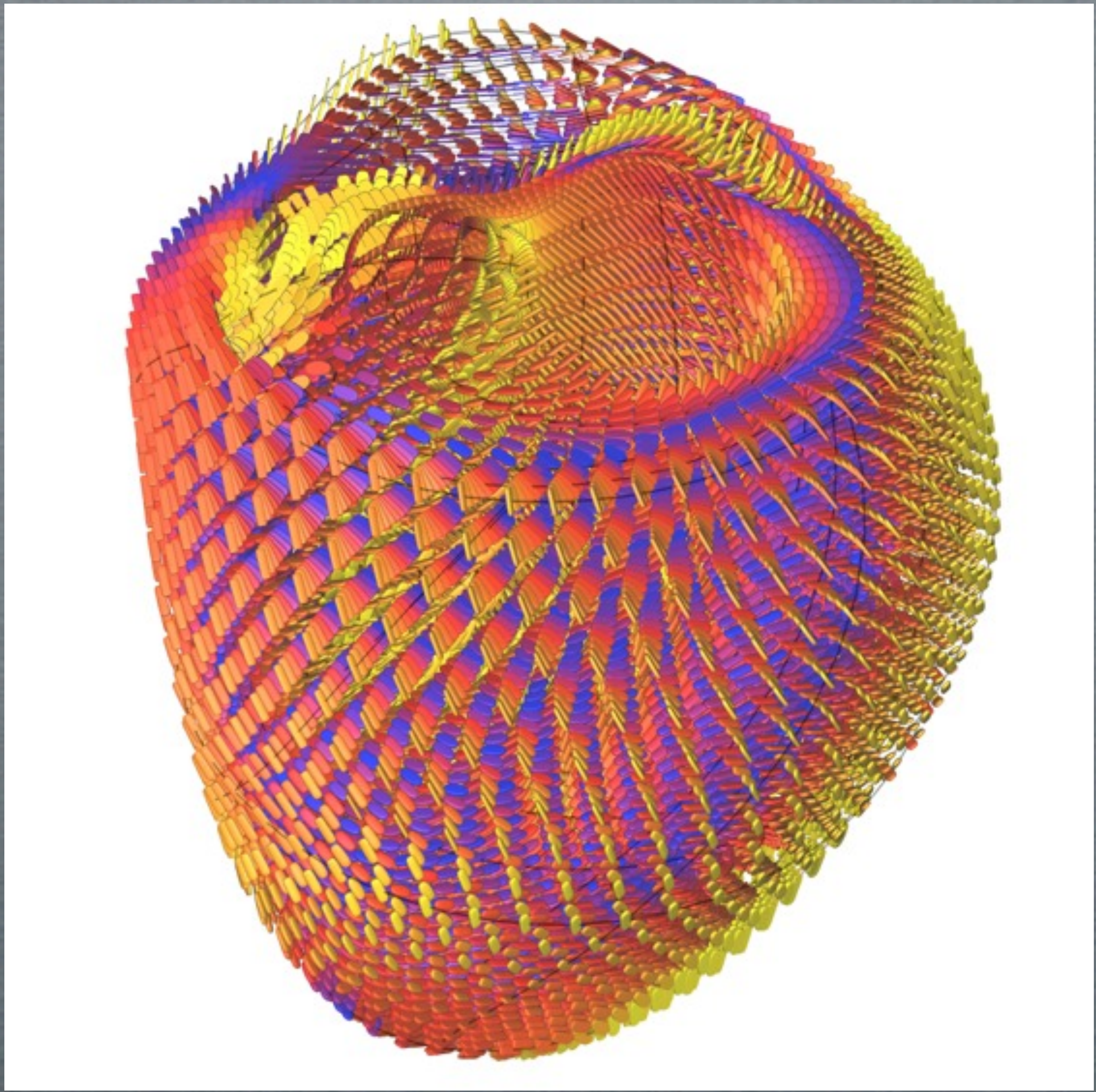




Streeter, 1969

Excised canine heart (Howard, UCSD Cardiac Biomechanics Group 2011) using 3D Spiral FSE DTI sequence (Frank et al, Neuroimage 2010)

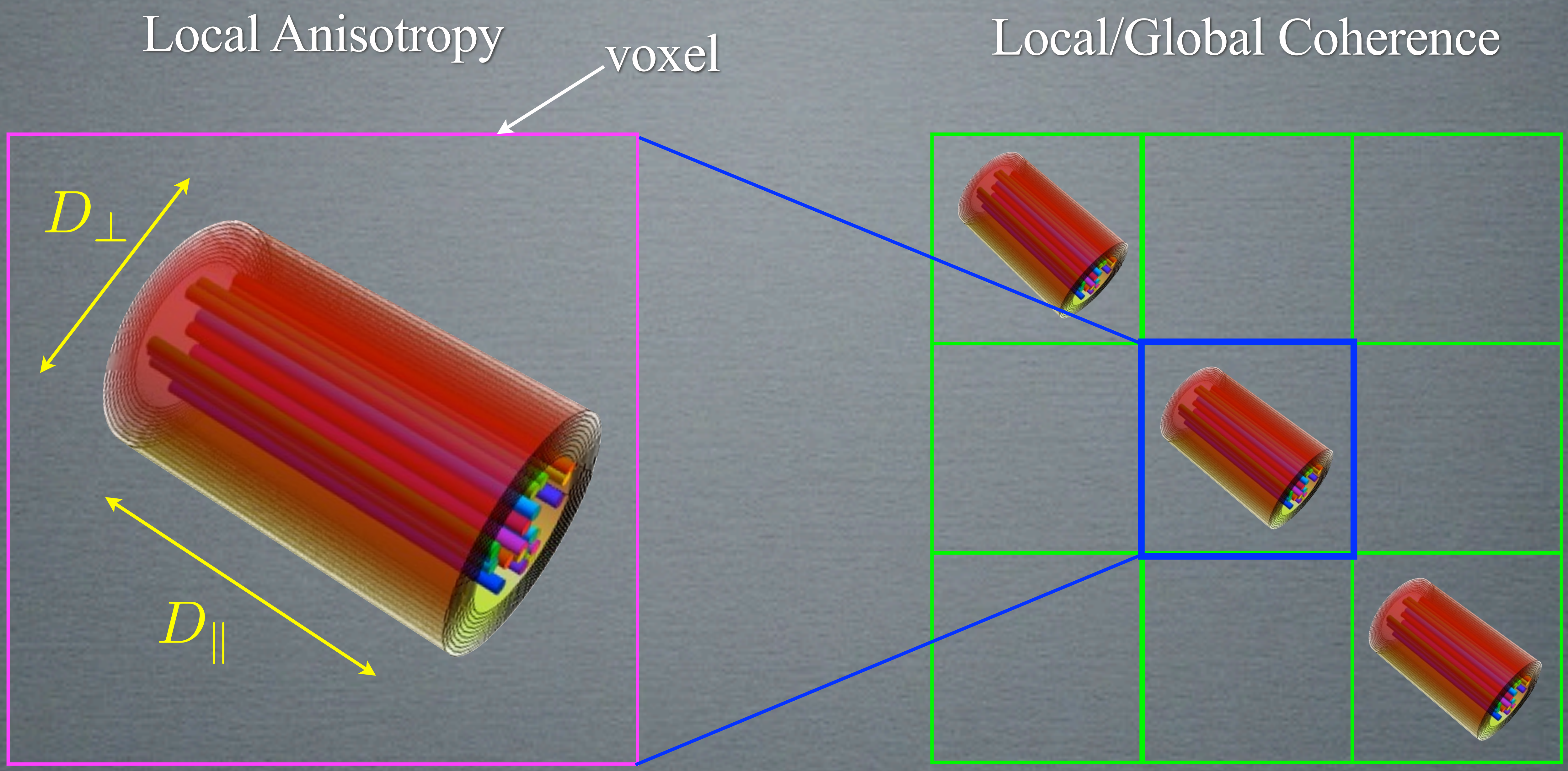
The uses of anisotropy: Cardiac Mechanics



Our first whole human heart DTI (ex vivo)

From local (voxel) anisotropy to extended spatially coherent anisotropy: Tractography

From local (voxel) anisotropy to extended spatially coherent anisotropy: Tractography

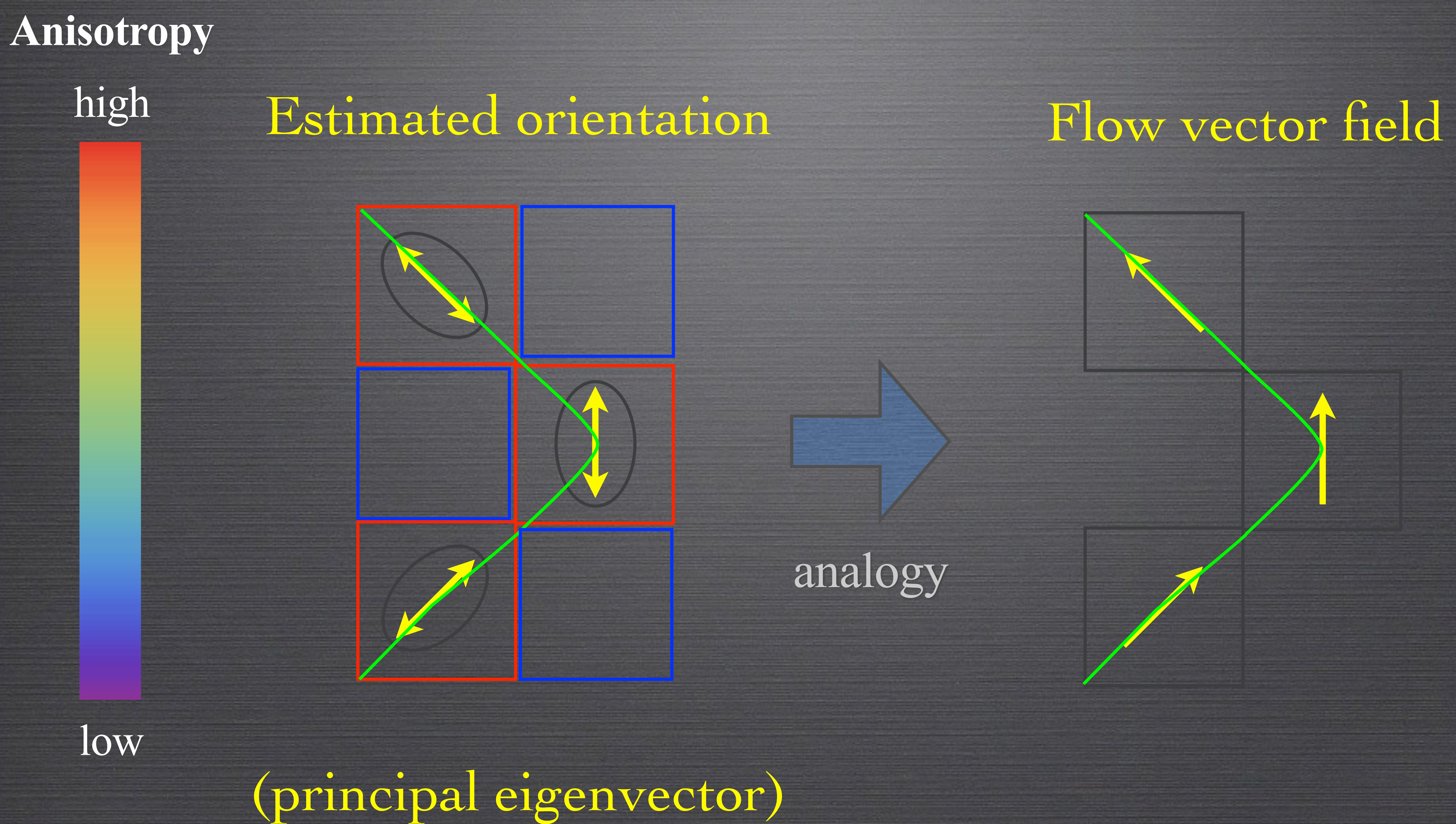


$$D_\| \approx 3D_\perp$$

$(1.2\mu^2/ms)$ $(0.4\mu^2/ms)$

Streamlines

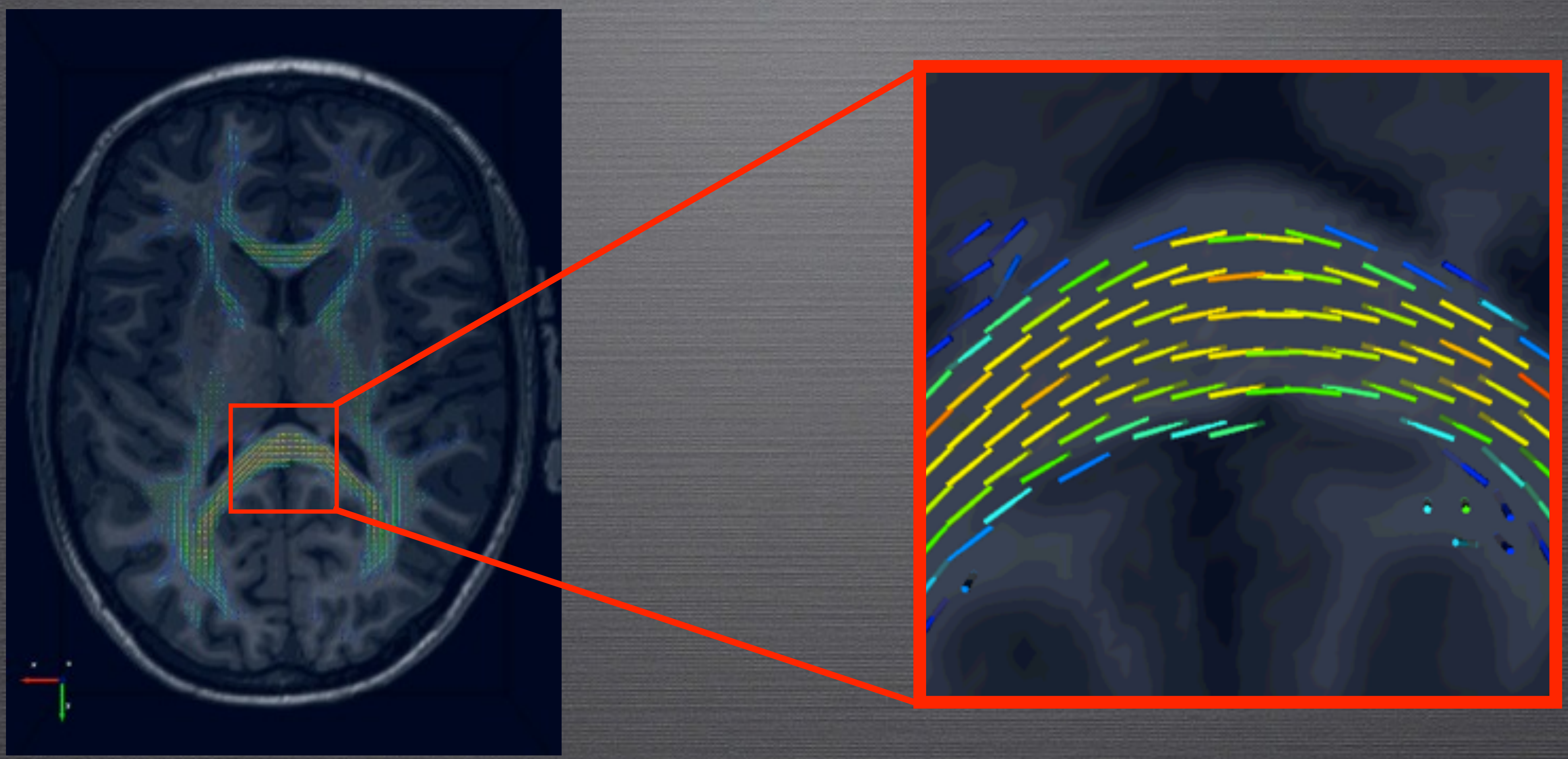
Streamlines



What we expect of Diffusion Imaging

What we expect of Diffusion Imaging

Some information about the microscopic structure



For voxels with aligned fibers (as in the corpus callosum)...

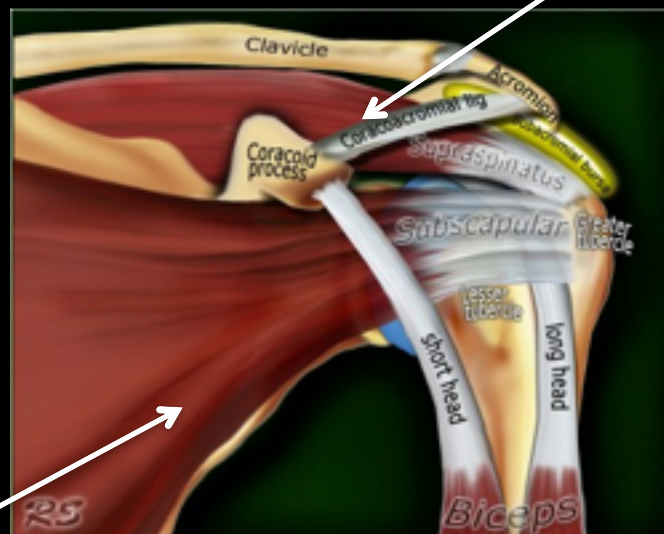
...the primary diffusion direction should be oriented in the same direction as the fiber.

