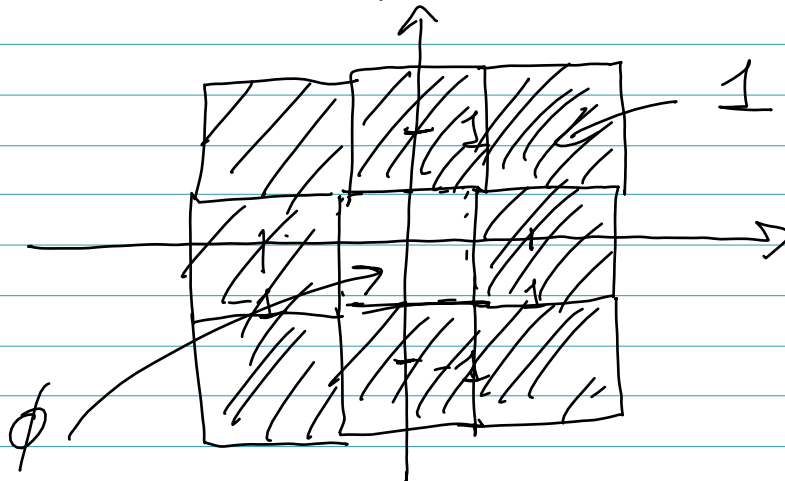


BEZBOA4 Exam

Problem 1

$$f(x, y) = \text{rect}(x, y) * \left[\begin{aligned} &\delta(x-1, y) + \delta(x+1, y) \\ &+ \delta(x, y-1) + \delta(x, y+1) \\ &+ \delta(x-1, y-1) \\ &+ \delta(x+1, y+1) \\ &+ \delta(x+1, y-1) \\ &+ \delta(x-1, y+1) \end{aligned} \right]$$

