BENG280A, Principles of Biomedical Imaging 11/18/13
Fall Quarter 2013

Exam

This exam is closed book and closed notes. The use of electronic devices is not permitted. By signing below, I am agreeing to these terms and confirming that I have not discussed the exam material with anyone else (aside from the instructor and the TA) and that the work on this exam is solely my own.

Printed Name _____ Signature _____

Please make sure to put your name at the top of each page and to number the pages.
When copying down the expressions, make sure that you've made an accurate transcription.

Problem 1 (50 pts)

A parallel beam CT imaging system is used to image an object defined as:

$$f(x,y) = rect(x,y) ** \left[\delta(x,y) + \delta(x-1,y) + \delta(x+1,y) + \delta(x,y-1) + \delta(x,y+1)\right]$$

(a) (5 pts) Sketch the object.
(b) (5 pts) Sketch the projection at 0 degrees.
(c) (5 pts) Sketch the projection at 45 degrees.
(d) (10 pts) Derive and sketch the Fourier transform of the object. For the sketch, you may leave it as the sum of separate plots.
(e) (10 pts) Show that the projection-slice theorem holds for the projection at 0 degrees.
(f) (15 pts) Show that the projection-slice theorem holds for the projection at 45 degrees.

Problem 2 (60 pts)

Consider the object $f(x,y) = rect\left(x/4\right) rect\left(y/4\right) \cos(2\pi x + 2\pi y)$

(a) (5 pts) Sketch the object.
(b) (5 pts) Compute and sketch the Fourier transform of the object.
(c) (5 pts) Determine the values of Δk_x and Δk_y needed to avoid aliasing
(d) (5 pts) Assume that the desired resolution in both the x and y directions is ½ cm. Determine $k_{x,\max}$ and $k_{y,\max}$ and also the number of voxels in the x and y directions.
(e) (15 pts) Assume that the voxels are arranged such that there is one voxel centered about the origin, so that there will be one more voxel along the negative directions than in the positive directions. Sketch the orientations of the spins along the x-axis (i.e. y=0) for each of the sample points along the k_x axis (you can assume $k_y = 0$). Assume that each voxel contains one phasor. Compute the vector sums at each of the sample points. For what sample values of k_x does the absolute value of the vector sum of spins achieve a maximum or minimum? NOTE: For each value of k_x, you will have a 1D quiver plot of the spins along the x-direction.
(f) (15 pts) Now draw the spin orientations at the values of $\left(k_x, k_y\right)$ that maximize the MRI signal. Explain how the interaction of the object and the spin orientations leads to this maximum. NOTE: for this part, you should have a 2-d quiver plot.
(g) (10 pts) Use the spin orientation diagrams from part (e) to explain why there is aliasing when the sampling interval is doubled along the k_x axis. NOTE: for this part, you can refer back to the diagrams you drew for part (e).