A. Rodrigues-Soto,
Ward group

Supraspinatus DTI

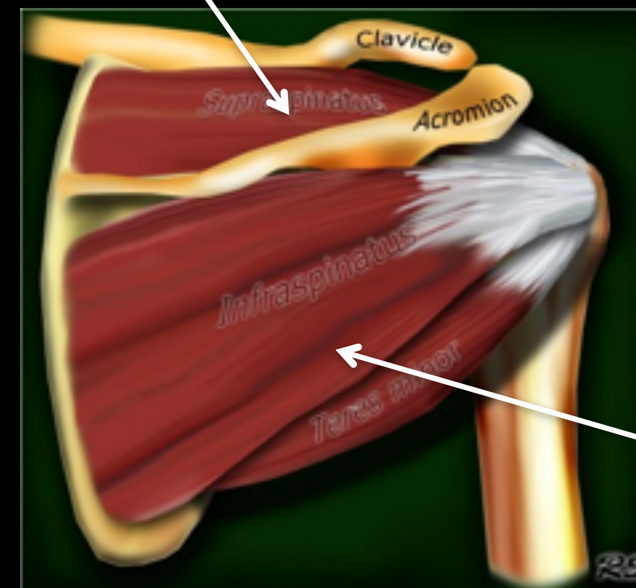
Supraspinatus

Anterior

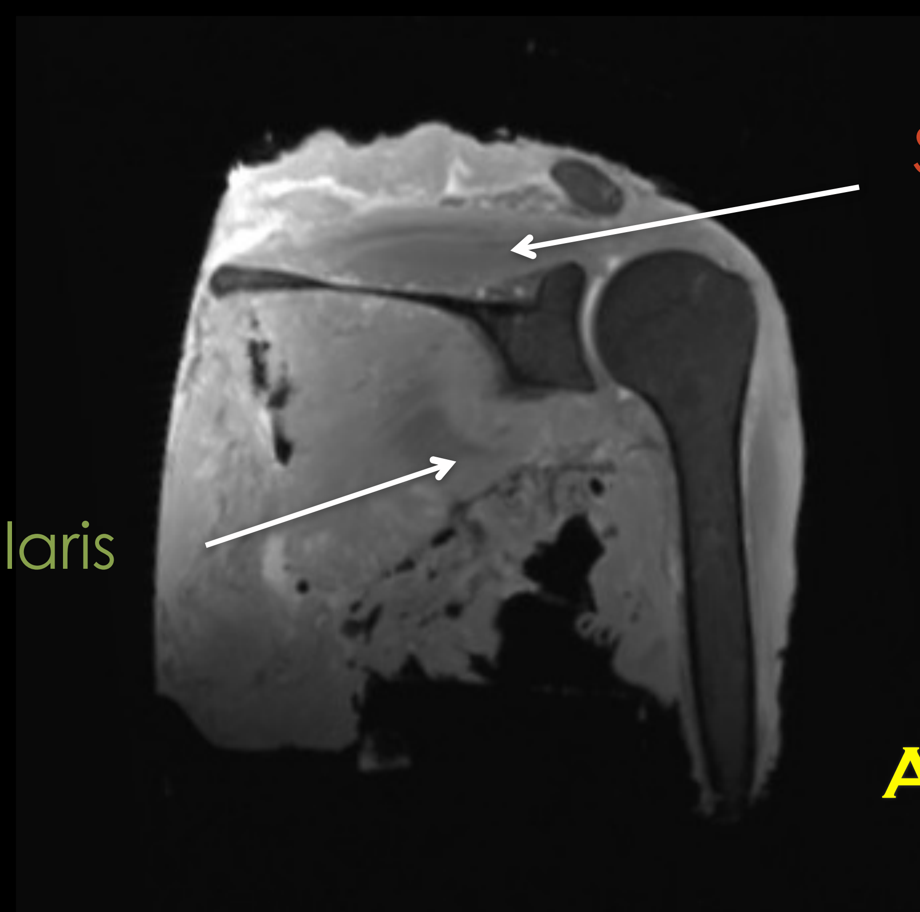


Subscapularis

Posterior



Infraspinatus
+ Teres Minor

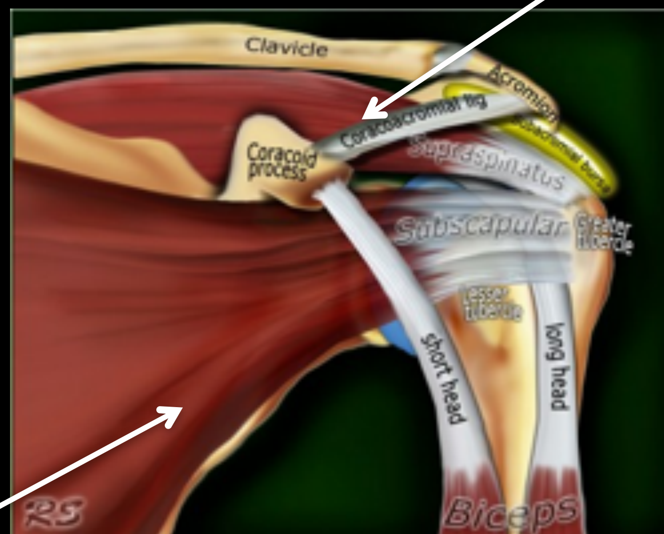


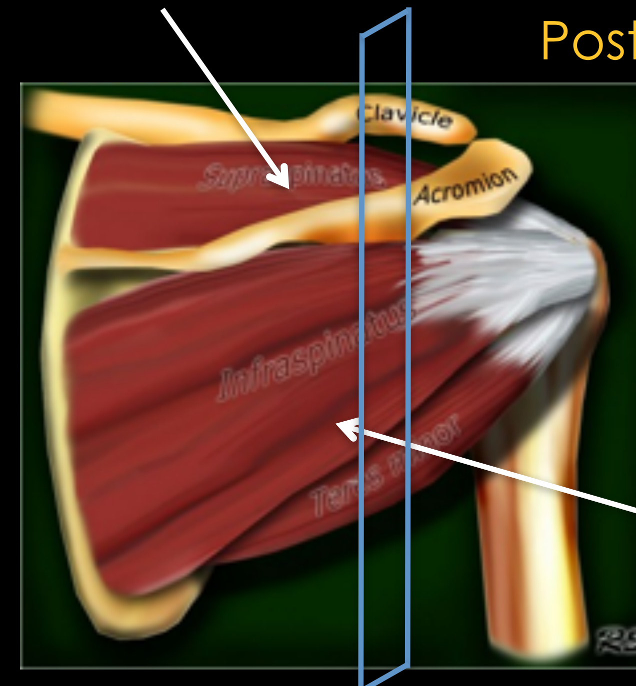
Supraspinatus

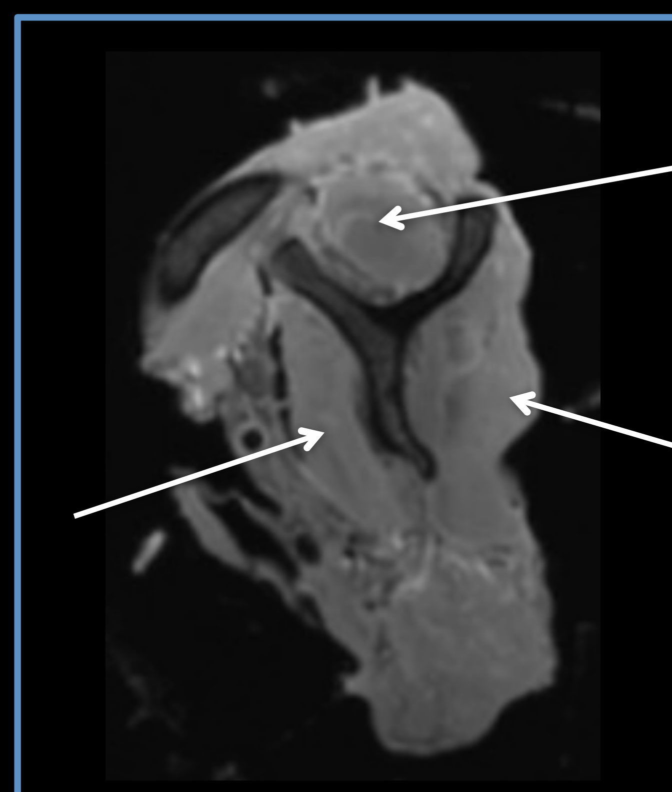
Subscapularis

A. Rodrigues-Soto, Ward group

Supraspinatus DTI





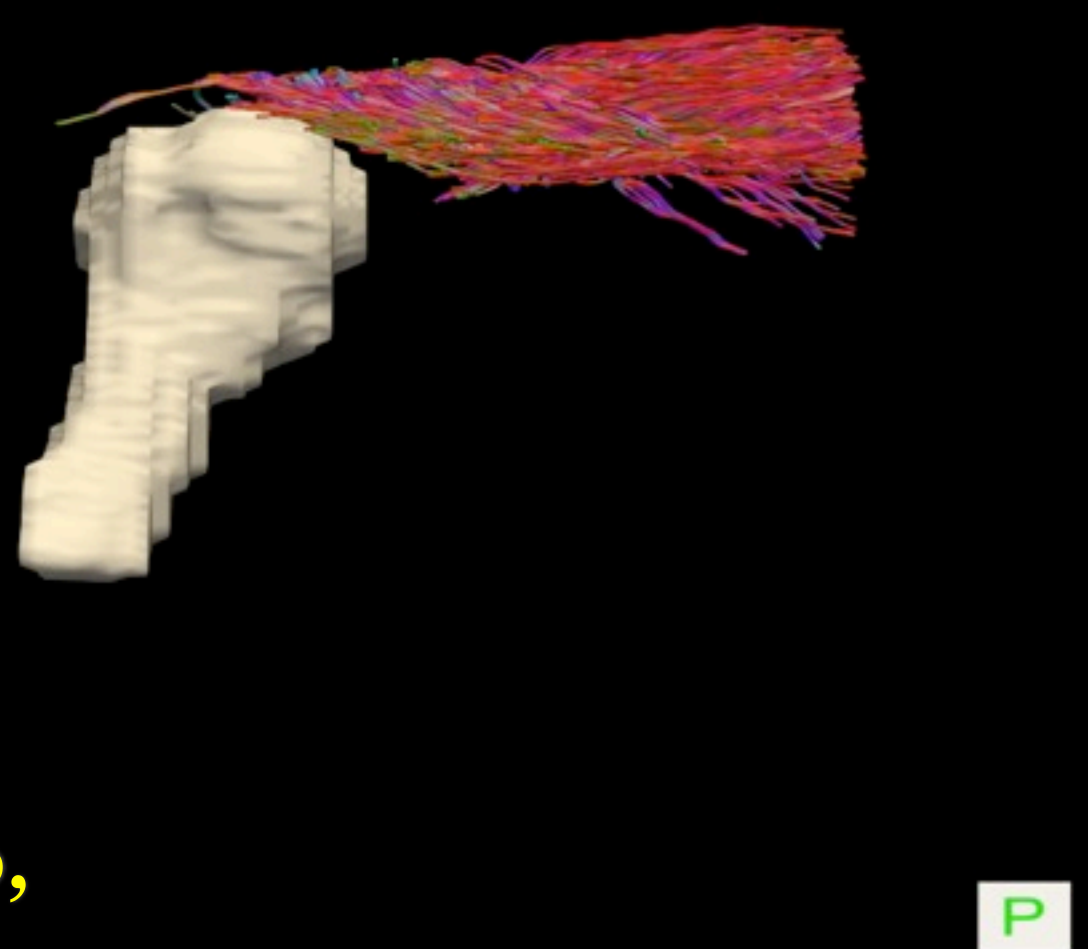


A. Rodrigues-Soto, Ward group

Supraspinatus Tractography @60 directions

A. Rodrigues-Soto,
Ward group

Supraspinatus Tractography @60 directions



A. Rodrigues-Soto,
Ward group

Supraspinatus Tractography @60 directions

A. Rodrigues-Soto,
Ward group

What is the Neural Structure of Elasmobranchs?

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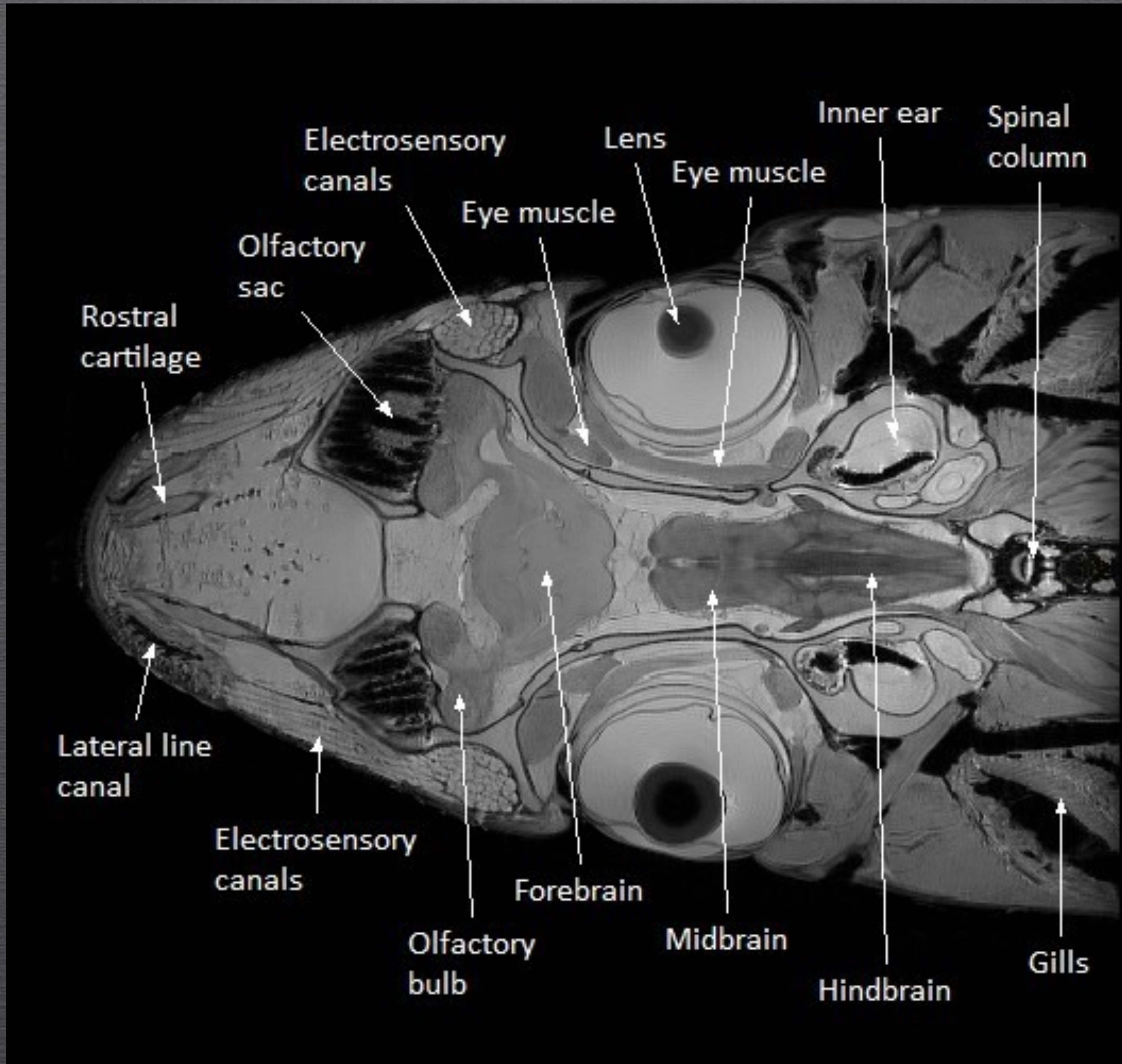
Mustelus henlei

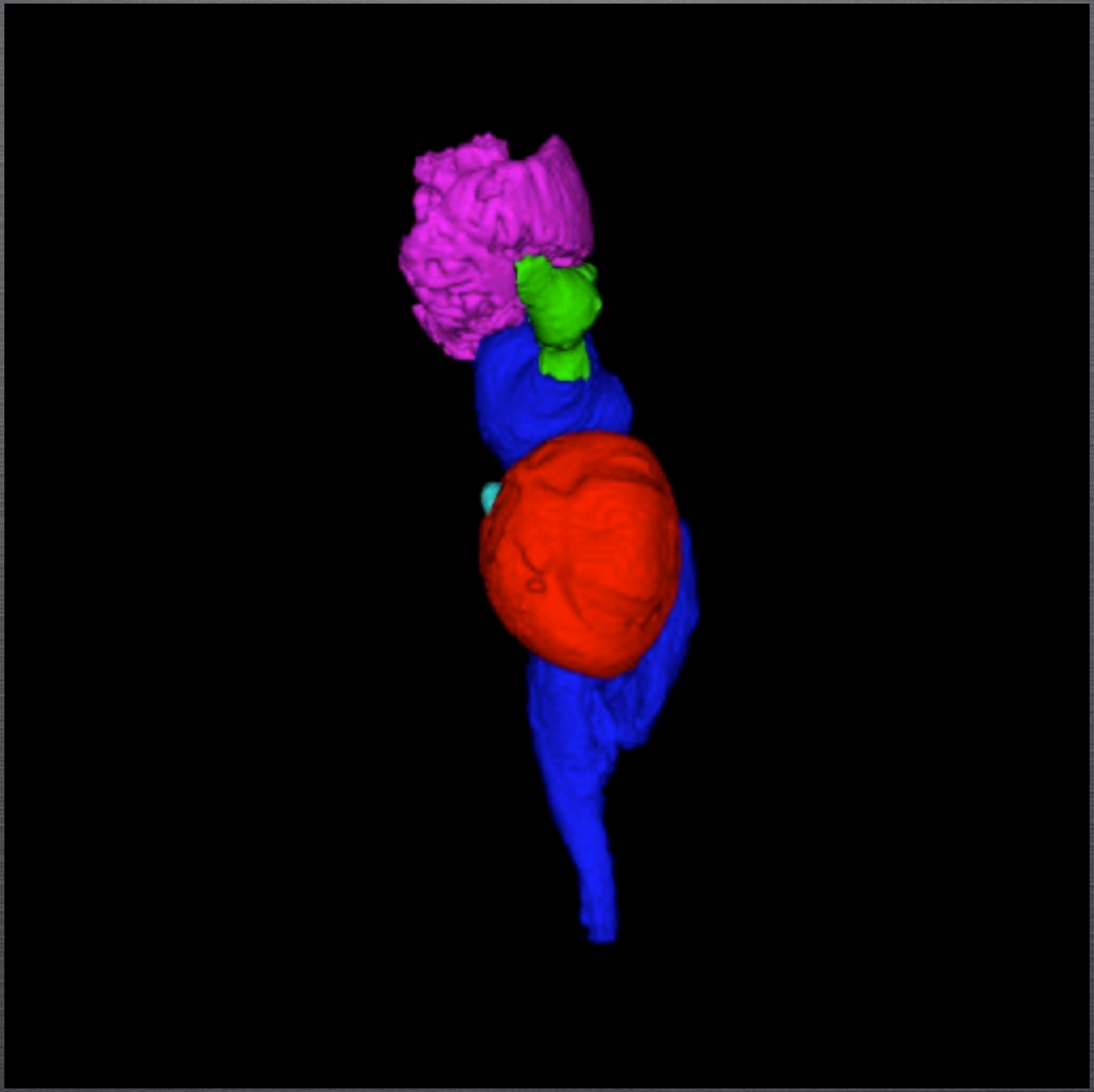
What is the Neural Structure of Elasmobranchs?



Mustelus henlei

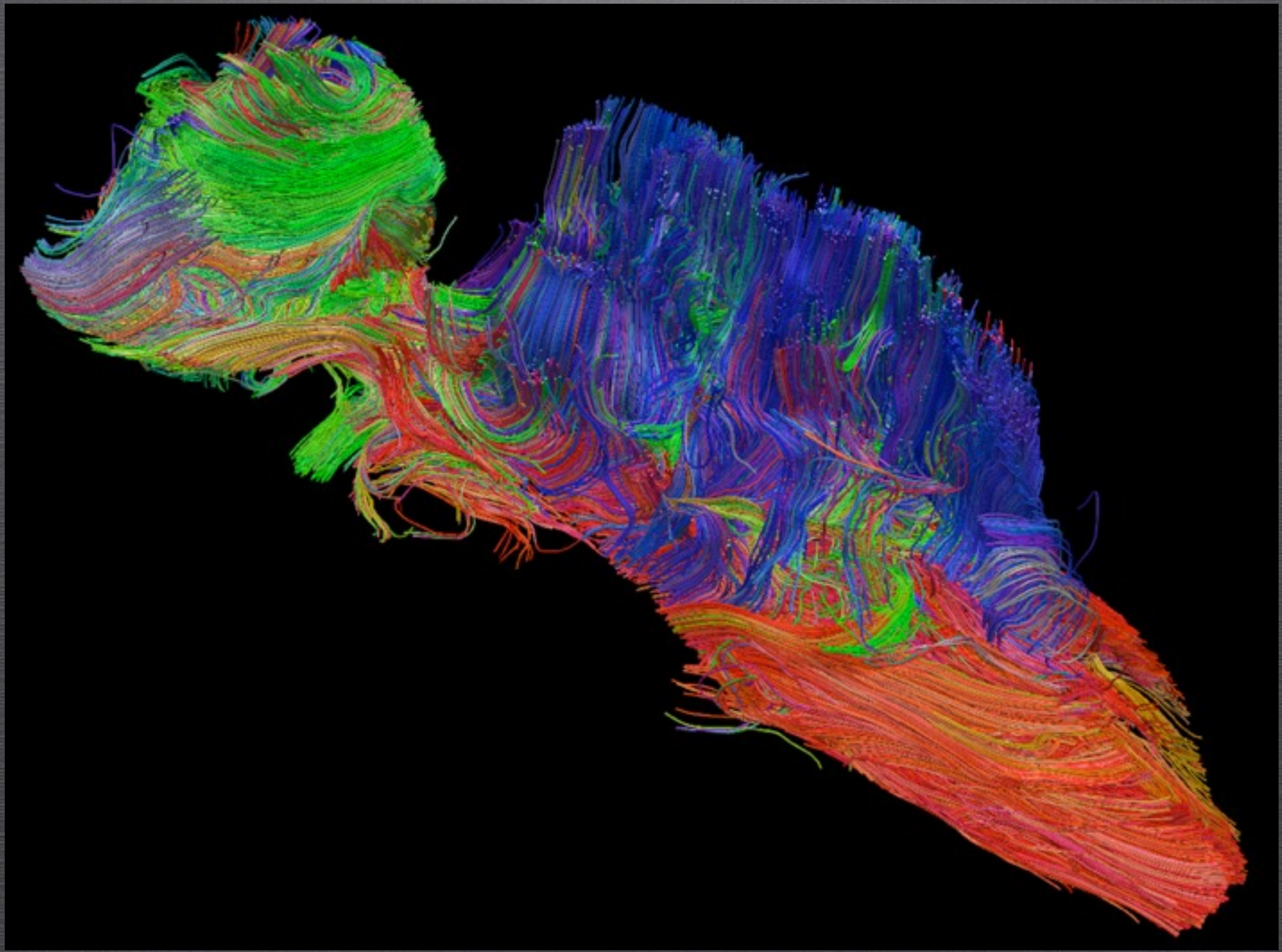
Data: M. Tyszka, CalTech
R. Berquist, CSCI

What is the Neural Structure of Elasmobranchs?



Segmentation: K. Yopak, CSCI

What is the Neural Structure of Elasmobranchs?

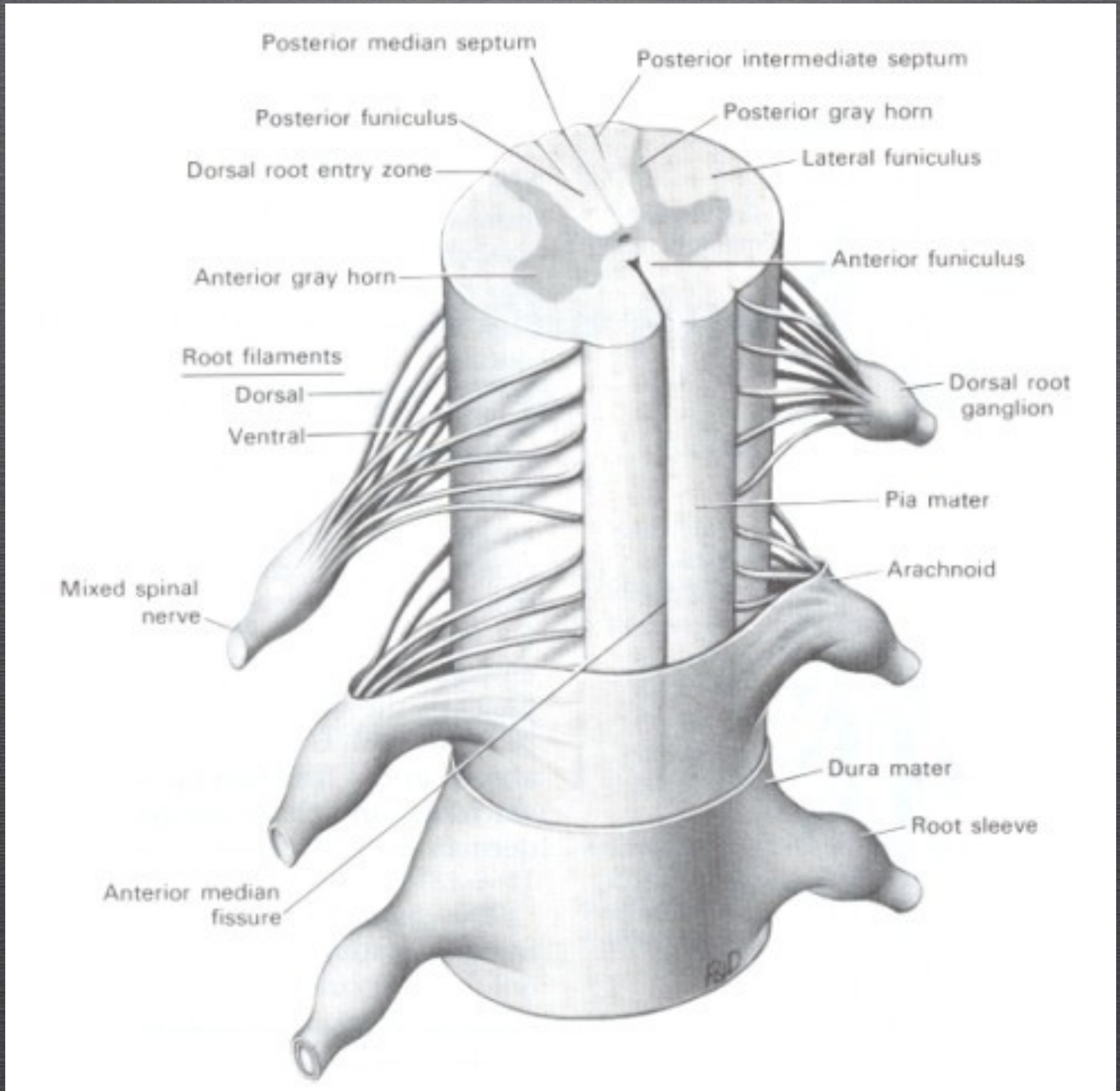


DTI @ 11.7T

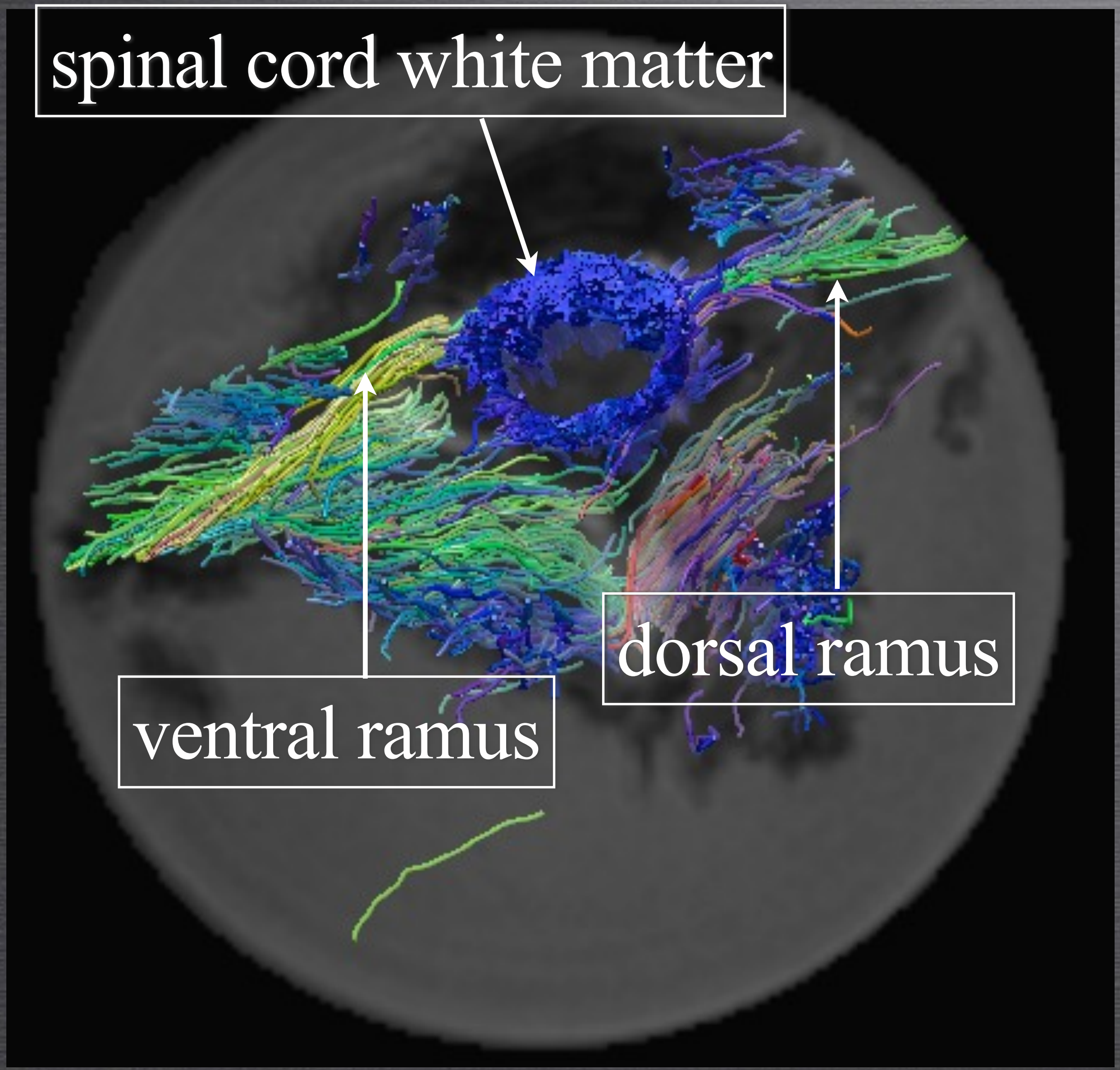
Data: M. Tyszka, CalTech
R. Berquist, CSCI