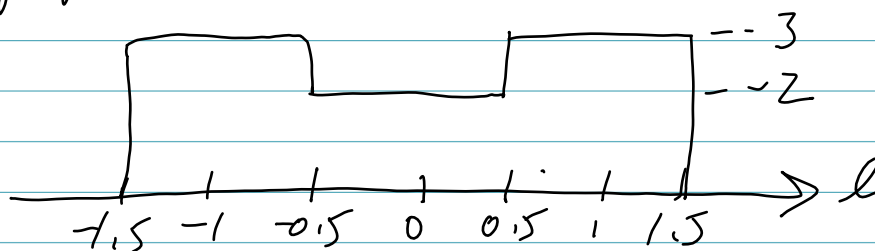
a) sketch the object



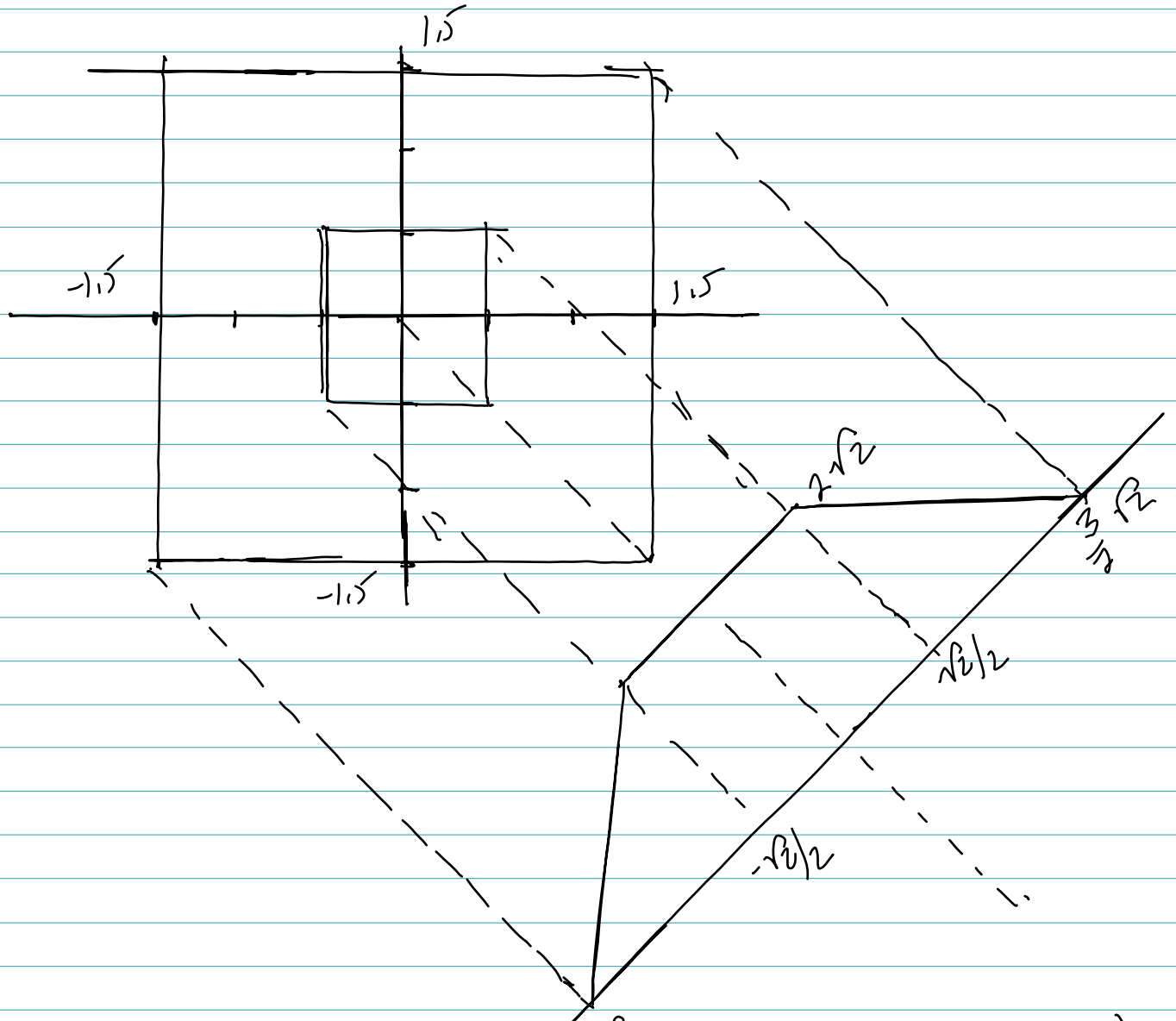
Note: we can rewrite the object as

$$f(x, y) = \text{rect}\left(\frac{x}{3}, \frac{y}{3}\right) - \text{rect}(x, y)$$

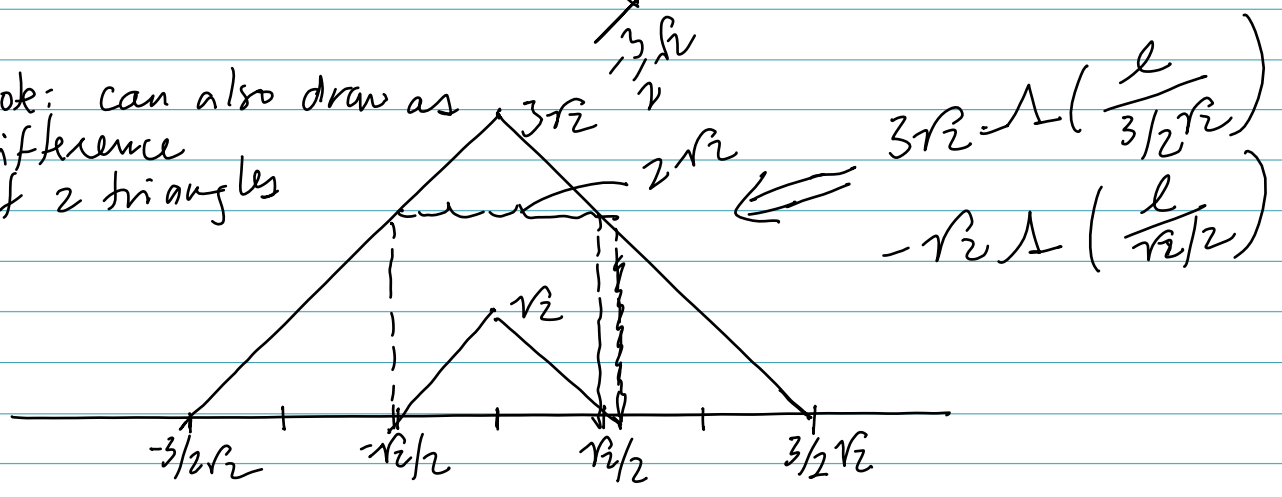
b) projection at 0°



c] projection at 45°



Note: can also draw as difference of 2 triangles



$$3\sqrt{2} \triangle \left(\frac{l}{3/2\sqrt{2}} \right)$$

$$-\sqrt{2} \triangle \left(\frac{l}{\sqrt{2}/2} \right)$$

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d)

