BENG280A, Principles of Biomedical Imaging 10/15/14
Fall Quarter 2014

HOMEWORK #2
Due at 5 pm on Friday 10/24/14

Homework policy: Homeworks can be turned in during class prior to the due date or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material. It is recommended that you make a copy of the homework for yourself (e.g. scan it in) before you turn it in.

Required Readings:
Principles of CT and CT Technology; available at http://tech.snmjournals.org/content/35/3/115.full.pdf
CT Tutorial available at http://pubs.rsna.org/doi/pdf/10.1148/radiographics.22.4.g02jl14949

Optional Readings:
Prince and Links, page 150-161 and pages 181-203 (Note this is on reserve at the library).
Pages 49 to 56 of Chapter 3 from Slaney and Kak available at http://www.slaney.org/pct/pct-toc.html

Optional Videos:

Some nice background videos on Dirac Delta Functions:
https://www.youtube.com/watch?v=J-oyM1GyyDk
https://www.youtube.com/watch?v=fhpR6sqbcr8
https://www.youtube.com/watch?v=4qfdCwys2ew

Graphic example of convolution of two rectangular pulses:
https://www.youtube.com/watch?v=4-FS5GN_vFE

Problems:

1. Derive and sketch the answers for the following expressions where * denotes convolution:
   a) $m(x) = \left[2\delta(-x-3) + 2\delta(-x+3)\right] * rect(-x^2/7)$
   b) $m(x) = \left[2\delta(x-2003) - 2\delta(x+2003)\right] * \sin(\pi x)$
   c)
   $$m(x,y) = \left[\delta(x,y) + \delta(x-1/2,y) + \delta(x+1/2,y) + \delta(-x,y-4) + \delta(x,y+4)\right] ** \left(rect(2x,y/4)\right)^2$$
   d) $m(x,y) = \left[\delta(x,y) + \delta(x-0.5,y) + \delta(x+0.5,-y)\right] ** \left[\delta(x-0.5,y+7) + \delta(-x,y+4)\right]$

2. Consider an x-ray imaging system with z = 40 cm and d = 60 cm. The x-ray source can be modeled as the 2D-function $s(x,y) = S_0 \exp(-x^2)\delta(y)$. Let the object be defined as the 2D-transmission function $t(x,y) = \delta\left(x - \frac{w}{2}\right) + \delta\left(x + \frac{w}{2}\right)$ [Adapted from Prince and Links 5.21]
   a) Compute the object magnification M
   b) Compute the source magnification *m*
   c) Sketch the object.

d) Derive an expression for and sketch the image of the object at the detector plane.
e) Determine the minimum value of *w* such that the two lines in the image at the detector plane can be distinguished from each other. You may assume that this occurs when the peaks of the lines are separated by the FWHM.

3. Consider an X-ray imaging system with source distribution *s(x,y)* and object transmission function *t(x,y)*. The distance from the source to the detector is d; and the distance from the source to the object is z. Let $s(x,y) = rect(2x,y)$ and $t(x,y) = rect(2x,y)$. Write down a general expression for the intensity of the image at the detector (explicitly evaluate the convolution). Your answer should be a function of x,y, z, and d. You may define magnification variables (e.g. m and M) that are functions of z and d and use these in your answer. Sketch the answer for $z = d/2$.

4. Consider a 6G CT scanner with a table that moves at a speed of 3 cm/s. The x-ray source-detector apparatus rotates at a speed of $4\pi / s$. It takes 1 ms to measure a projection. The pitch of the helix is defined as the distance traveled during a full 360 degree rotation. [Adapted from problem 6.2 in Prince and Links].
   a) What is the pitch of the helix?
   b) How many projections does the system measure over a 2π angle?
   c) How long does it take to scan a 60 cm torso?

5. (OPTIONAL BONUS PROBLEM) Use the basic properties of the Dirac delta function to show that $\frac{1}{M}\delta\left(\frac{x - Mx_0}{M}\right) = \delta\left(x - Mx_0\right)$ (i.e. show that the areas are equivalent). Next use this identity and the definition of the convolution integral to show that
$$s\left(\frac{x}{m}\right) * \frac{1}{M}\delta\left(\frac{x - Mx_0}{M}\right) = s\left(\frac{x - Mx_0}{m}\right).$$

Matlab Exercise:

Define a 281x281 object that is zero everywhere except for a point target at a location that you define. Treat the middle of your object as the origin (0,0). Use the **radon** function to compute the projections (use the MATLAB help feature to get more information on how to use the radon function). Use the imagesc function to display your answer for the following 3 locations: (a) (30, 0); (b) (0, -20); and (c) (30, -20). Note: you will have one image for each location, for a total of 3 different images. Please turn in your code and your images.