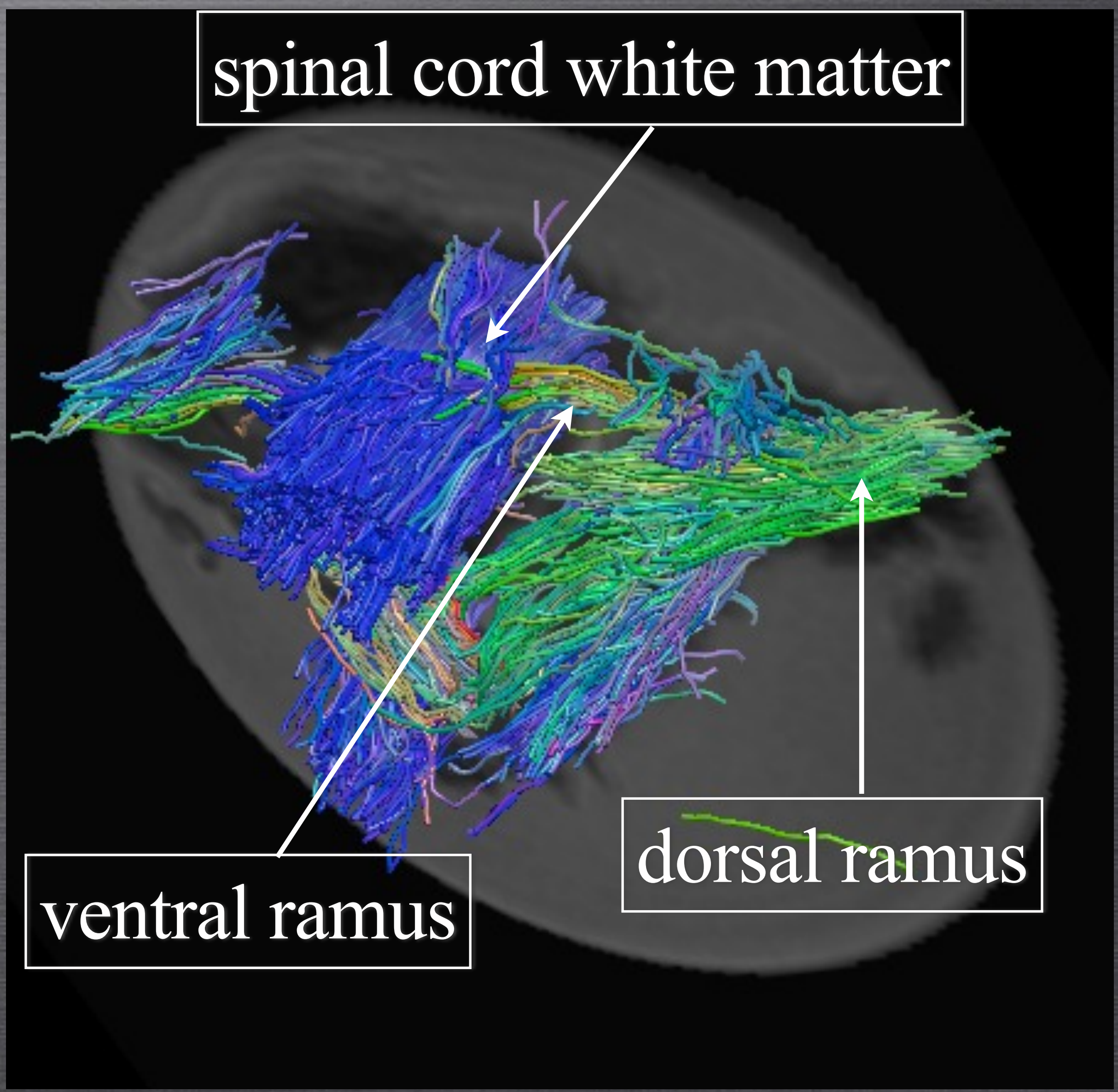
Spinal Cord Injury
(Rat Model at 7T)

Spinal Cord Injury (Rat Model at 7T)



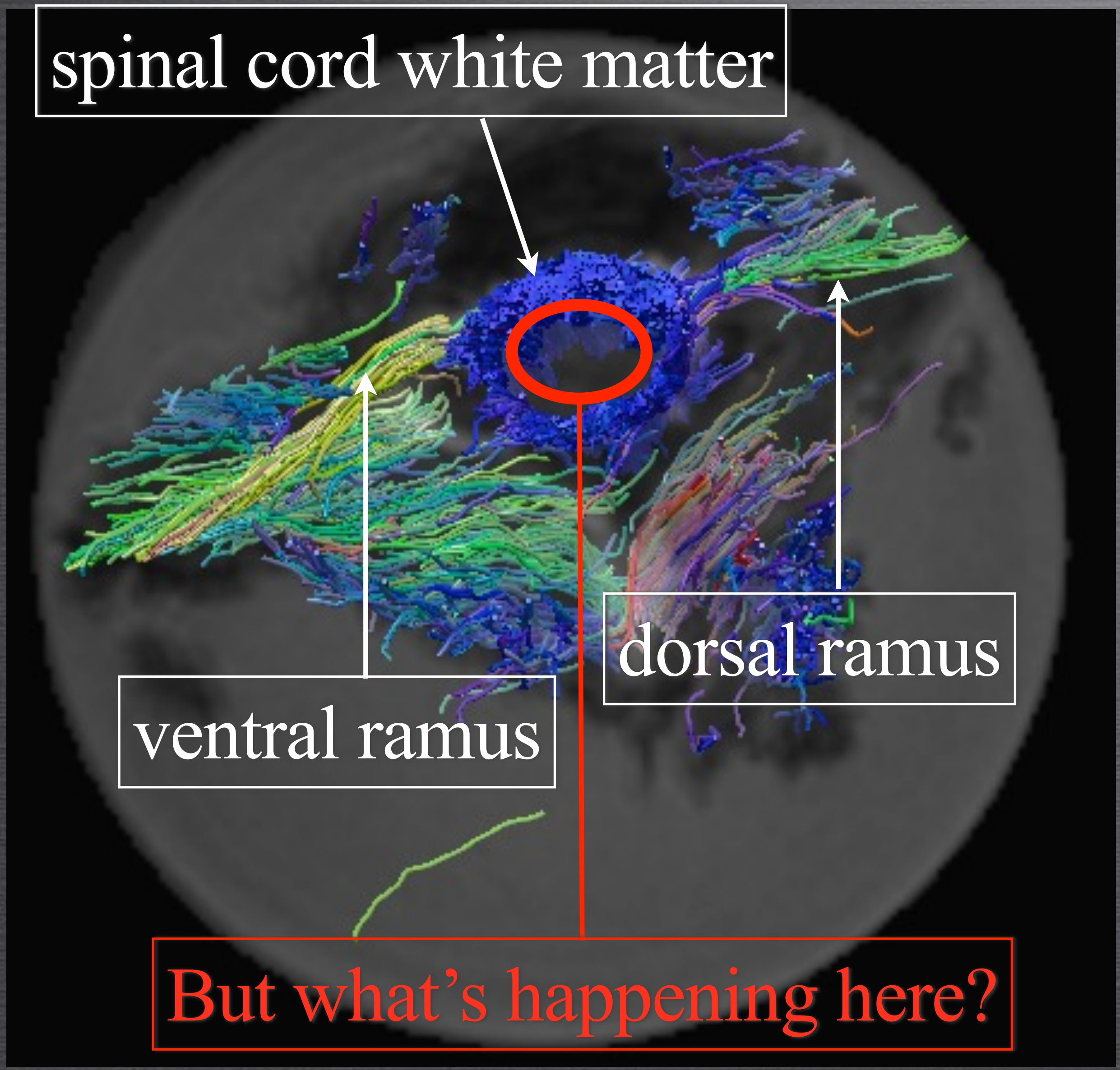
Spinal Cord Injury (Rat Model at 7T)

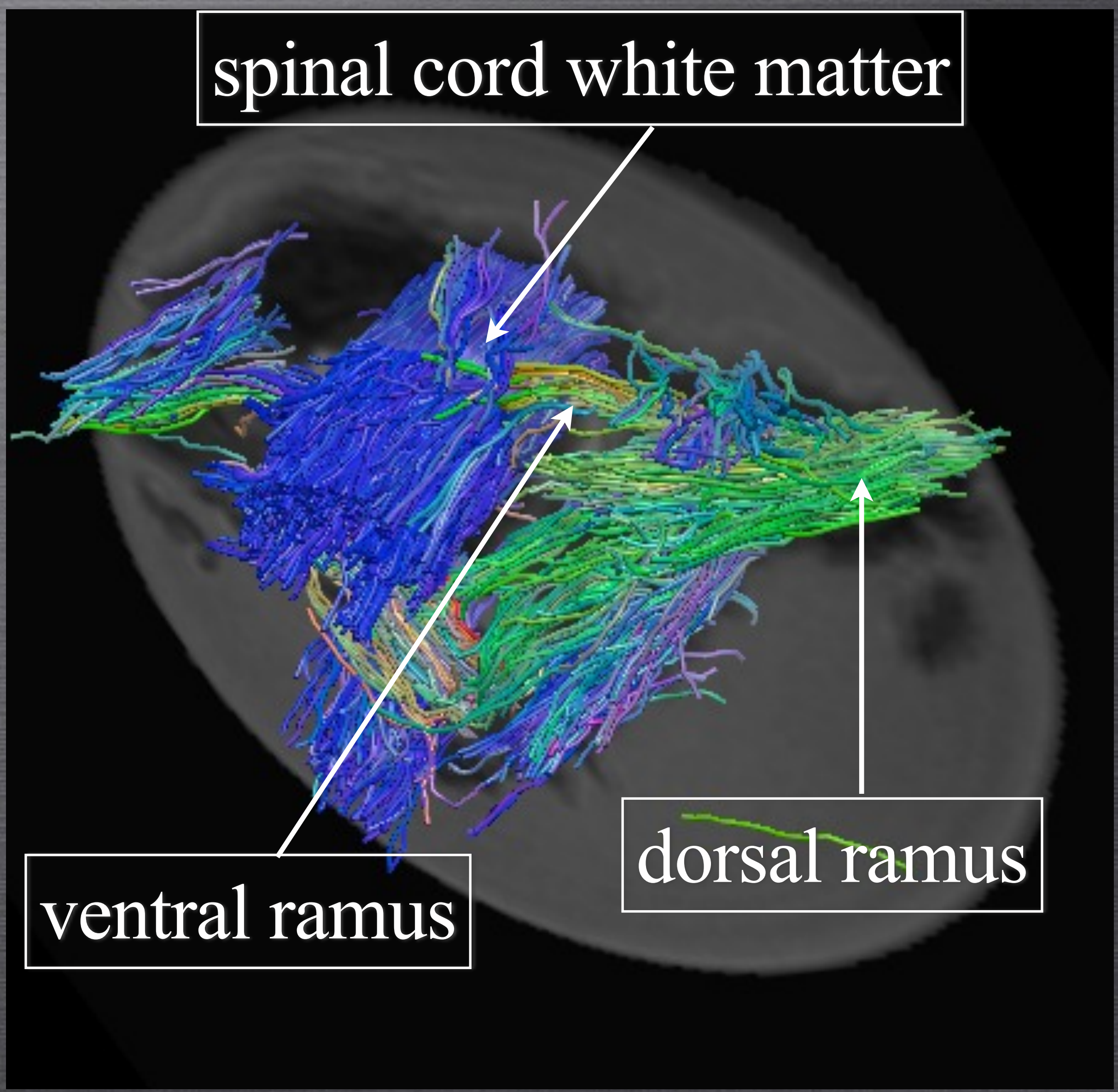




Jacob Koffler, Ph.D.
Mark H. Tuszynski, M.D., Ph.D.
Center for Neural Repair
University of California, San Diego

Spinal Cord Injury
(Rat Model at 7T)





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University of California, San Diego

How much information can we extract?

How much information can we extract?

UFODIGEST

UFO AND PARANORMAL NEWS FROM AROUND THE WORLD

Giant Monolith Photographed On Mars

Submitted by Dirk Vander Ploeg on Mon, 04/16/2012 - 09:02

Dirk Vander Ploeg is the publisher of UFODigest.com and other paranormal and UFO related websites. He is the author of the non-fiction book "Quest for MIddle-earth" and is currently writing a new book. He has worked in marketing for the Toronto Star and the Hamilton Spectator and as a publisher and writer for travel related and other magazines.

By: Natalie Wolchover

Published: 04/11/2012 05:50 PM EDT on Lifes Little Mysteries

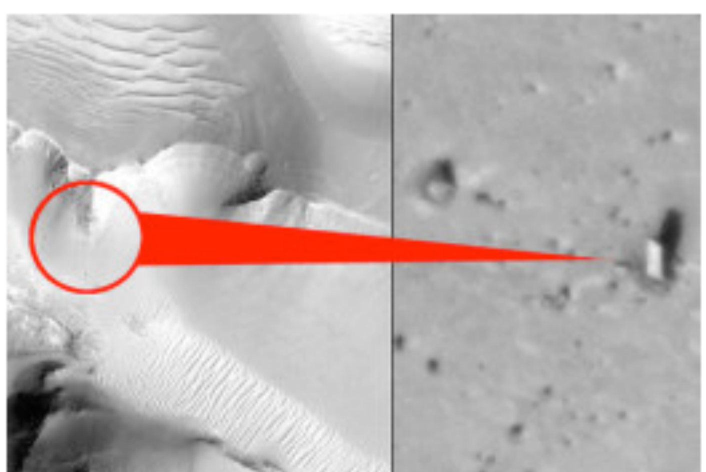
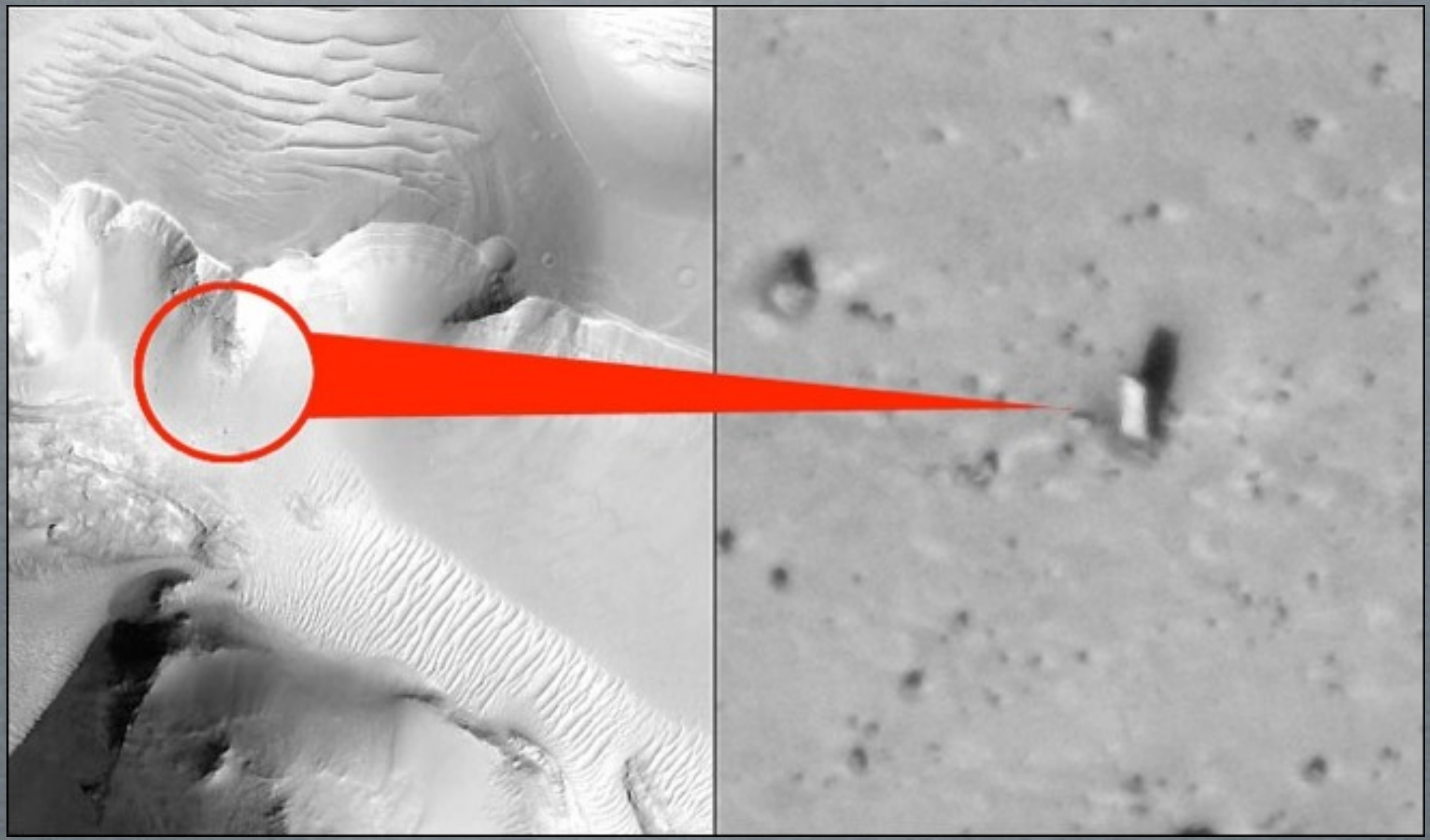


Image: NASA HiRISE. Arrow: thesun.co.uk

How much information can we extract?



How much information can we extract?

Science on msnbc.com

Mars 'monolith' isn't the work of Martians

Object in NASA images looks just like one in '2001: A Space Odyssey' — but it's just a rock

How much information can we extract?

Science on msnbc.com

Mars 'monolith' isn't the work of Martians

Object in NASA images looks just like one in '2001: A Space Odyssey' — but it's just a rock

According to Jonathon Hill of Arizona State University, the reason that the rock looks like an artificial construction is very simple: *lack of resolution in the image.*

What's the problem?