$$F(k_x, k_y) = \mathcal{F} \left[\text{rect} \left(\frac{x}{3}, \frac{y}{3} \right) - \text{rect}(x, y) \right]$$

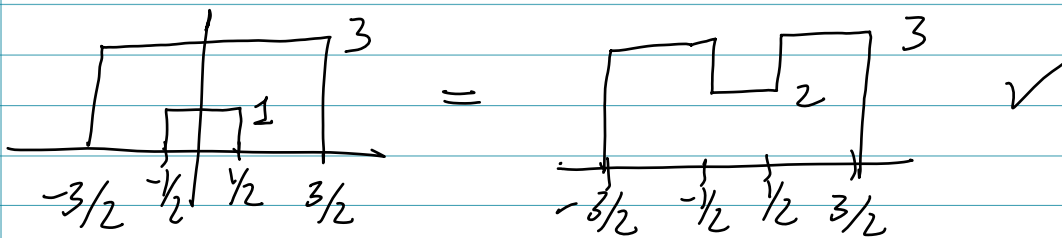
$$= 3 \text{sinc}(3k_x) \cdot 3 \text{sinc}(3k_y) - \text{sinc}(k_x, k_y)$$

$$= 9 \text{sinc}(3k_x, 3k_y) - \text{sinc}(k_x, k_y)$$

e) at zero degrees $k_x = k$ $k_y = \phi$

$$F(k, 0) = 9 \text{sinc}(3k) - \text{sinc}(k)$$

$$\overset{\mathcal{F}^{-1}}{\rightarrow} 3 \text{rect} \left(\frac{l}{3} \right) - \text{rect}(l)$$

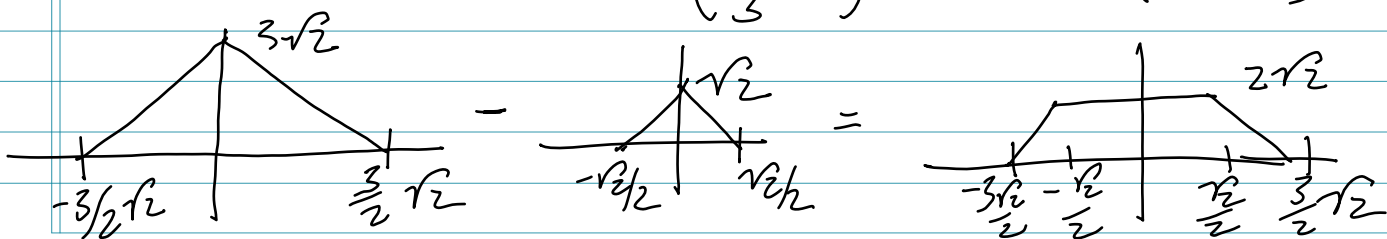


f) at 45 degrees. $k_x = k_y = k/\sqrt{2}$

$$p(k) = 9 \text{sinc}^2 \left(\frac{3k/\sqrt{2}}{2} \right) - \text{sinc}^2 \left(\frac{k/\sqrt{2}}{2} \right)$$

$$\overset{\mathcal{F}^{-1}}{\mathcal{H}} (p(k)) = \frac{3 \cdot 2}{\sqrt{2}} \Lambda \left(\frac{2}{3\sqrt{2}} l \right) - \frac{2}{\sqrt{2}} \Lambda \left(\frac{2}{\sqrt{2}} l \right)$$

$$= 3\sqrt{2} \Lambda \left(\frac{\sqrt{2}}{3} l \right) - \sqrt{2} \Lambda \left(\sqrt{2} l \right)$$