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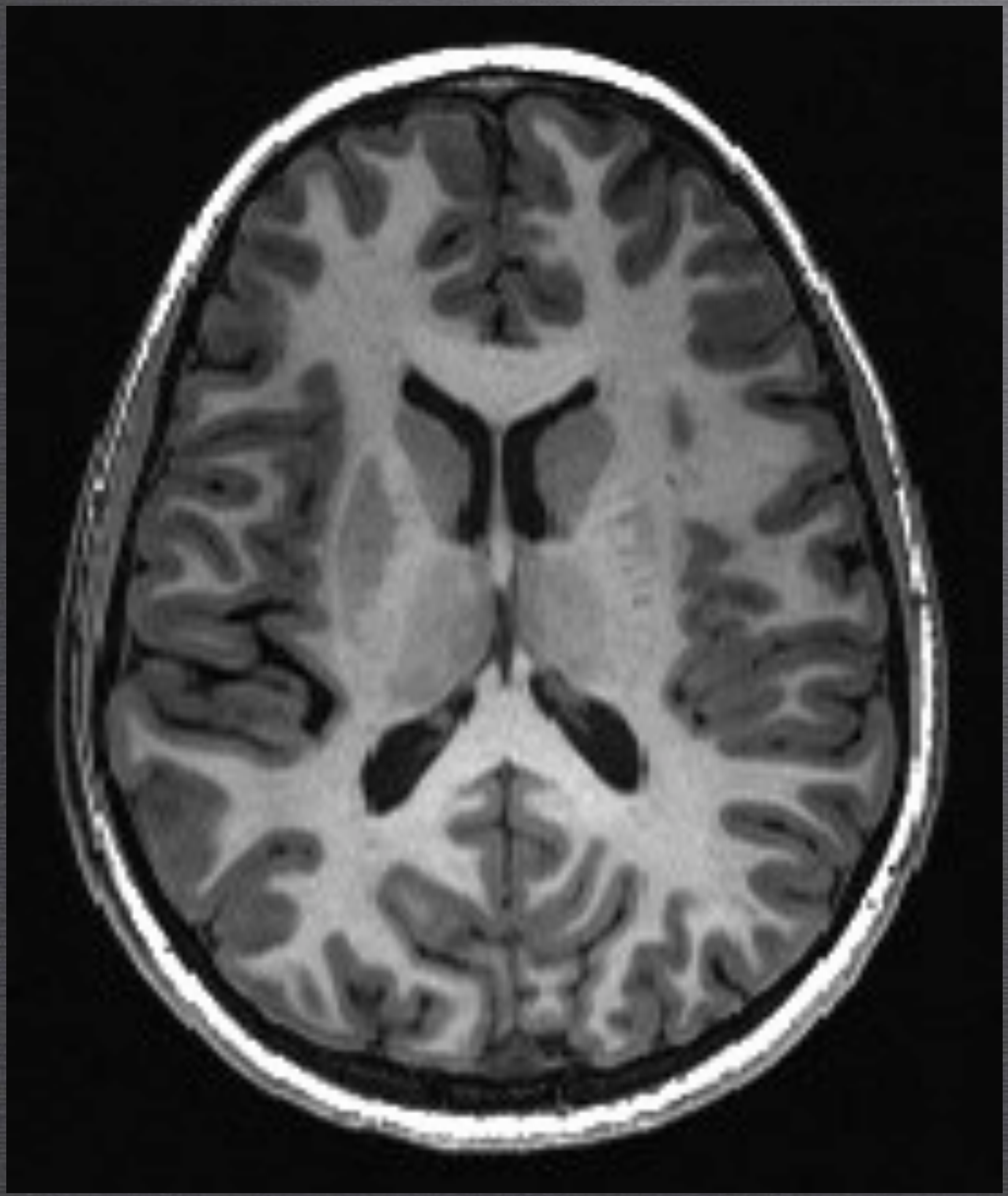
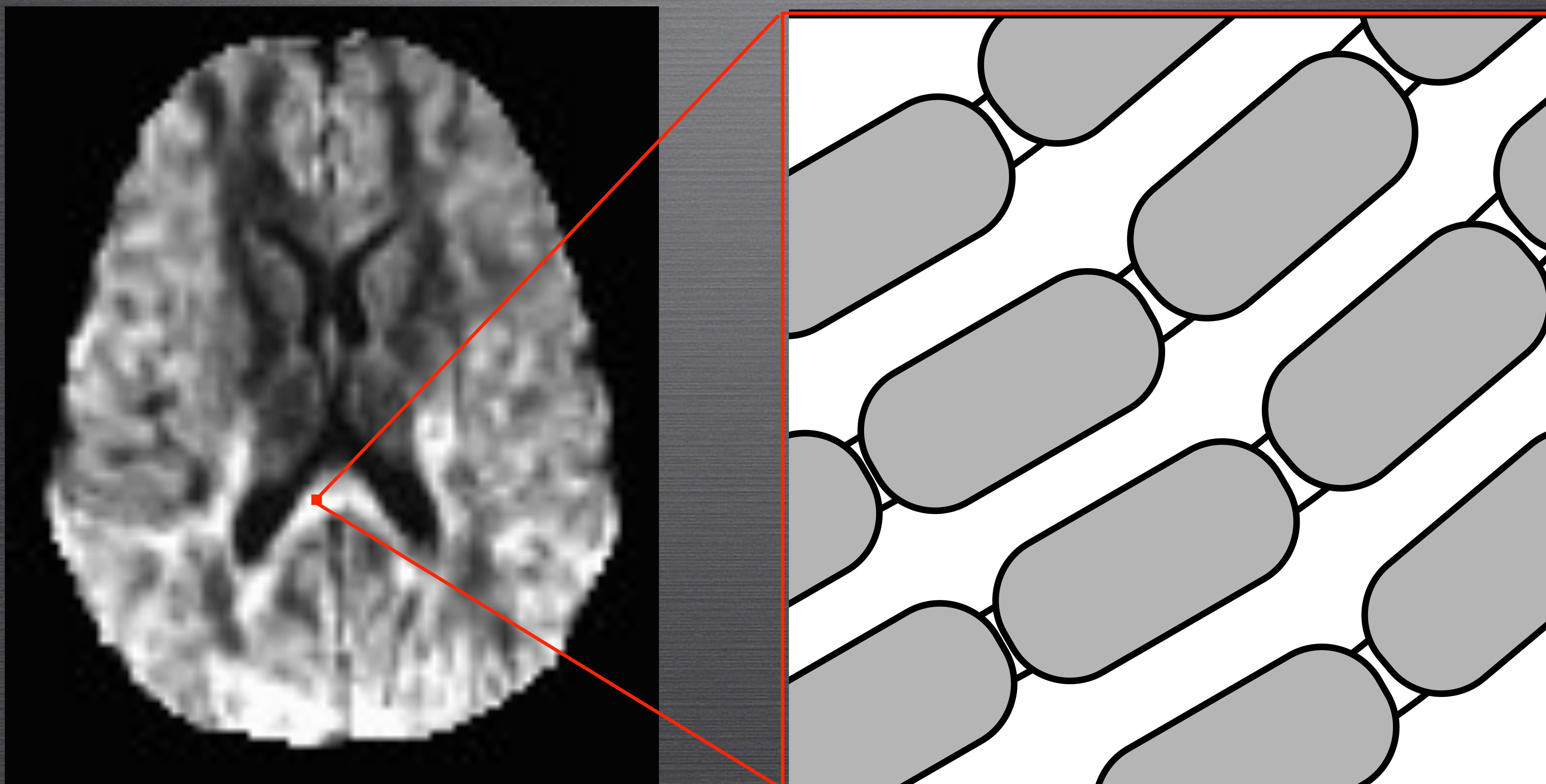
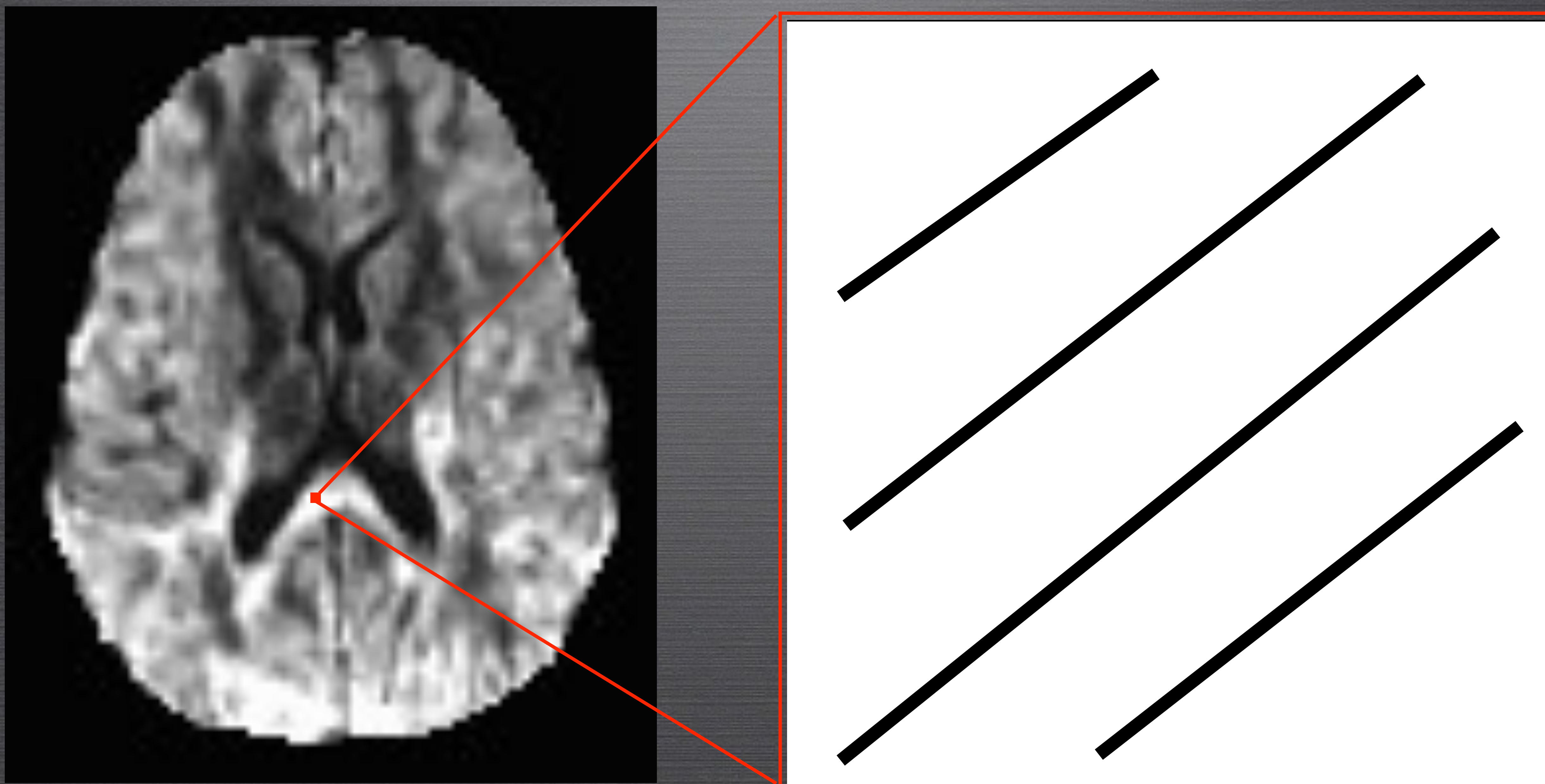


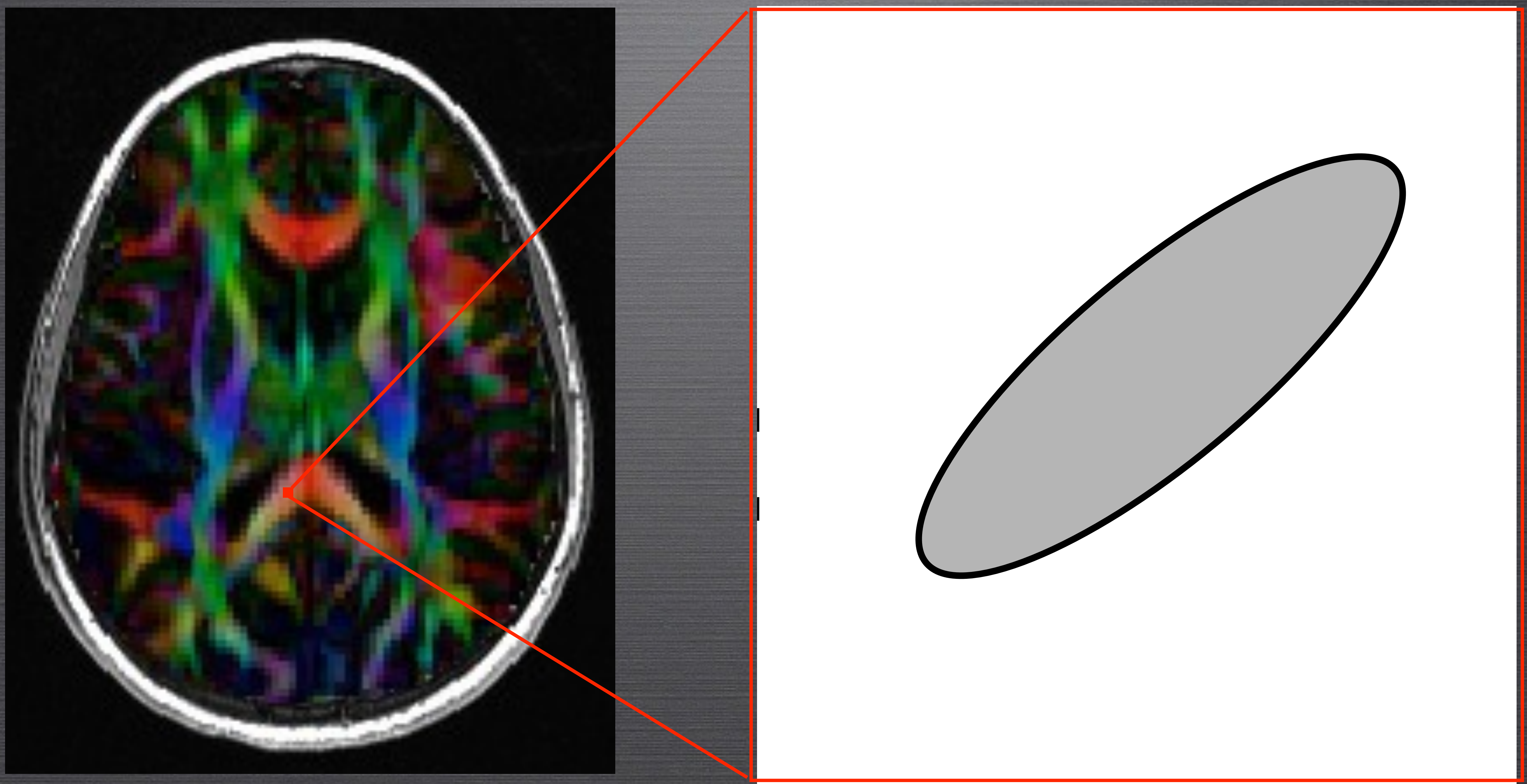
What's the problem?



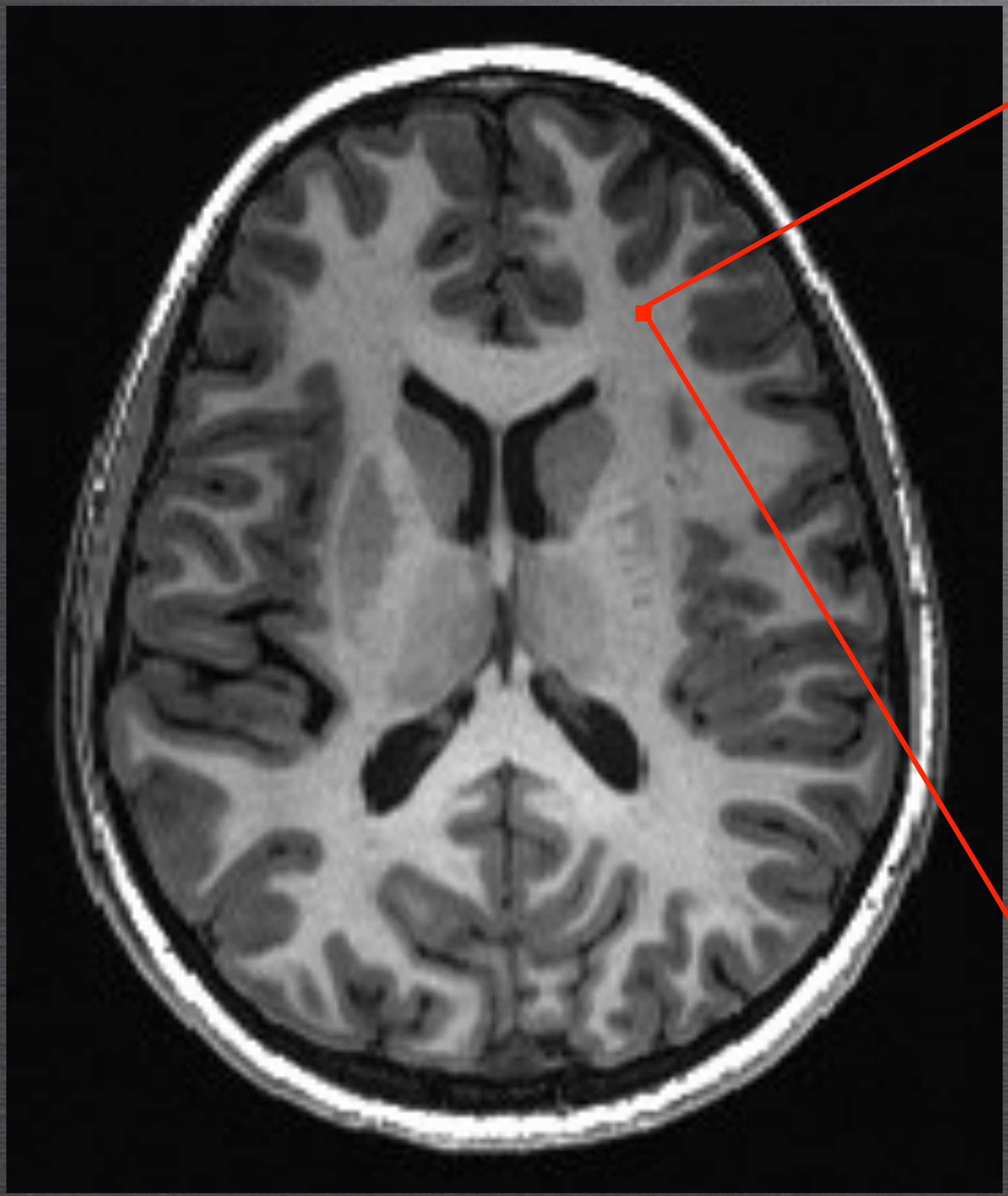
What's the problem?

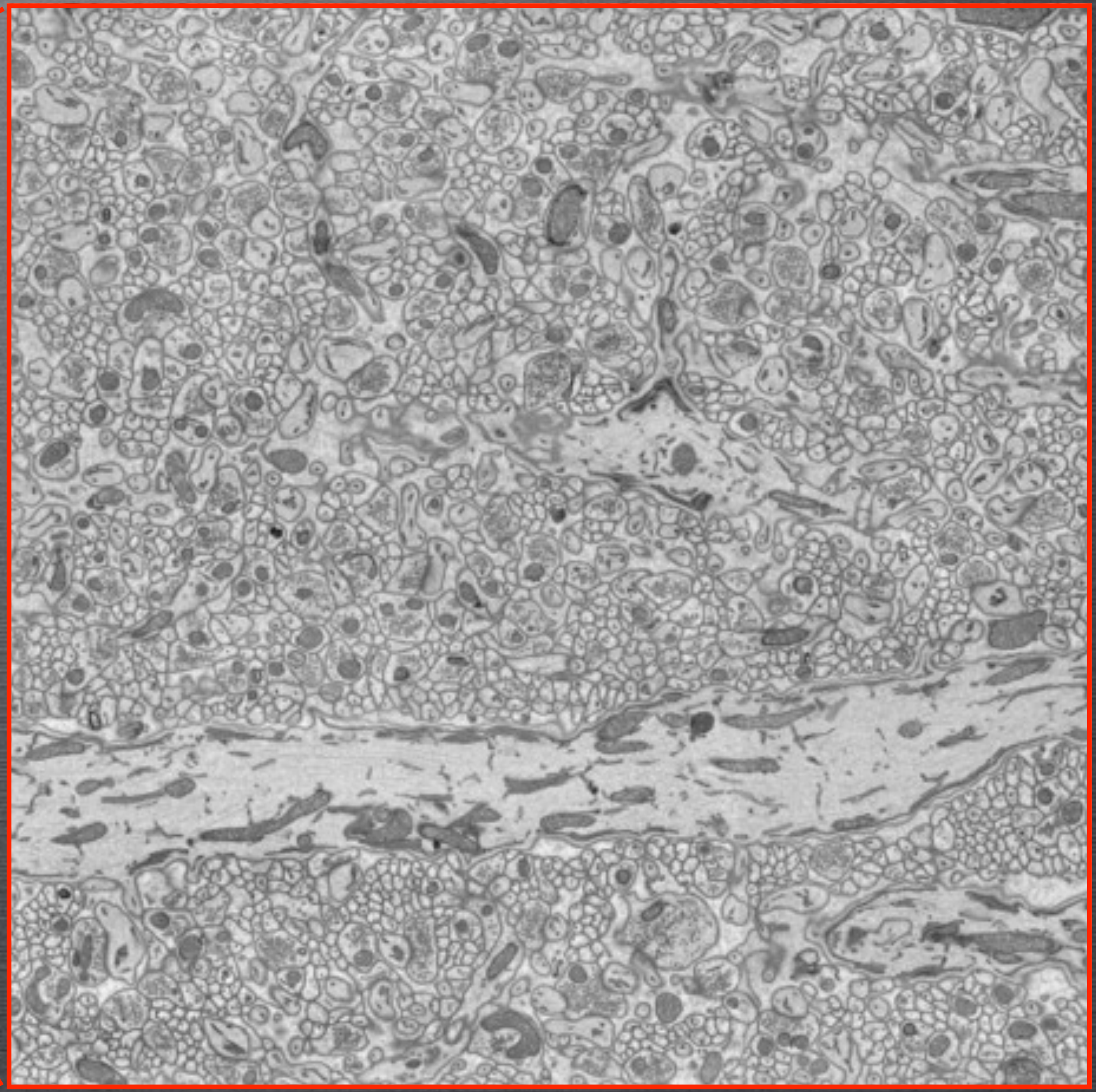


What's the problem?



But we know Neural Tissues aren't that simple



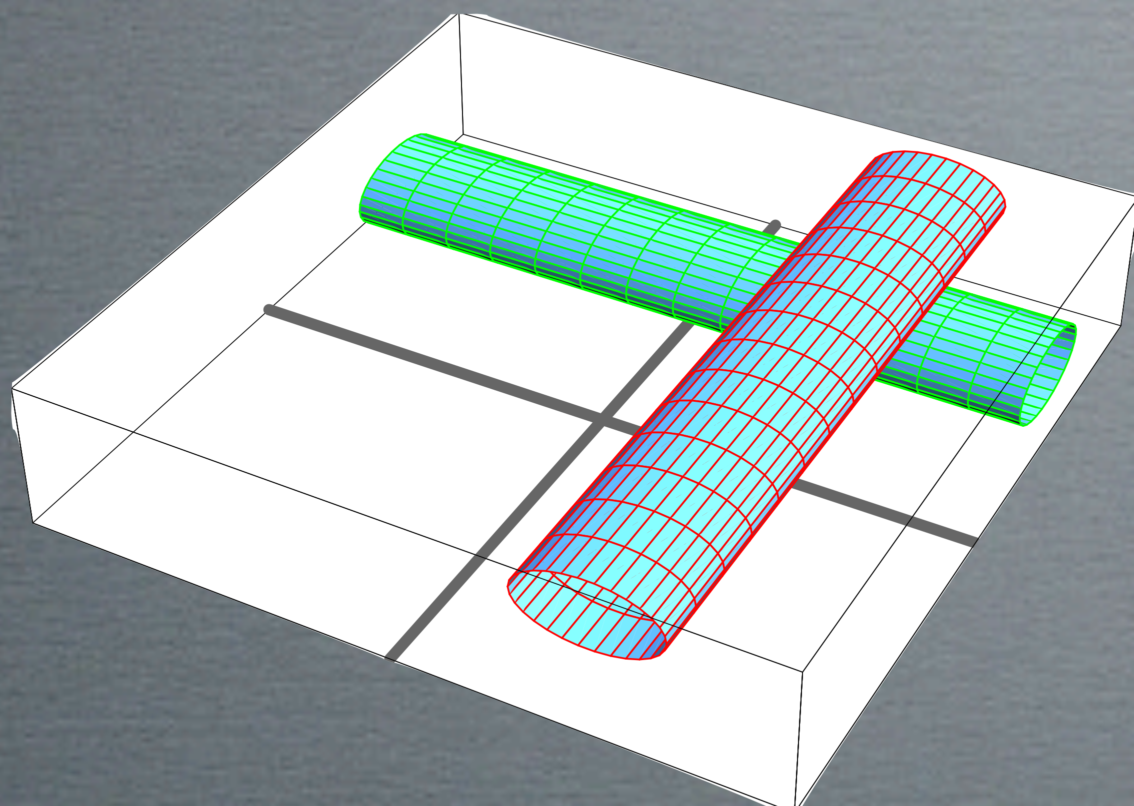


Rat WM electron microscopic image
Courtesy, M. Ellisman, UCSD

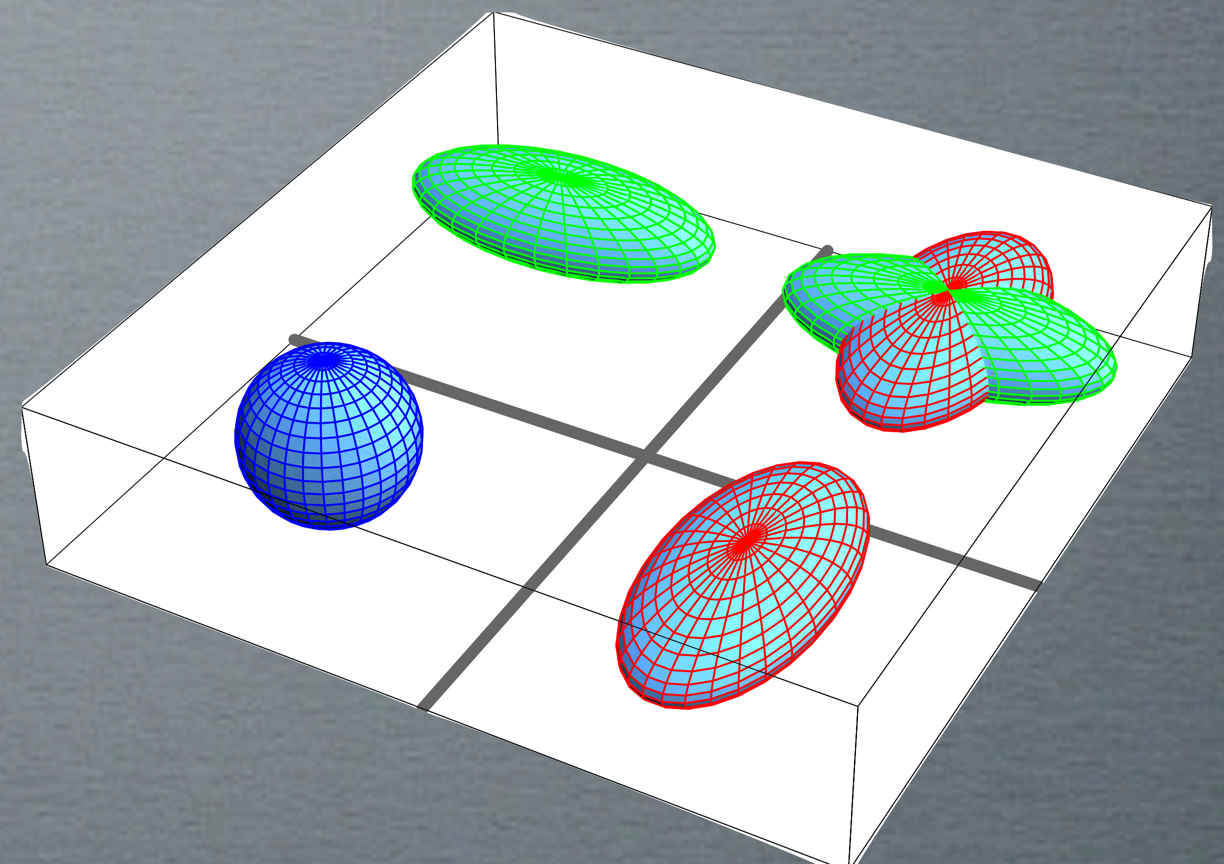
Failure of the standard model

Failure of the standard model

A simple partial-volume model



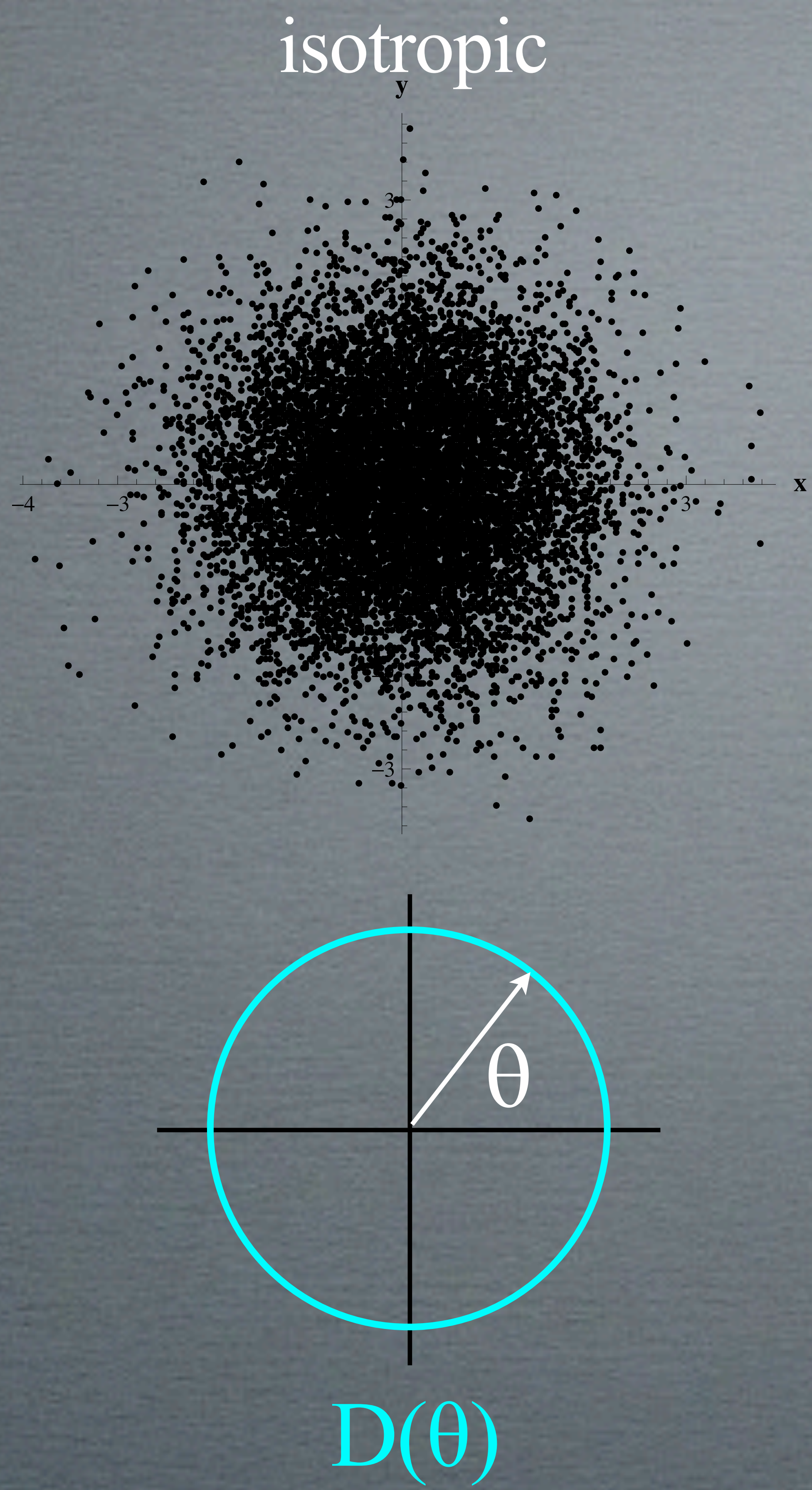
Two crossing fibers

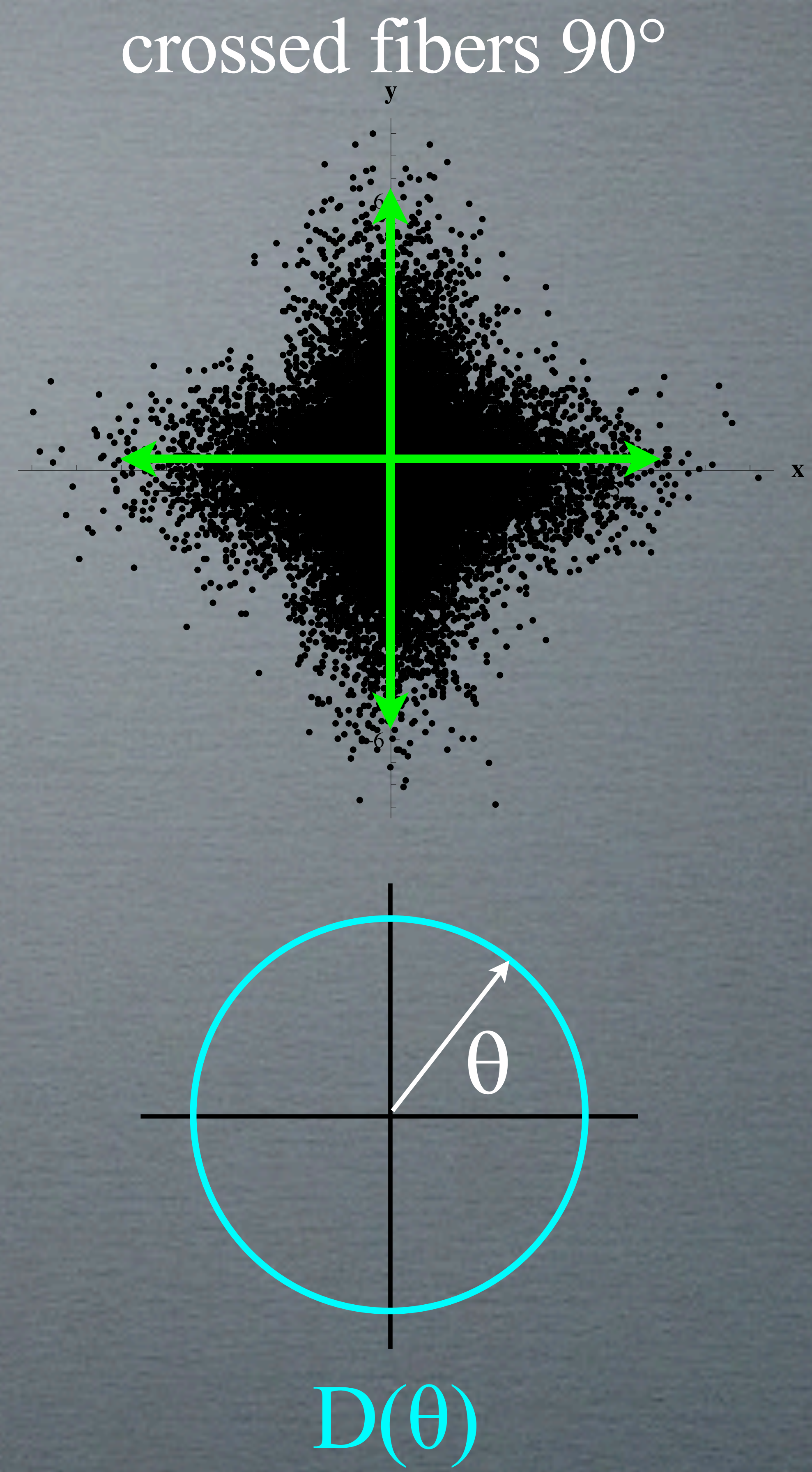


resulting distributions

Ambiguities in the standard model

Ambiguities in the standard model

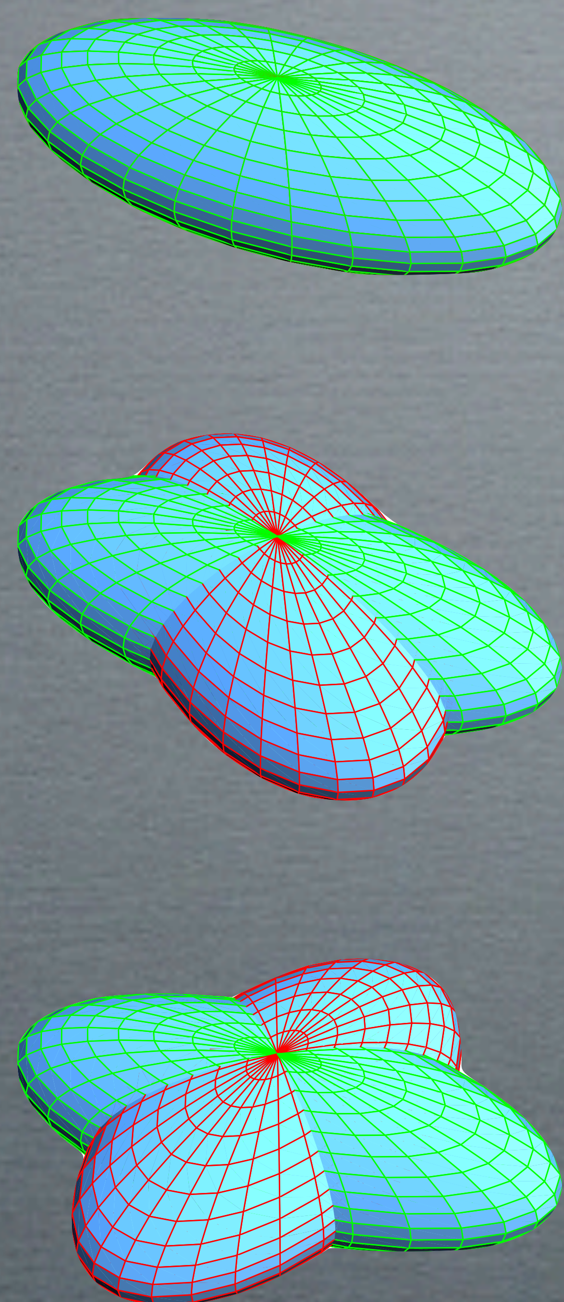




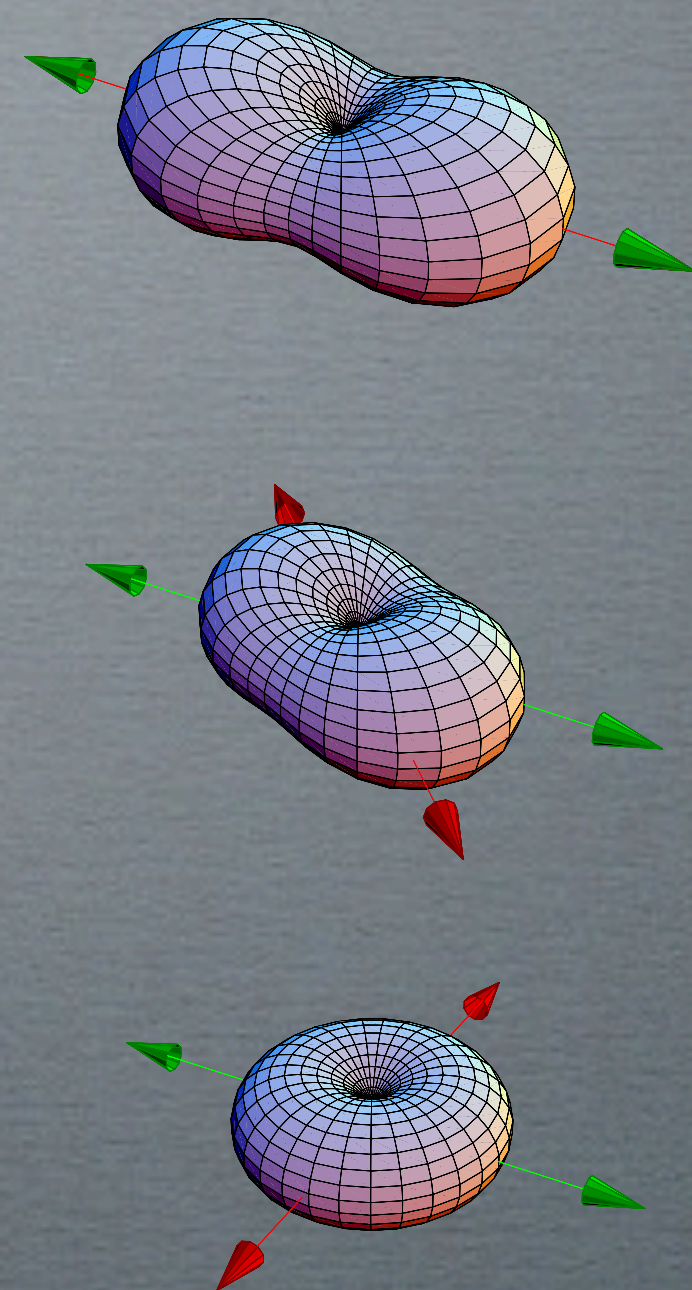
Failure of the standard model

Failure of the standard model

Distribution of spins



Estimated D



The Major Problem: Heterogeneous Voxels

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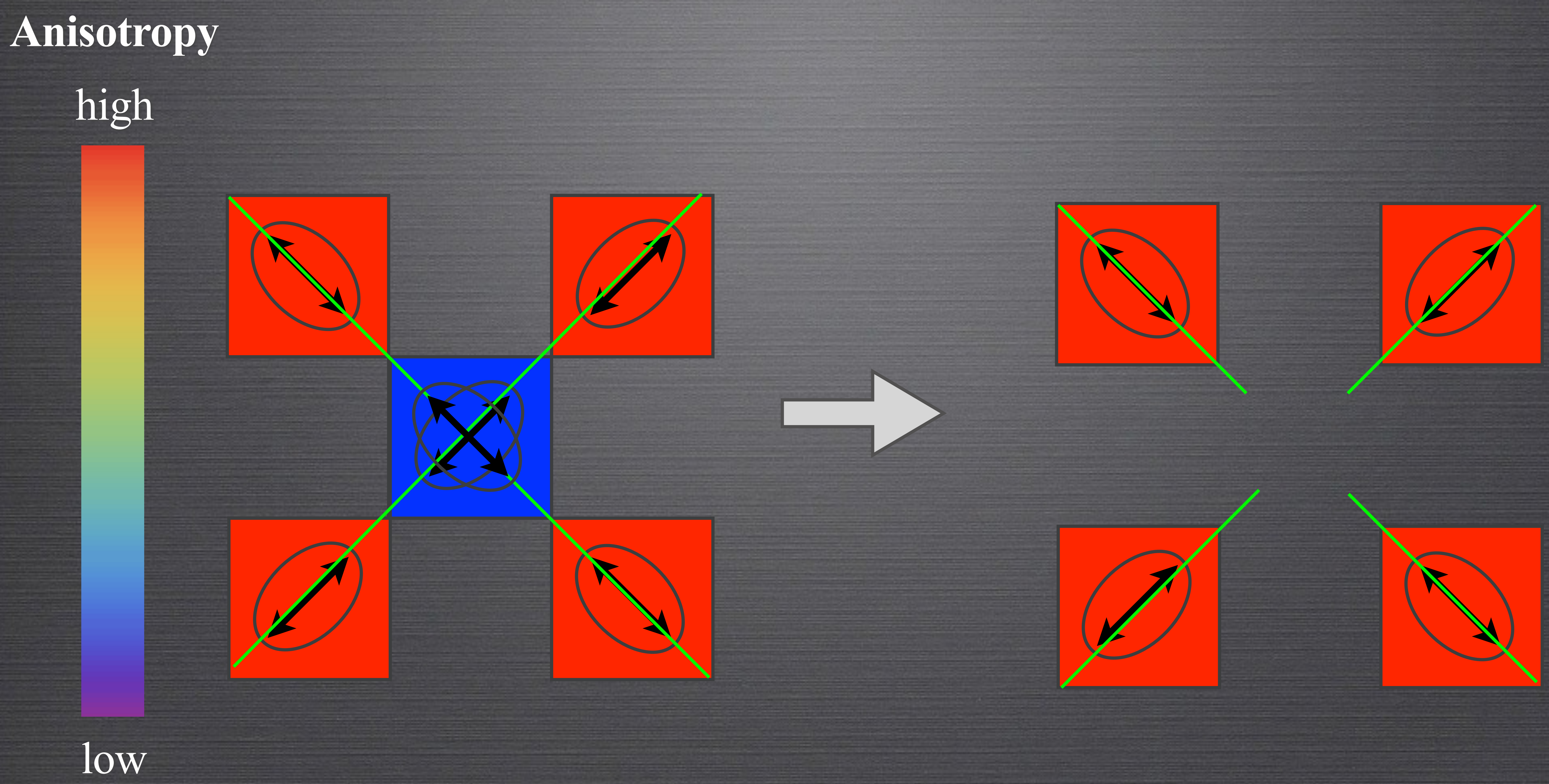
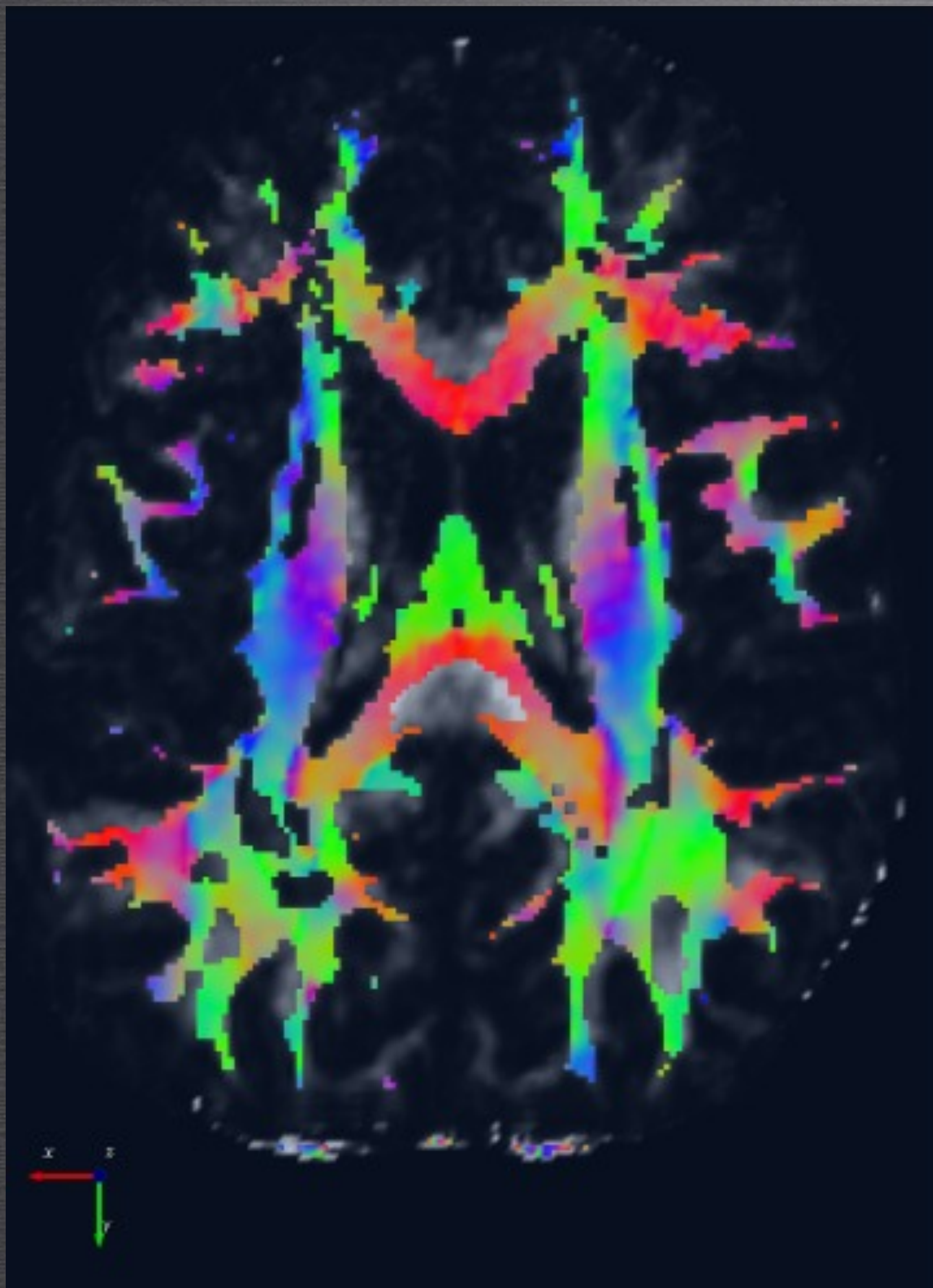