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g)

$$\begin{array}{|c|c|c|} \hline & & \\ \hline 3 & 2 & 3 \\ \hline & & \\ \hline \end{array} + \begin{array}{|c|} \hline 3 \\ \hline 2 \\ \hline 3 \\ \hline \end{array}$$

$$= \begin{array}{|c|c|c|} \hline 6 & 5 & 6 \\ \hline 5 & 4 & 5 \\ \hline 6 & 5 & 6 \\ \hline \end{array}$$

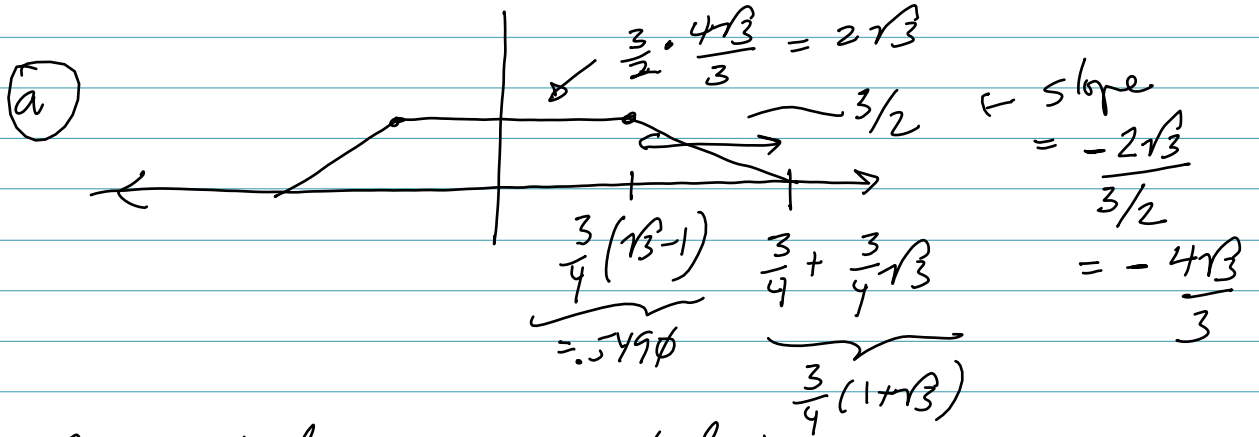
h) Bonus: $K_x = K \cos \theta = K \frac{\sqrt{3}}{2}$ $K_y = K \sin \theta = \frac{K}{2}$

$$G(k) = 9 \operatorname{sinc}\left(\frac{3\sqrt{3}}{2}k\right) \operatorname{sinc}\left(\frac{3k}{2}\right) - \operatorname{sinc}\left(\frac{\sqrt{3}}{2}k\right) \operatorname{sinc}\left(\frac{k}{2}\right)$$