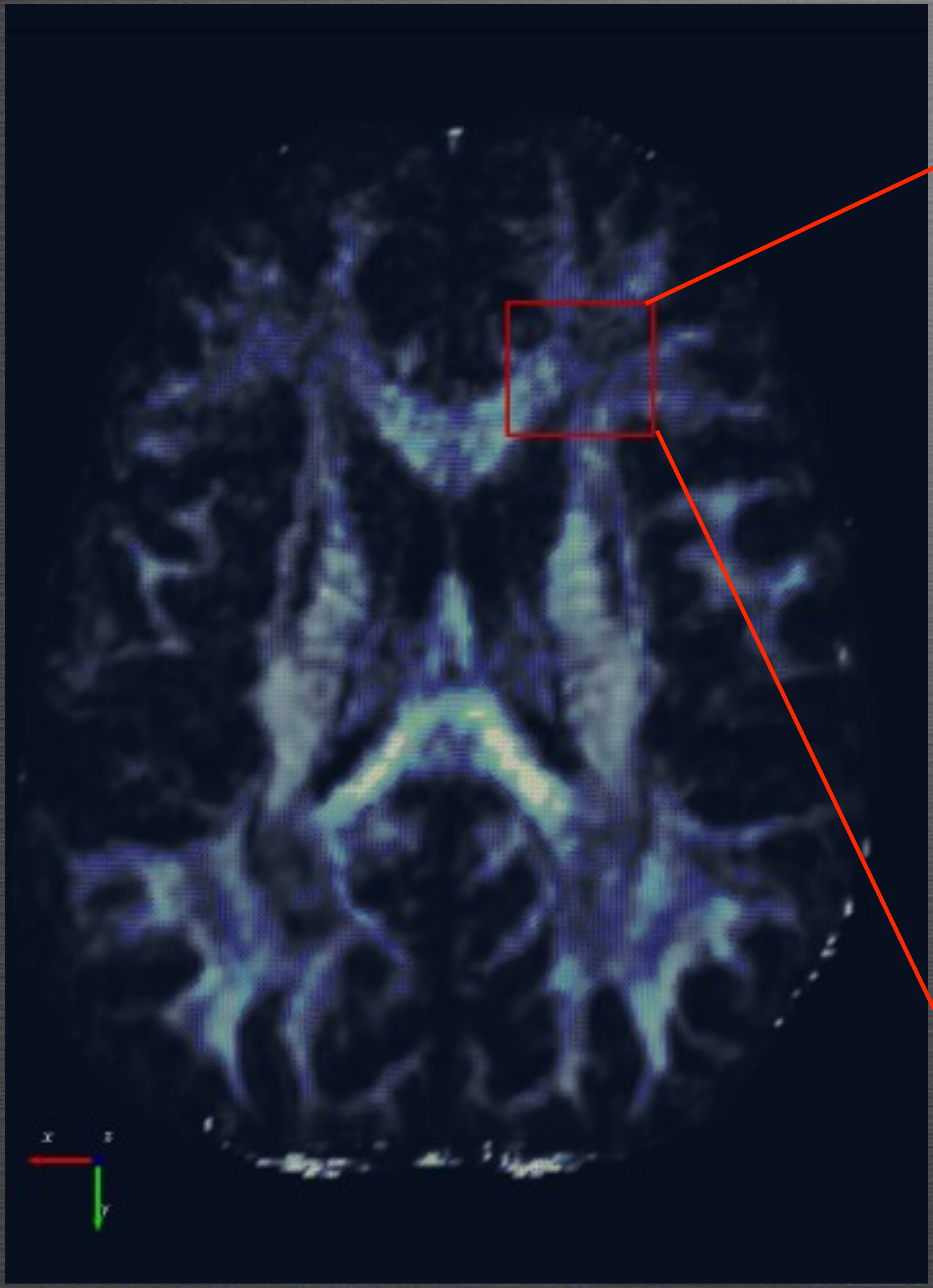
Tractography Problem

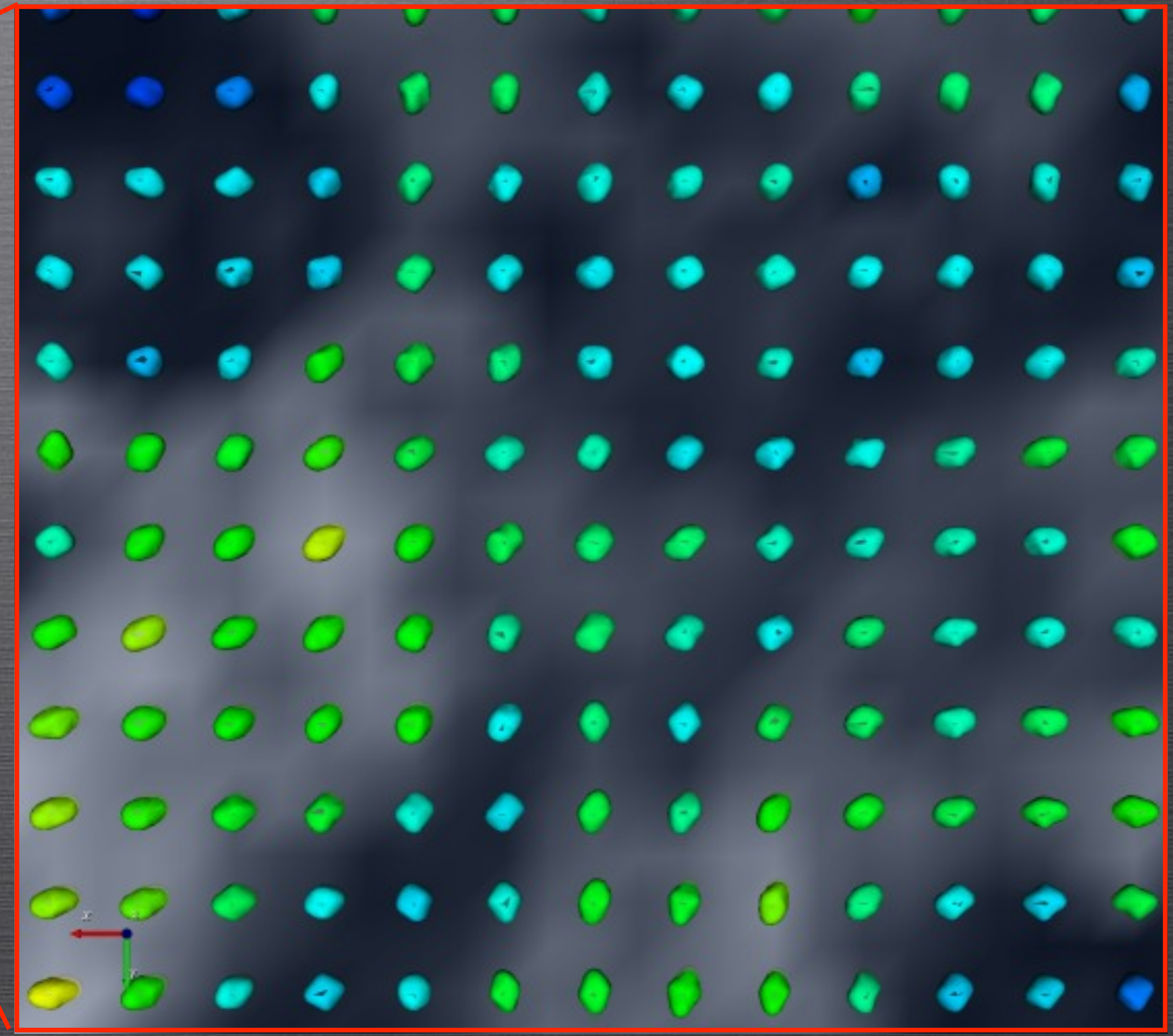
Tractography Problem



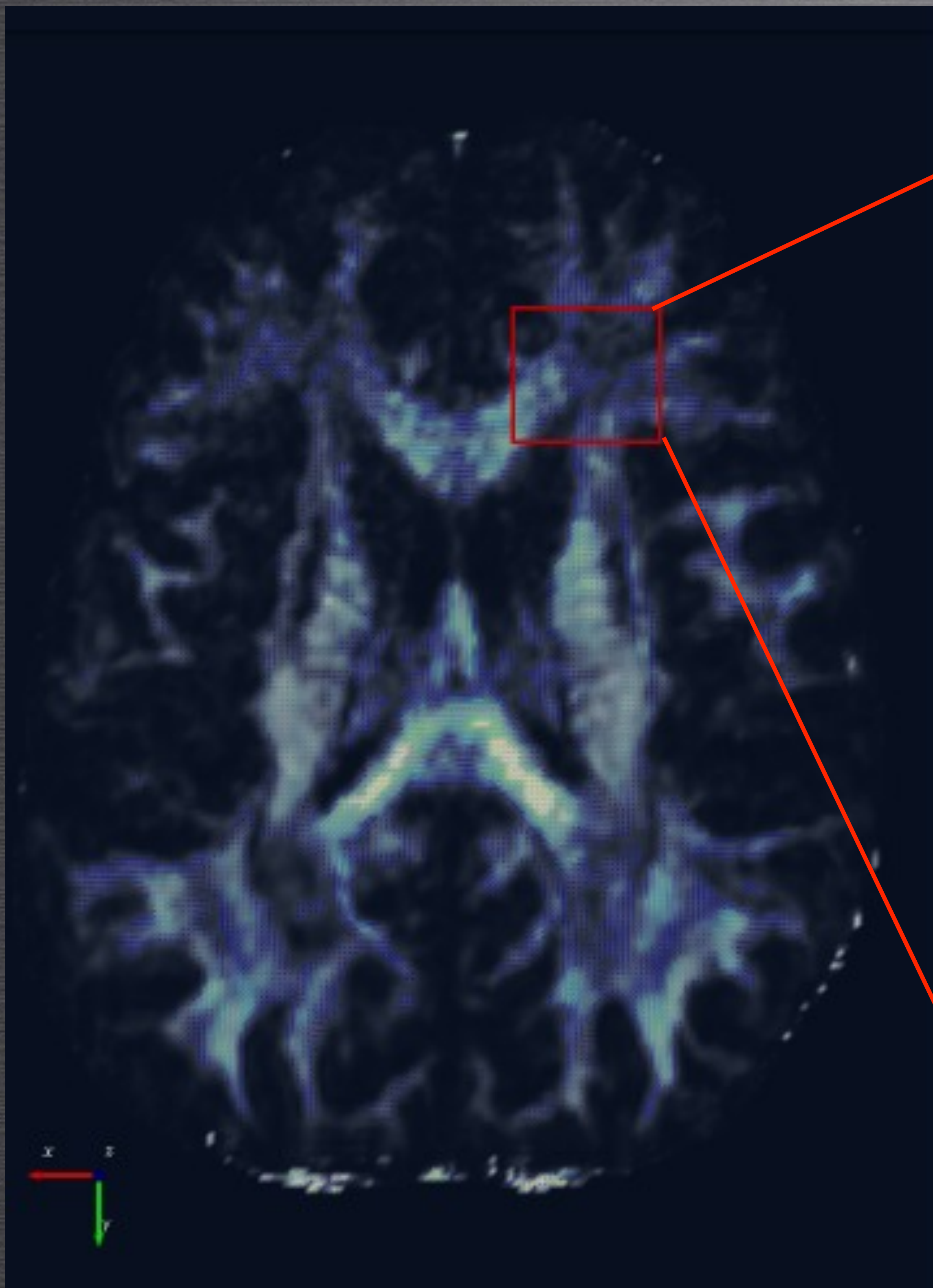
Monday, November 25, 13

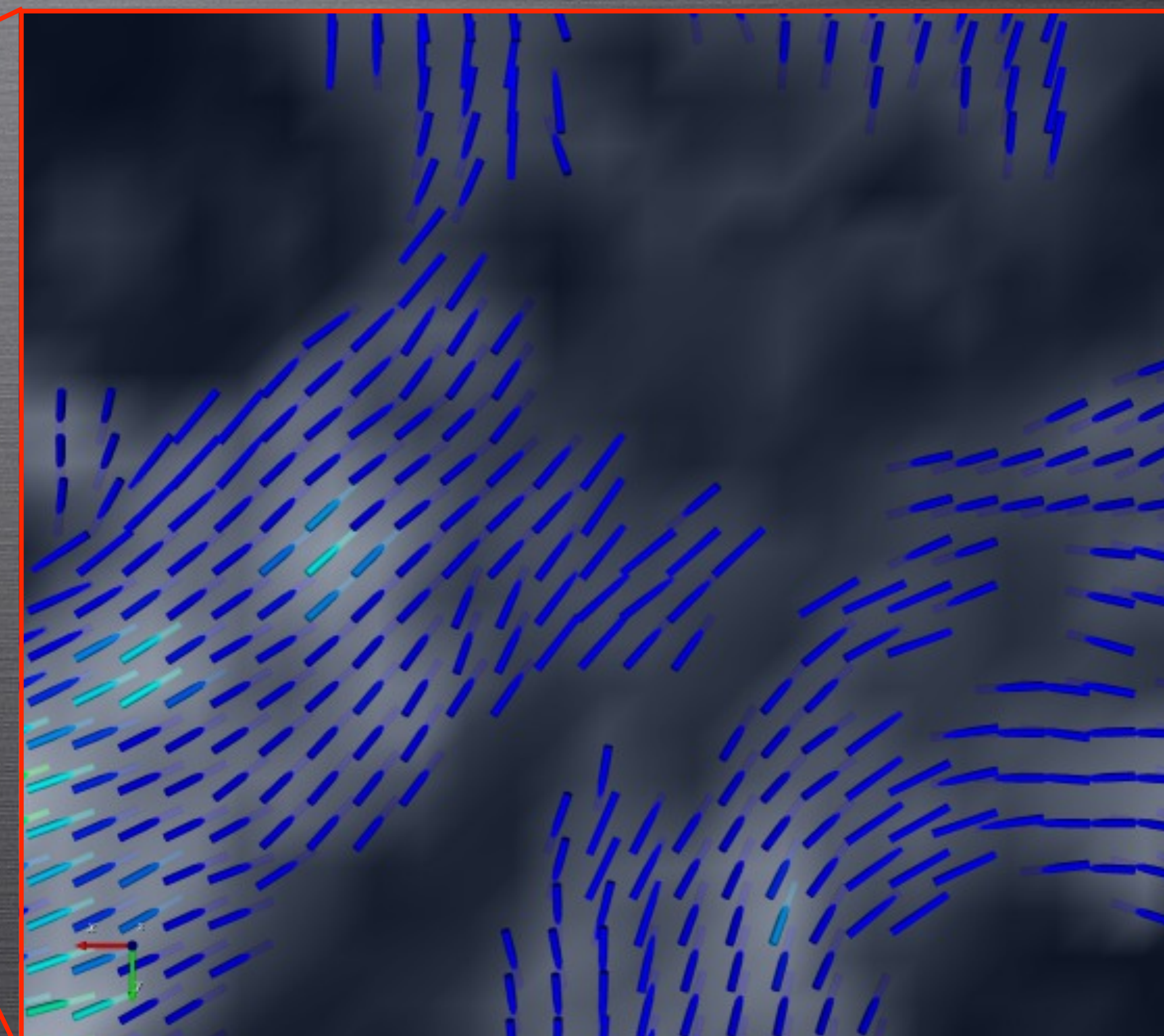
Tractography Problem





Tractography Problem

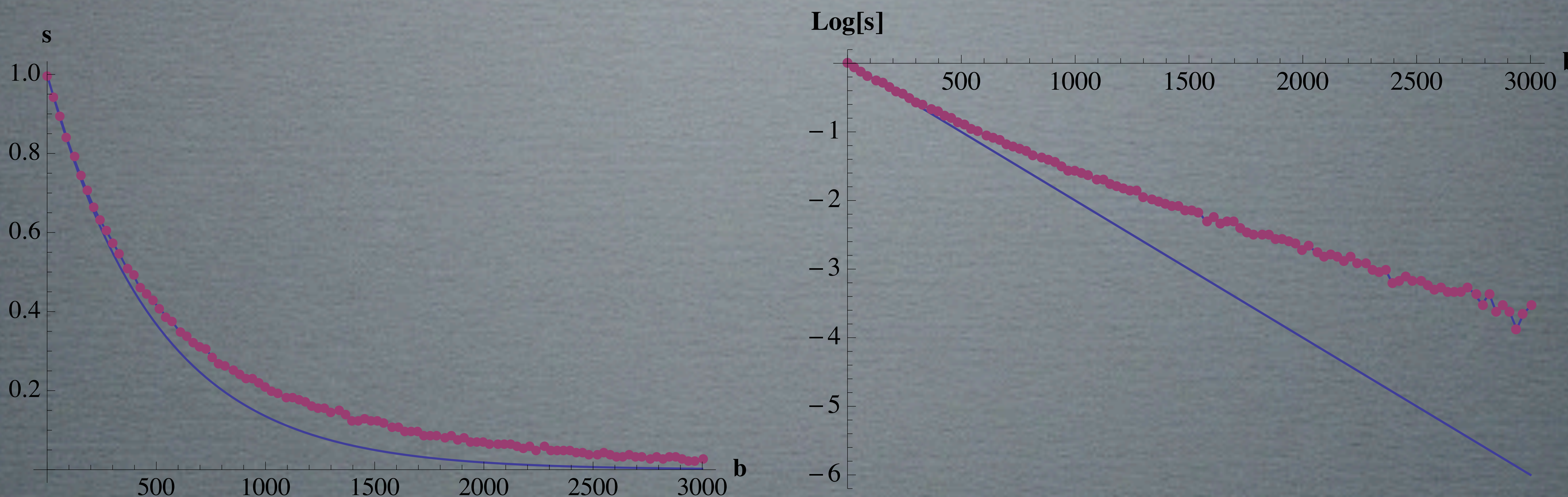




Failure of the standard model

Failure of the standard model

Not only angular issues, but b-value dependencies as well!

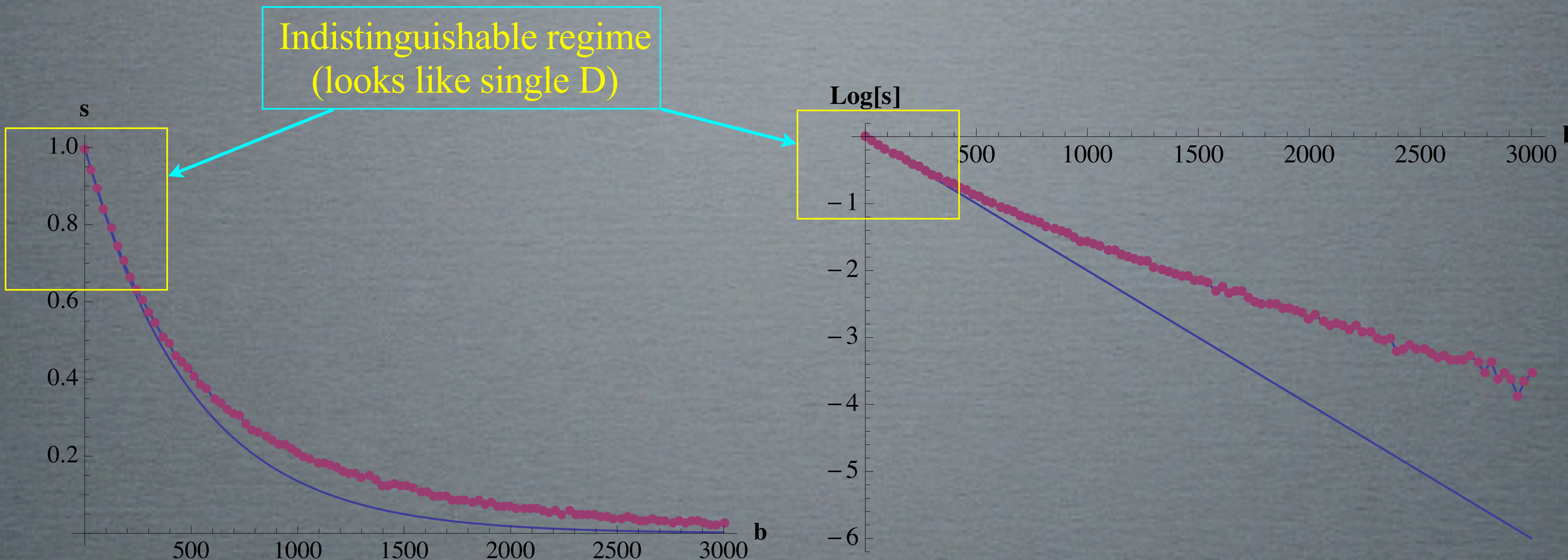


$$\frac{S(b)}{S(0)} = fe^{-bD_1} + (1-f)e^{-bD_2}$$

simple two diffusion coefficient model

Failure of the standard model

Not only angular issues, but b-value dependencies as well!



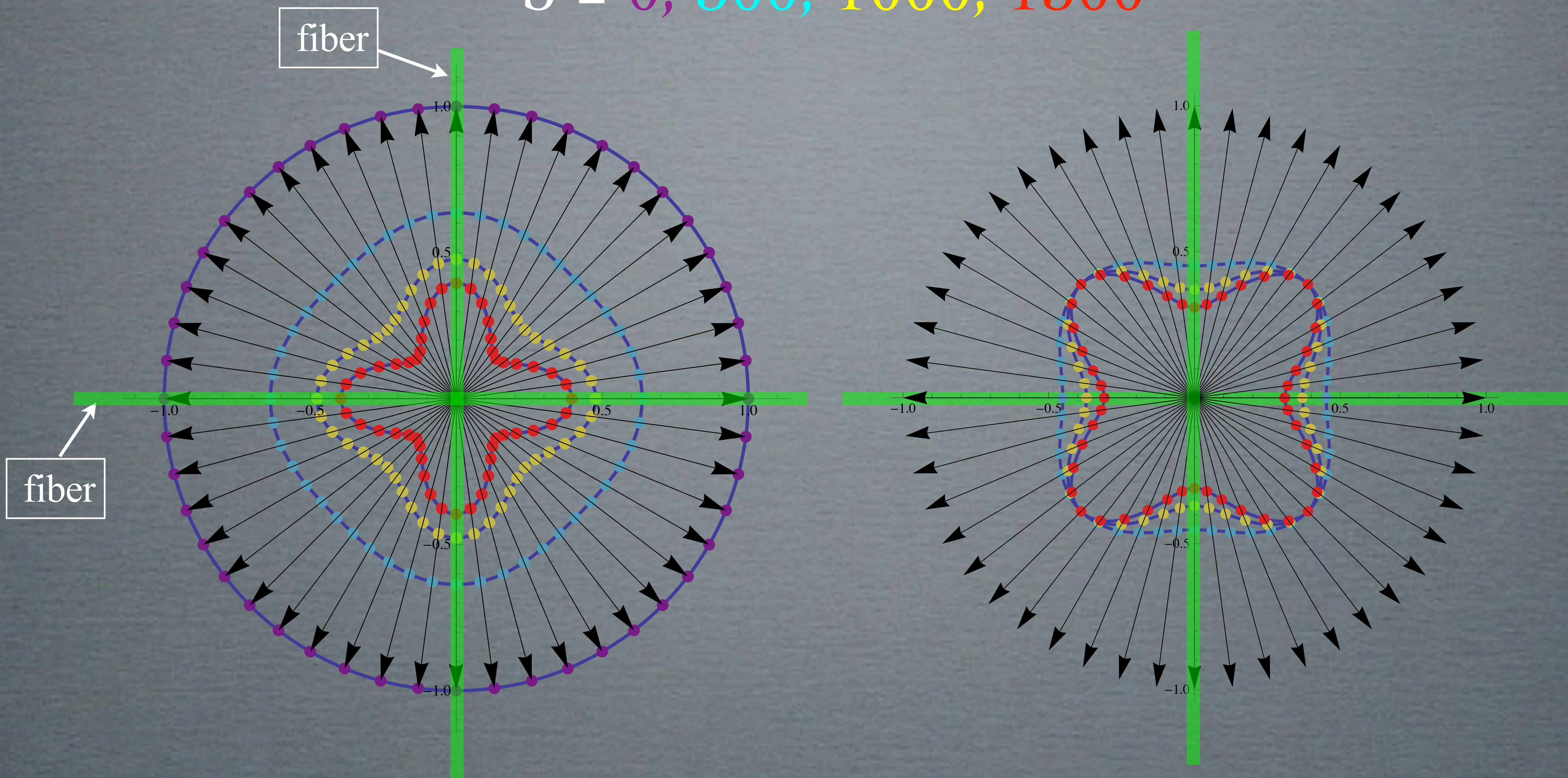
$$\frac{S(b)}{S(0)} = f e^{-bD_1} + (1-f)e^{-bD_2}$$

simple two diffusion coefficient model

High Angular Resolution DTI (HARDI)

High Angular Resolution DTI (HARDI)

b = 0, 500, 1000, 1500

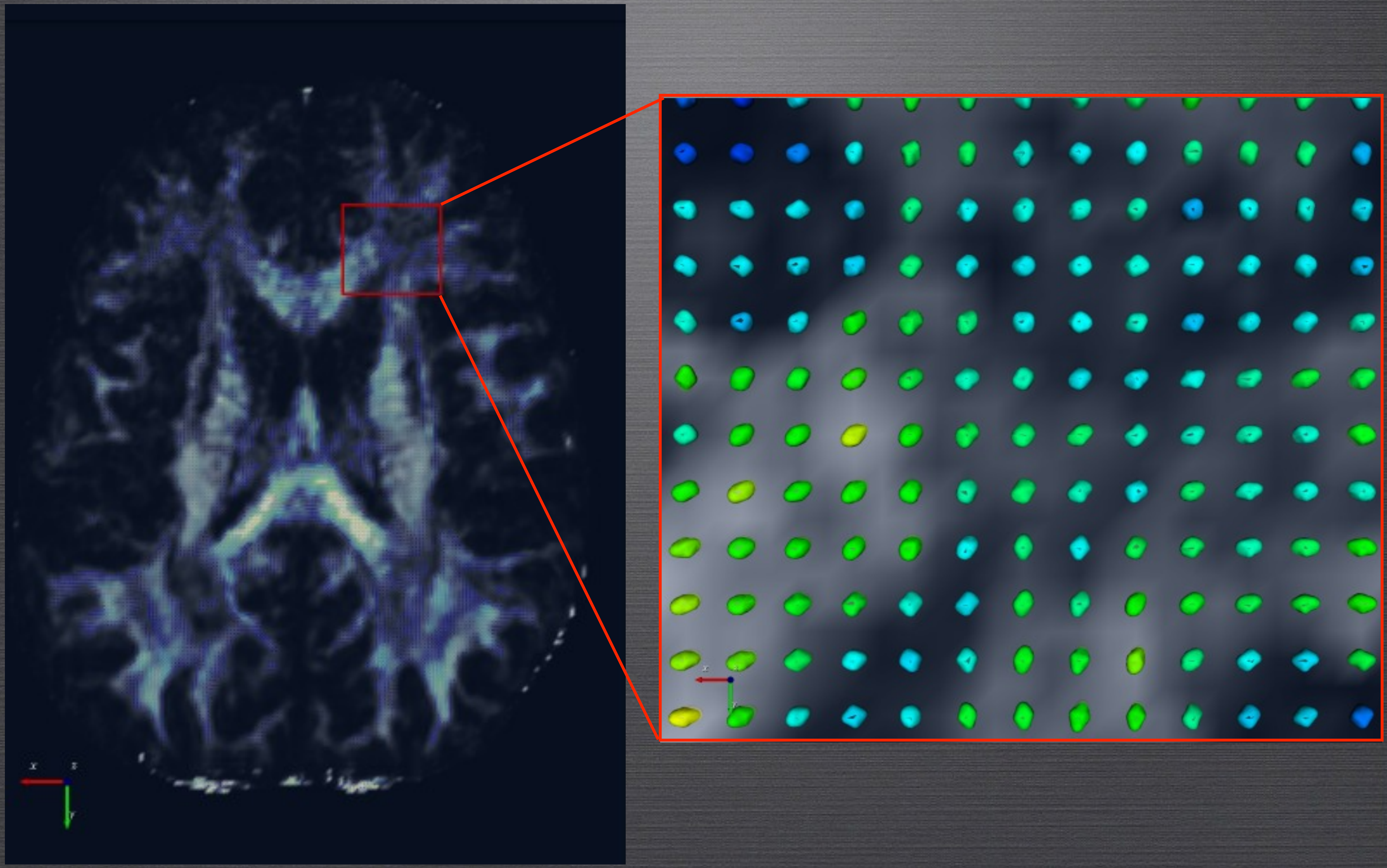


signal

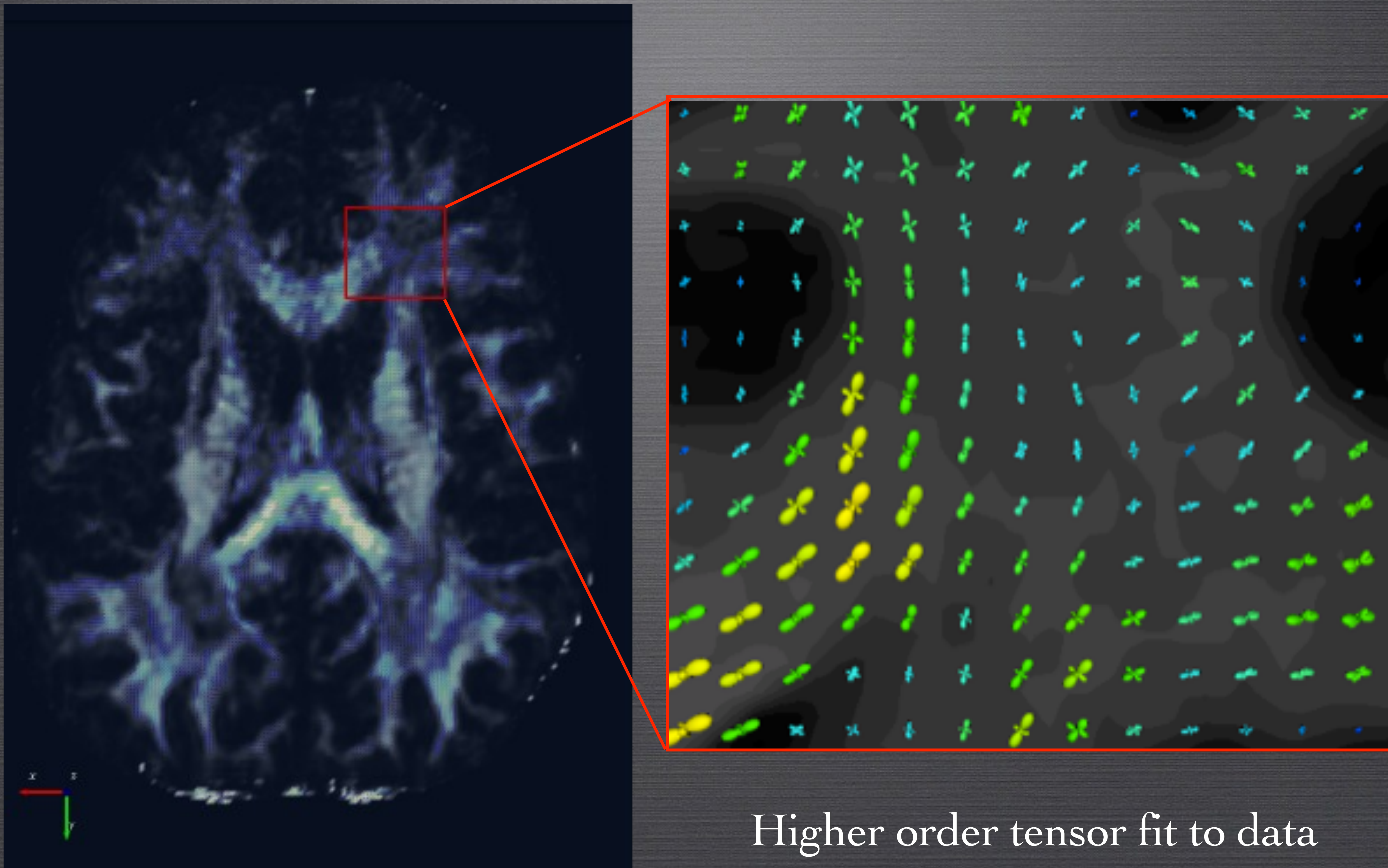
$D_{app}(\theta)$

Structure of lobes relative to fiber orientation is “non-intuitive”!

Tractography Problem, revisited

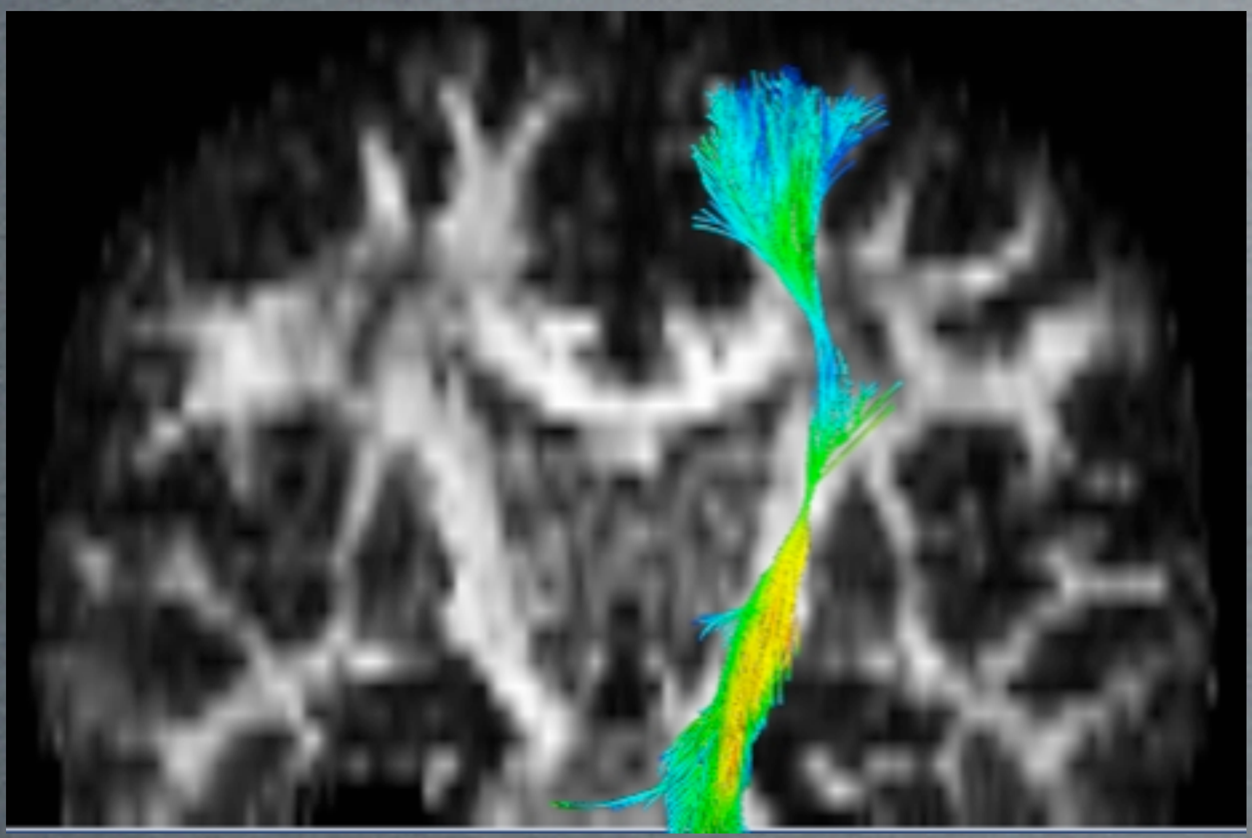


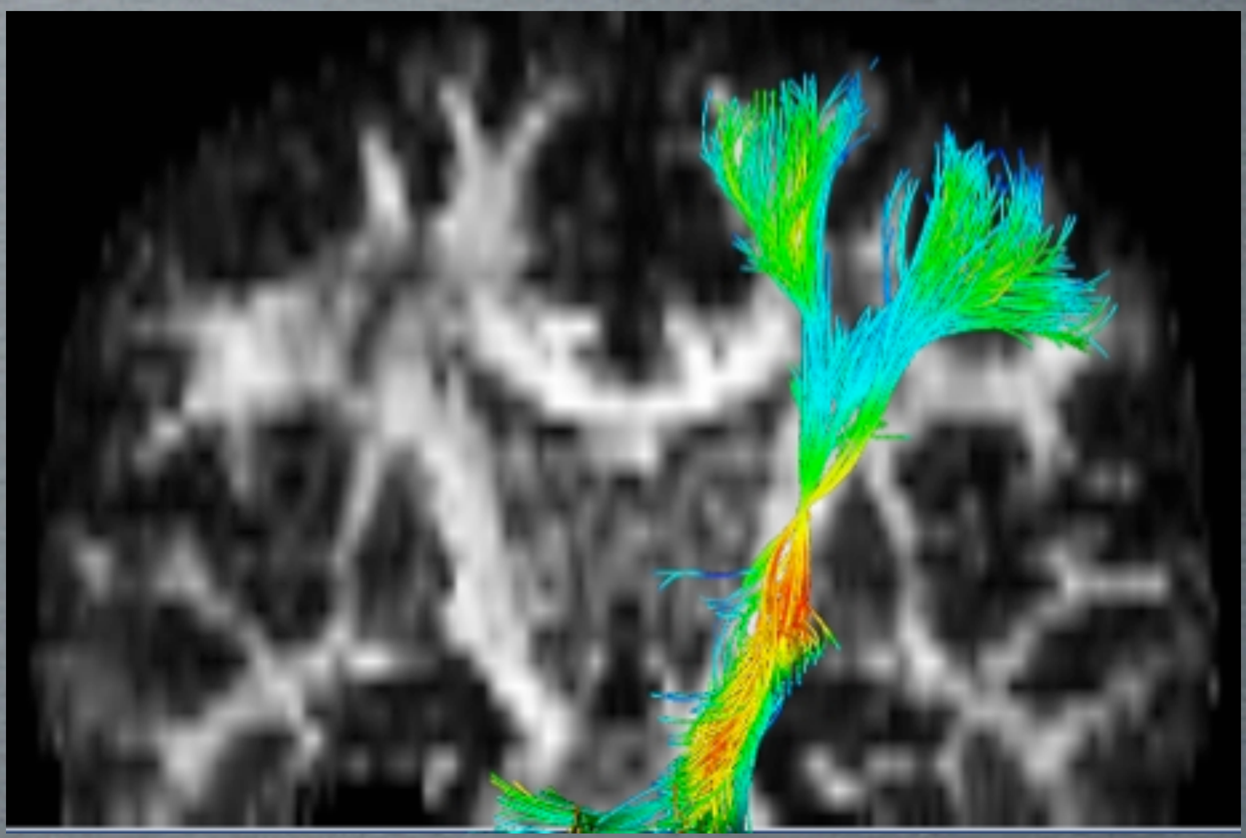
Tractography Problem, revisited



Higher order tensor fit to data

High Angular Resolution DTI (HARDI)

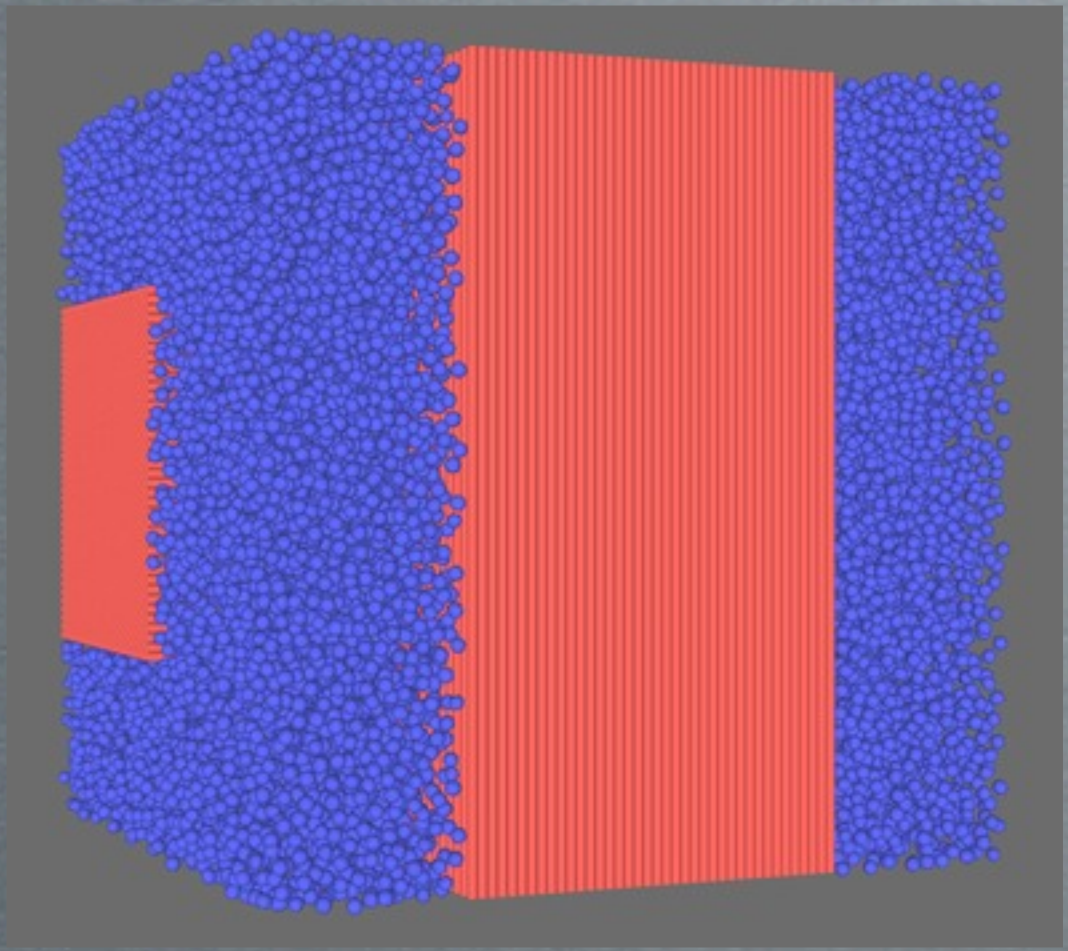




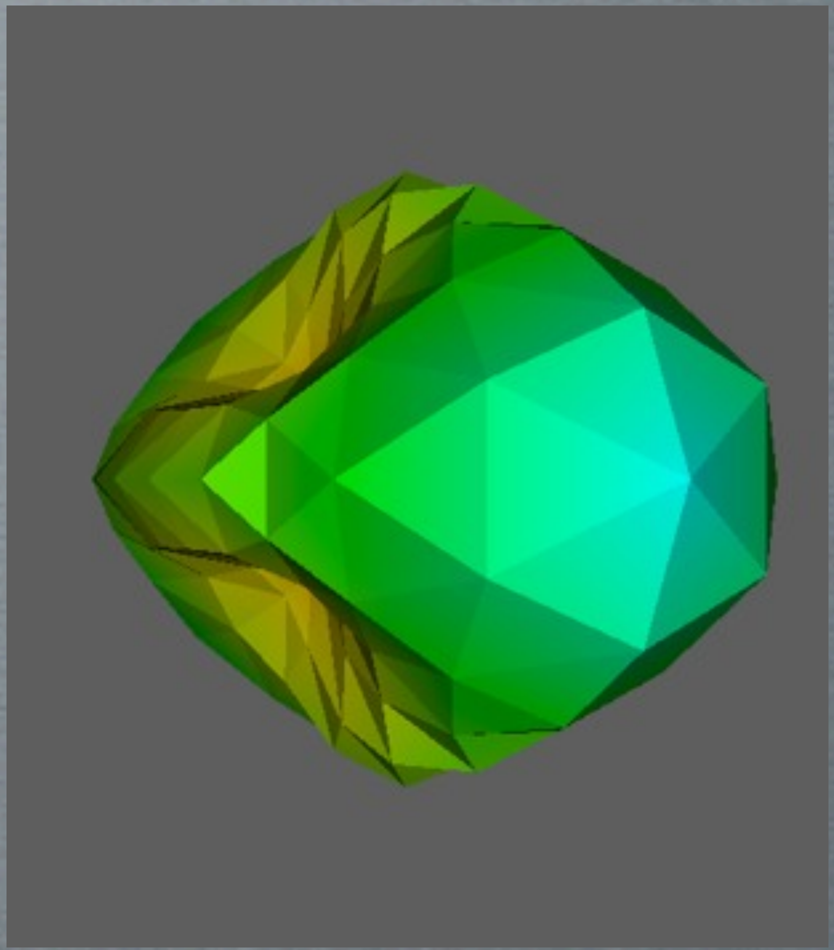
Standard DTI HARDI

Heterogeneous Voxels and High Angular Resolution Sampling

Heterogeneous Voxels and High Angular Resolution Sampling



a voxel with crossing fiber bundles and random spherical cells...



signal from 162 directions

Conclusion

Diffusion MRI has a unique sensitivity
to tissue architecture and physiology

...and diffusion sensitivity is relatively easy
to incorporate into standard sequences

However ...

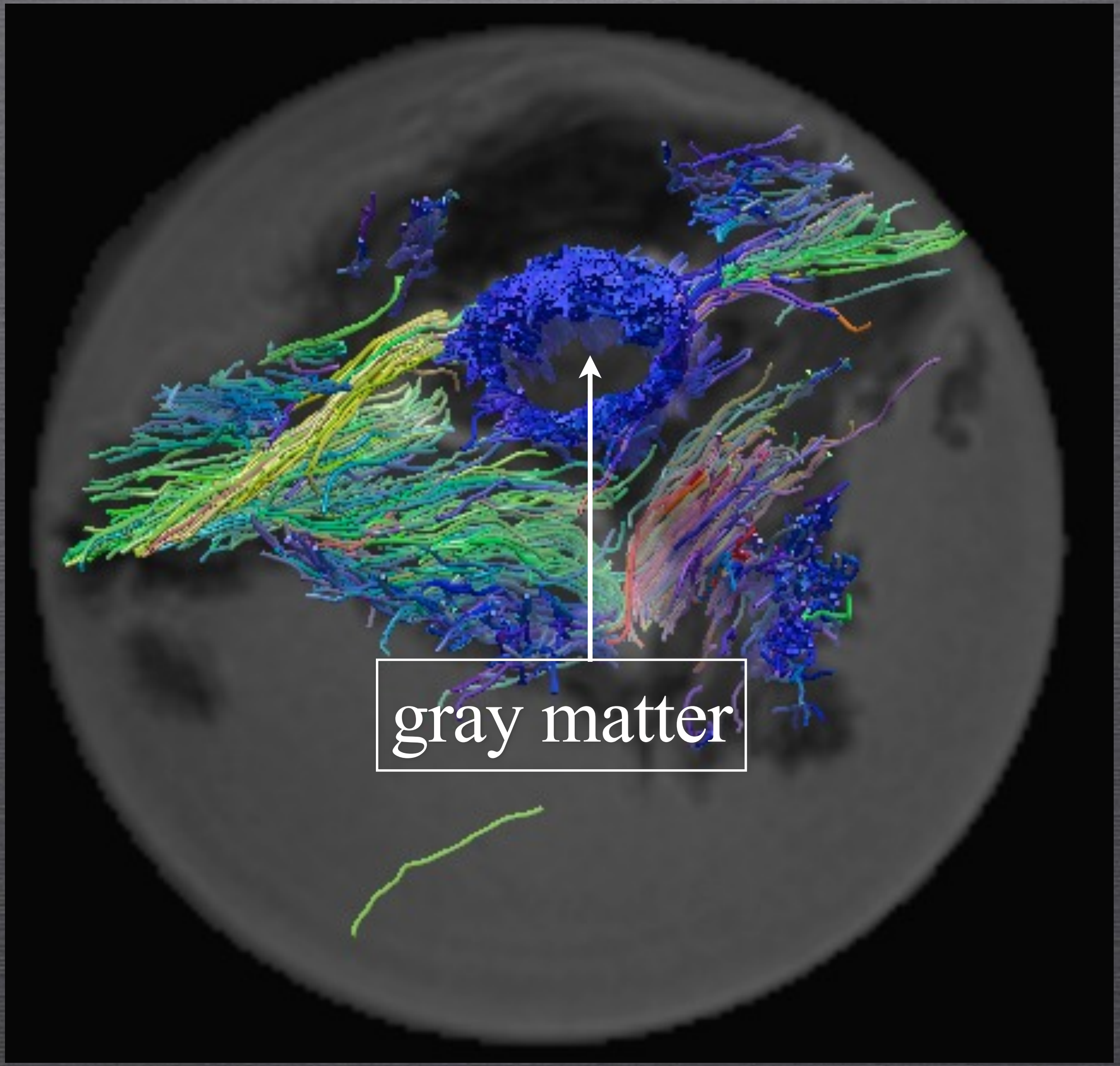
- Data artifact correction non-trivial
- Analysis is complicated
- Interpretation is difficult

But it's really cool!