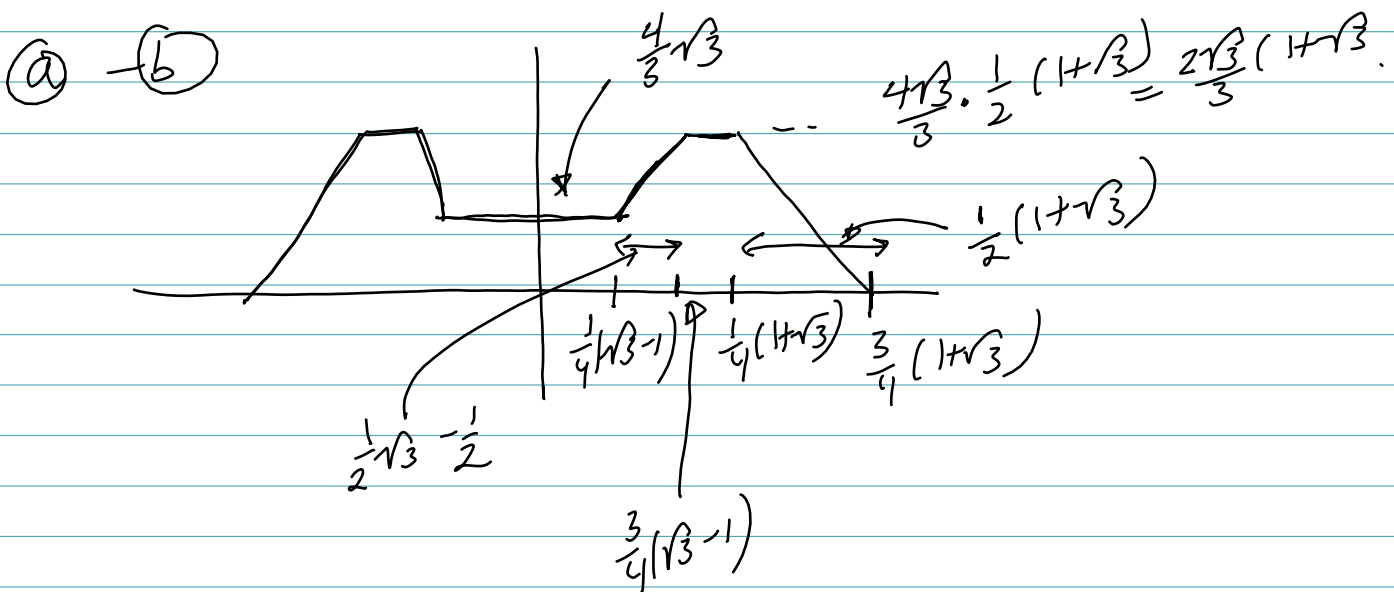
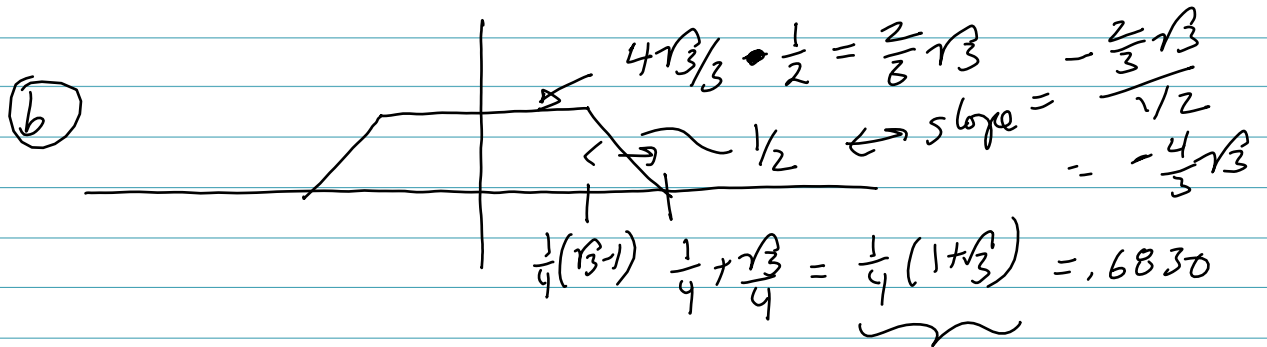
$$\begin{aligned} g(l) &= 9 \cdot \frac{2}{3\sqrt{3}} \cdot \frac{2}{3} \cdot \operatorname{rect}\left(\frac{2}{3\sqrt{3}}l\right) * \operatorname{rect}\left(\frac{2}{3}l\right) \\ &\quad - \frac{2}{\sqrt{3}} \operatorname{rect}\left(\frac{2}{\sqrt{3}}l\right) \cdot \operatorname{rect}(2l) \\ &= \frac{4\sqrt{3}}{3} \operatorname{rect}\left(\frac{l}{\sqrt{3}/2}\right) * \operatorname{rect}\left(\frac{l}{3/2}\right) \\ &\quad - \frac{4\sqrt{3}}{3} \operatorname{rect}\left(\frac{l}{\sqrt{3}/2}\right) * \operatorname{rect}\left(\frac{l}{1/2}\right) \end{aligned}$$

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$$\frac{4\sqrt{3}}{3} \operatorname{rect}\left(\frac{l}{\frac{3\sqrt{3}}{2}}\right) + \operatorname{rect}\left(\frac{l}{3/2}\right)$$



$$\frac{4\sqrt{3}}{3} \operatorname{rect}\left(\frac{l}{\frac{2\sqrt{3}}{2}}\right) + \operatorname{rect}\left(\frac{l}{1/2}\right)$$