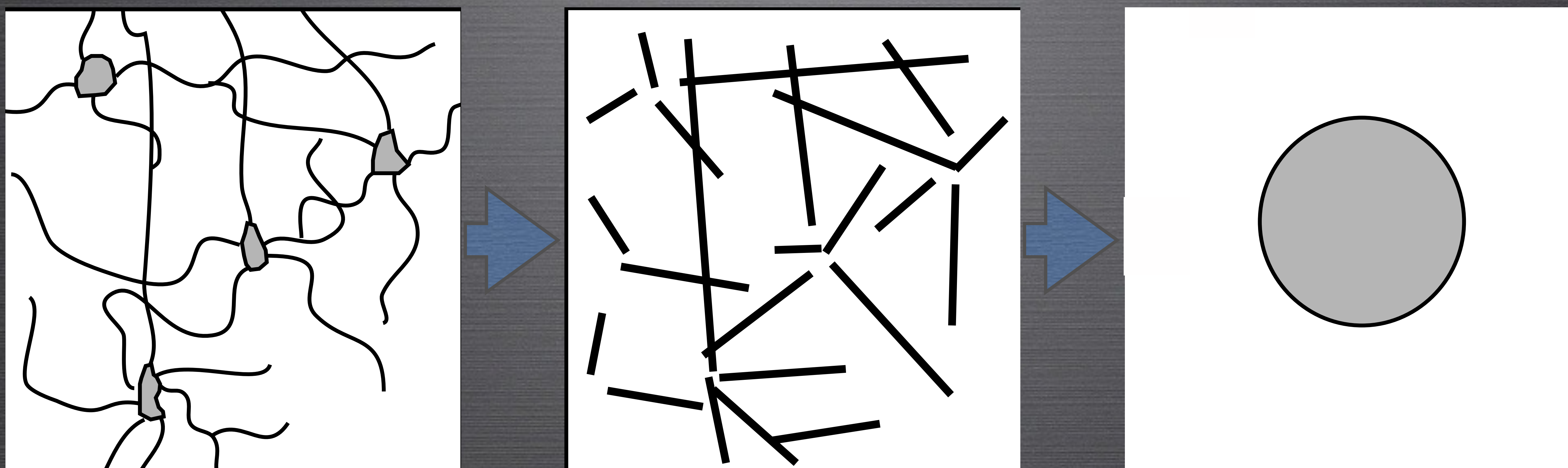
BREAK

Fundamental Limitation of DTI
Heterogeneous Voxels

Fundamental Limitation of DTI
Heterogeneous Voxels



Fundamental Limitation of DTI
Heterogeneous Voxels



Gray matter

microscopically anisotropic but macroscopically (voxel) isotropic

Recap

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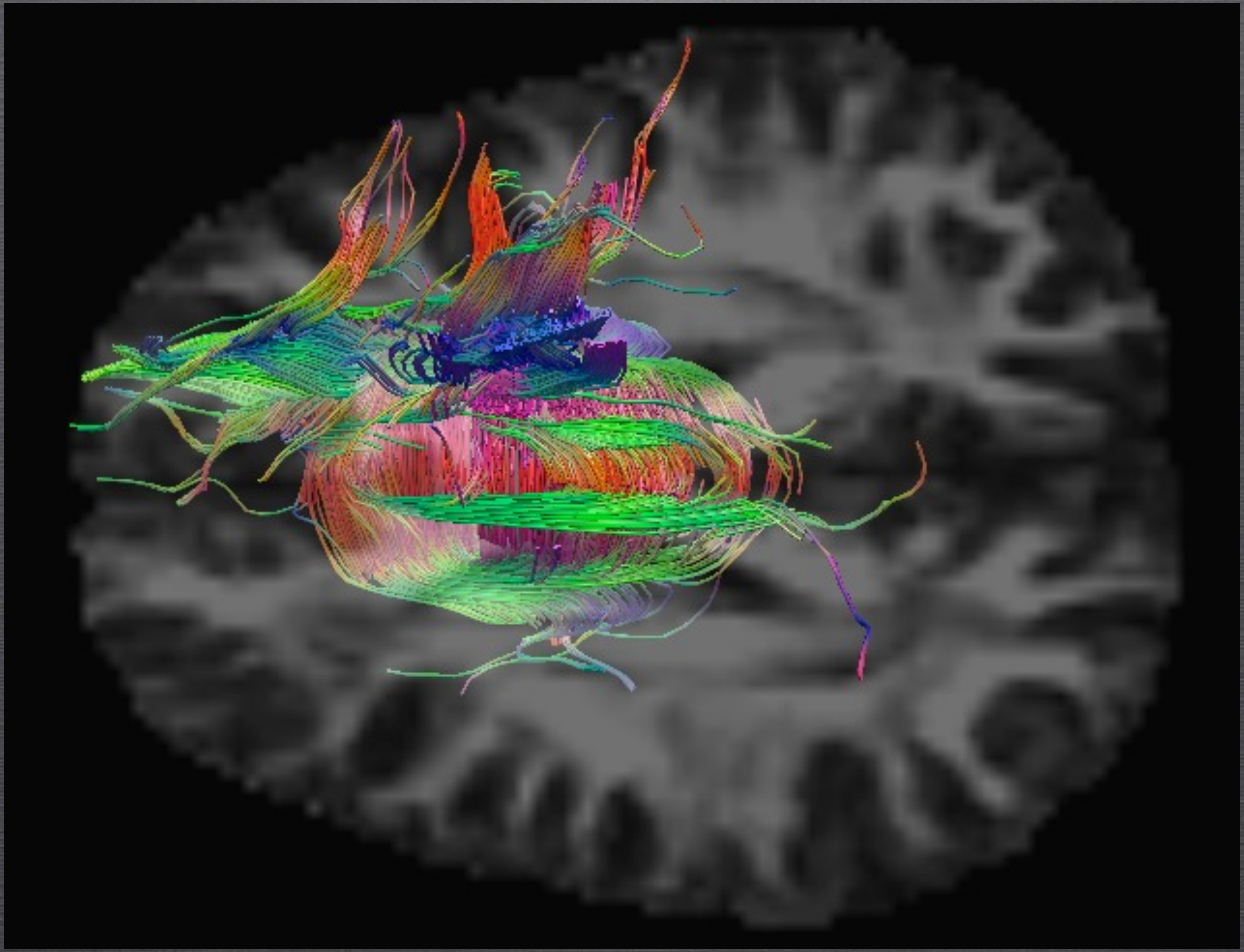
Streamlines

Data courtesy Drs S. Tapert and J. Jacobus, UCSD

Streamlines



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Tracking ambiguity

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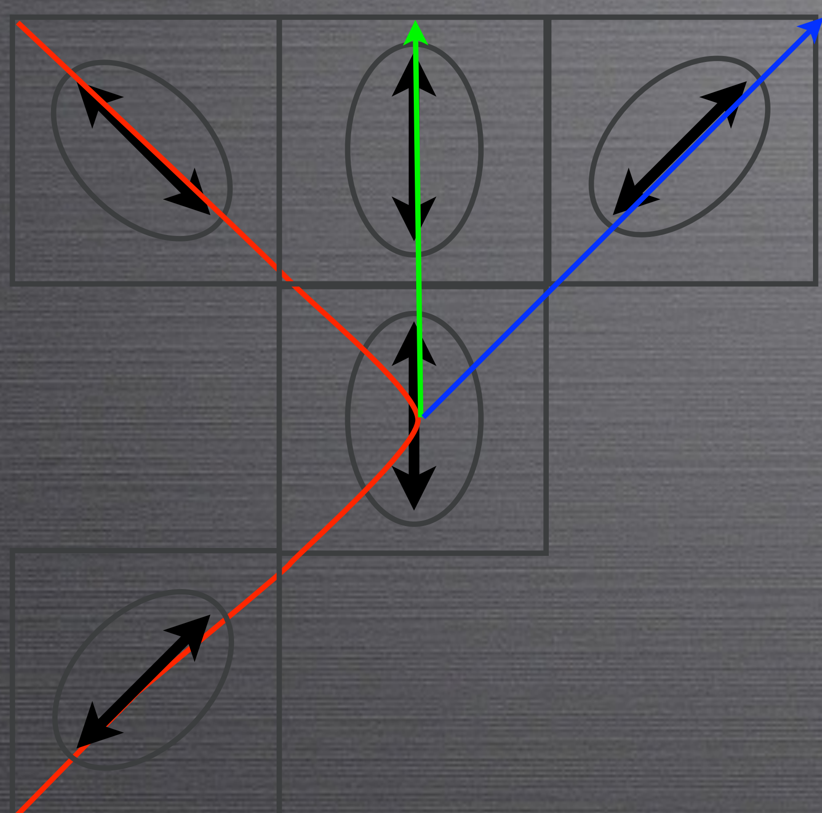
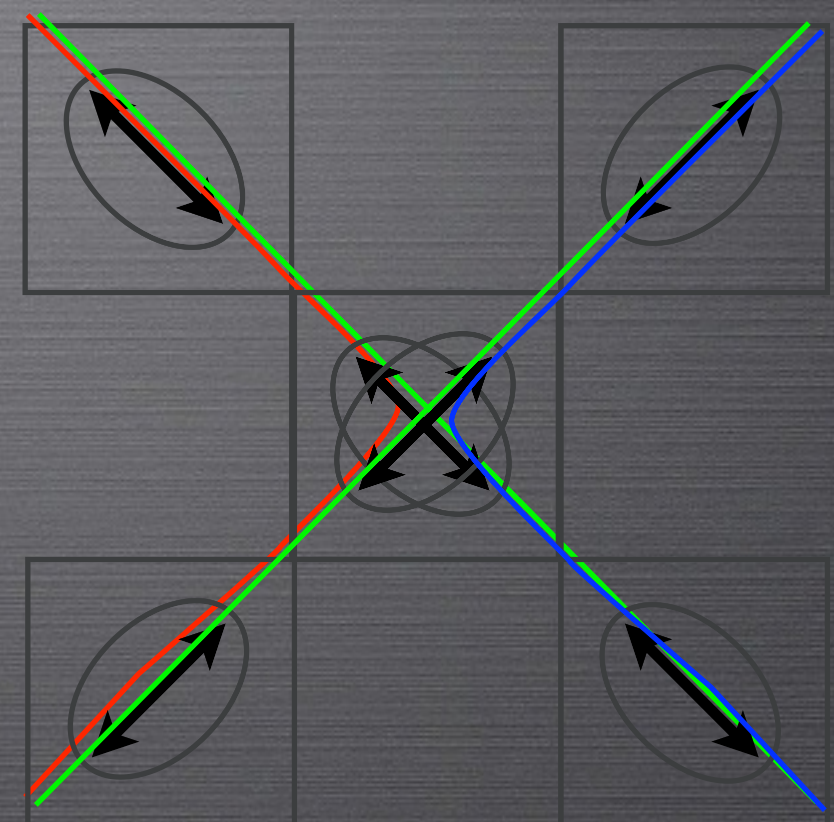


single fibers



crossing/kissing fibers

Diffusion Tensor Imaging (under the hood)

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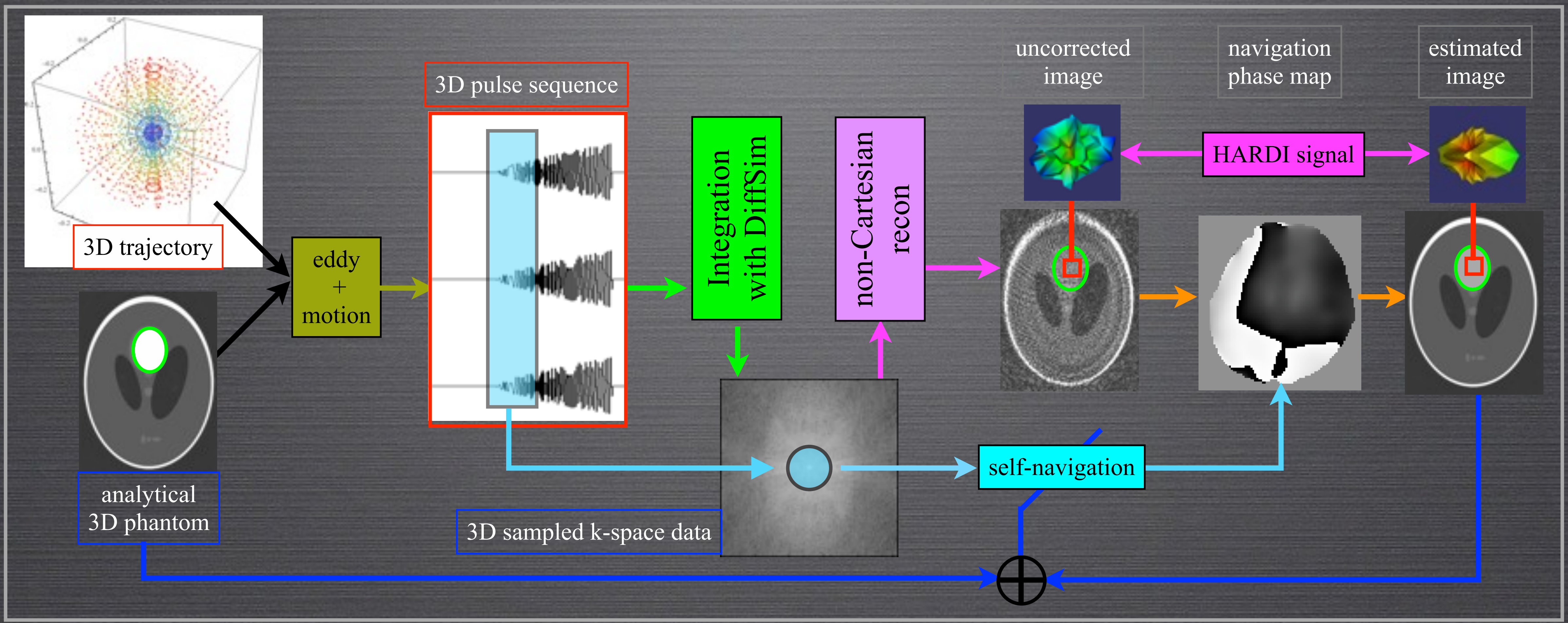
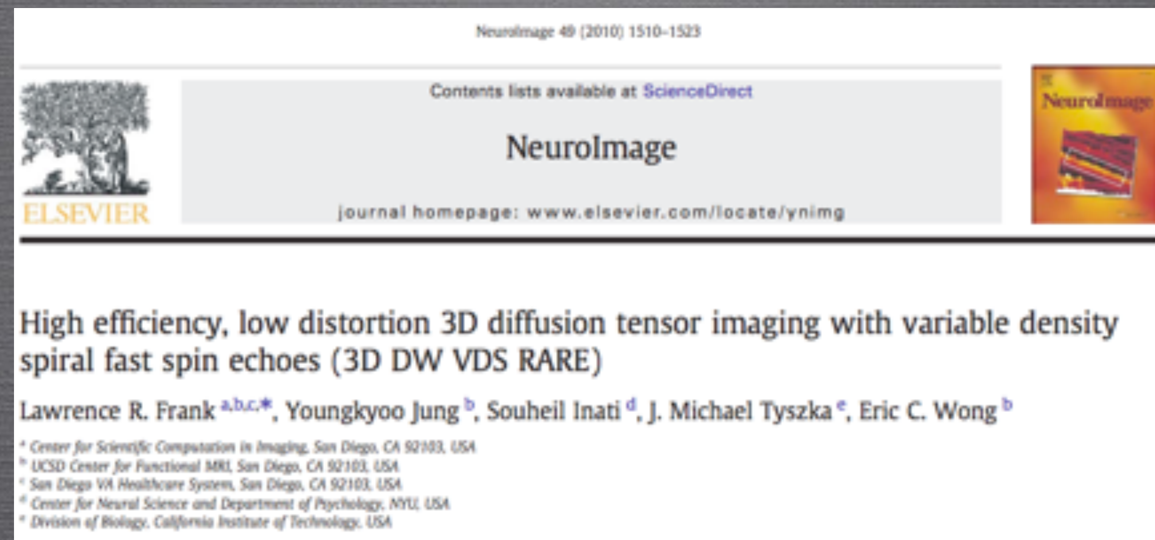


Pulse sequence, reconstruction, and analysis

Diffusion Tensor Imaging (under the hood)



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High efficiency, low distortion 3D diffusion tensor imaging with variable density spiral fast spin echoes (3D DW VDS RARE)

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[e] Division of Biology, California Institute of Technology, USA

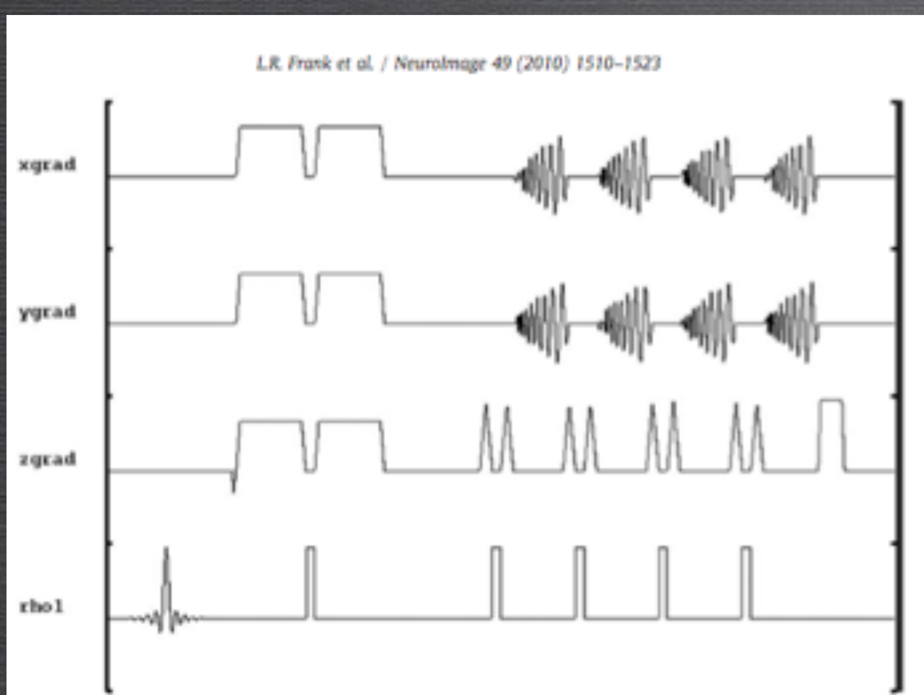
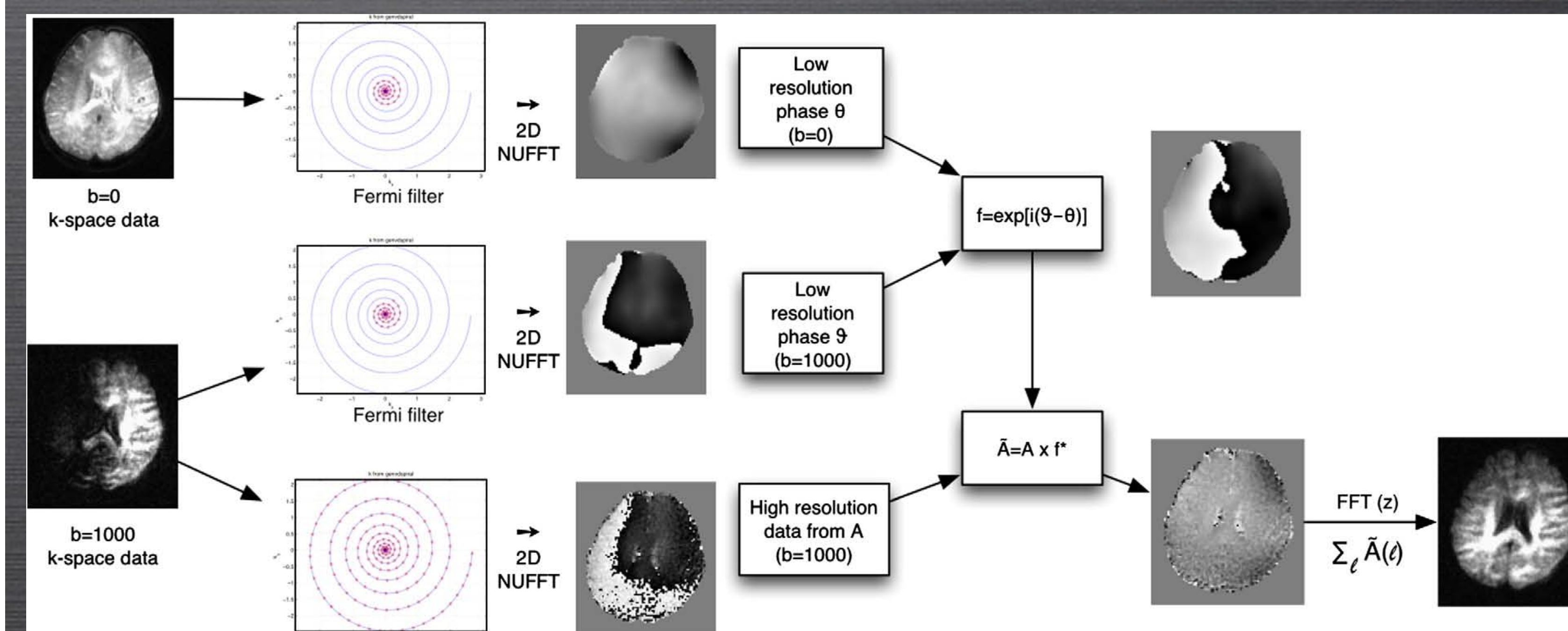


Fig. 1. 3D Variable Density Spiral Fast Spin Echo diffusion weighted pulse sequence.



Pulse sequence, reconstruction, and analysis

Diffusion Tensor Imaging (under the hood)

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