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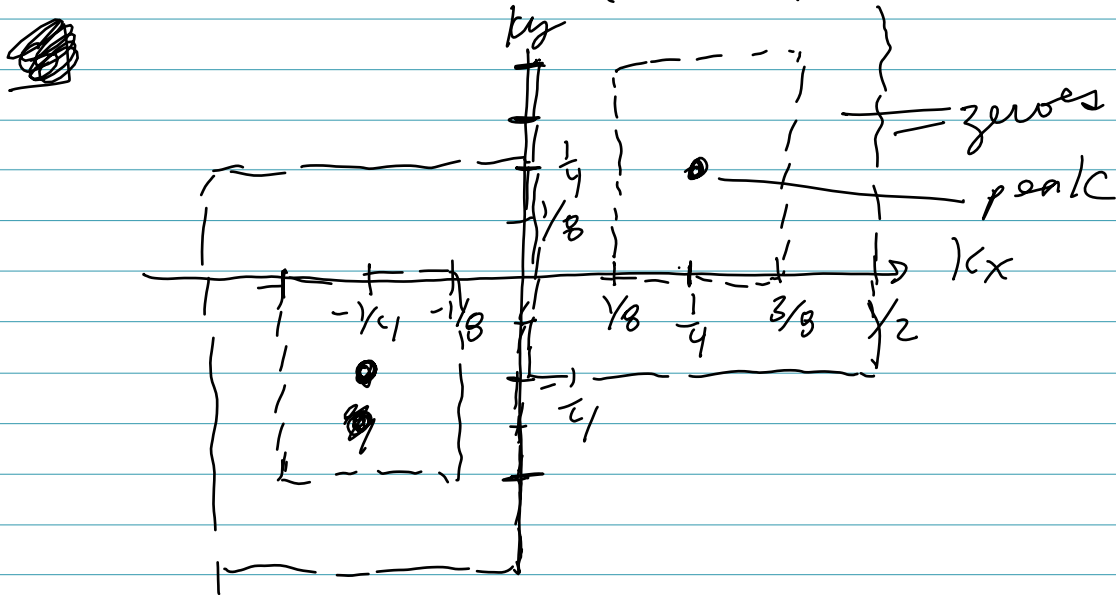
Problem 2

a) $m(x,y) = \text{rect}\left(\frac{x}{8}\right) \text{rect}\left(\frac{y}{4}\right) \cdot \cos\left(2\pi\left(\frac{x}{4} + \frac{y}{4}\right)\right)$

b) $M(k_x, k_y) = 8 \text{sinc}(8k_x) \cdot 4 \cdot \text{sinc}(4ky)$
 $\ast \frac{1}{2} \left[\delta\left(k_x - \frac{1}{4}, k_y - \frac{1}{4}\right) + \delta\left(k_x + \frac{1}{4}, k_y + \frac{1}{4}\right) \right]$

$= 16 \left[\text{sinc}\left(8\left(k_x - \frac{1}{4}\right)\right) \text{sinc}\left(4\left(k_y - \frac{1}{4}\right)\right) + \text{sinc}\left(8\left(k_x + \frac{1}{4}\right)\right) \text{sinc}\left(4\left(k_y + \frac{1}{4}\right)\right) \right]$

$= 16 \left[\text{sinc}(8k_x - 2) \text{sinc}(4k_y - 1) + \text{sinc}(8k_x + 2) \text{sinc}(4k_y + 1) \right]$



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c) $\Delta k_x = \frac{1}{Fov_x} = \frac{1}{8 \text{ cm}}$

$\Delta k_y = \frac{1}{Fov_y} = \frac{1}{4 \text{ cm}}$

d) $k_{x, \text{max}} = \frac{1}{2} W_{kx} = \frac{1}{2} \frac{1}{1 \text{ cm}} = \frac{1}{2 \text{ cm}}$

$k_{y, \text{max}} = \frac{1}{2} W_{ky} = \frac{1}{2} \frac{1}{1 \text{ cm}} = \frac{1}{2 \text{ cm}}$

e)

\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	row sum
\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	-1
\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	ϕ
\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	1
\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	ϕ
\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	\rightarrow	-1
ϕ						<u><u>sum = -1</u></u>

$\phi = -1$
 $\Delta = 1$

f)

\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	row sum
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	-1
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	0
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	1
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	0
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	-1
ϕ						<u><u>sum = -1</u></u>

g)

\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	row sum
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	-1
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	ϕ
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	1
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	ϕ
\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	\leftarrow	-1
ϕ						<u><u>sum = -1</u></u>

