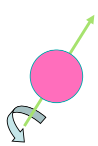
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2014
MRI Lecture 1

Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

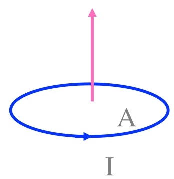
This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where γ is the gyromagnetic ratio and $\mathbf{S}$ is the spin angular momentum.

Nuclear Spin Rules

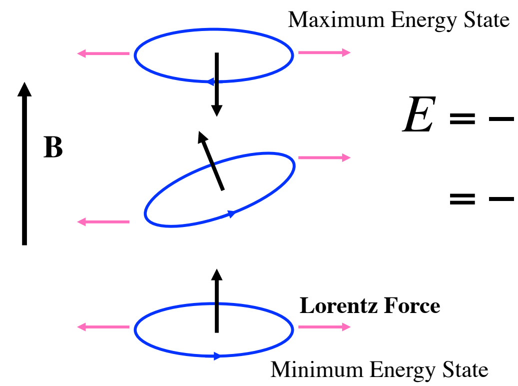
Number of Protons	Number of Neutrons	Spin	Examples
Even	Even	0	^{12}C, ^{16}O
Even	Odd	j/2	^{17}O
Odd	Even	j/2	^{1}H, ^{23}Na, ^{31}P
Odd	Odd	j	^{2}H

Classical Magnetic Moment



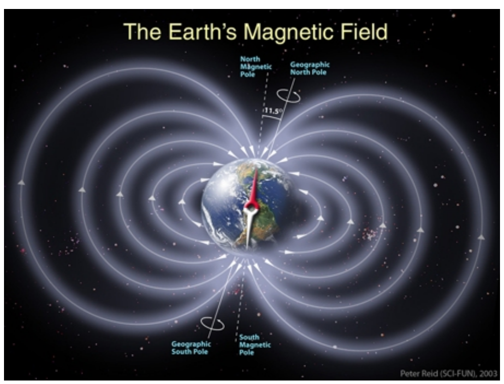
$$\vec{\mu} = IA\hat{n}$$

Energy in a Magnetic Field



$$E = -\vec{\mu} \bullet \mathbf{B}$$
$$= -\mu_z B$$

Energy in a Magnetic Field



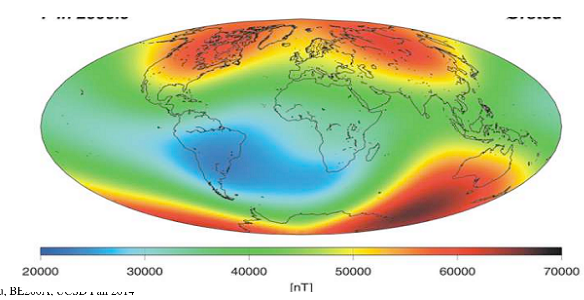
 www.qi-whiz.com/images/ Earth-magnetic-field.jpg

Magnetic Field Units

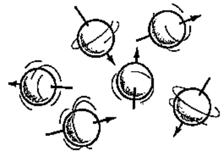
1 Tesla = 10,000 Gauss

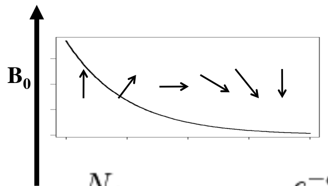
Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

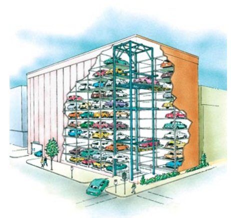


Boltzmann Distribution

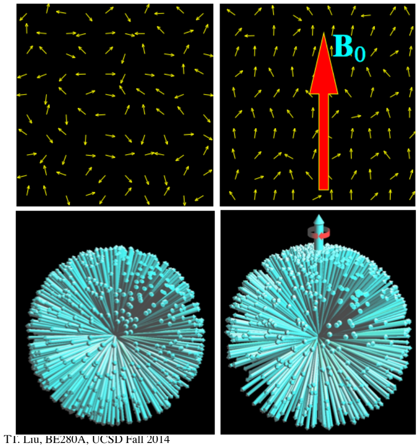




$$\frac{N_j}{N} \equiv \mathcal{P}(\epsilon_j) = \frac{e^{-\epsilon_j/(kT)}}{Q}.$$



Equilibrium Magnetization



$$\mathbf{M}_0 = N\langle\mu_z\rangle$$
$$\approx N\mu_z^2 B/(kT)$$
$$= N\gamma^2\hbar^2 B/(4kT)$$

N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

Hansen 2009

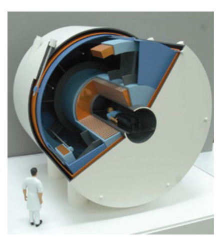
UCSD 3T

U. Minn. 7T

UIC 9.4T

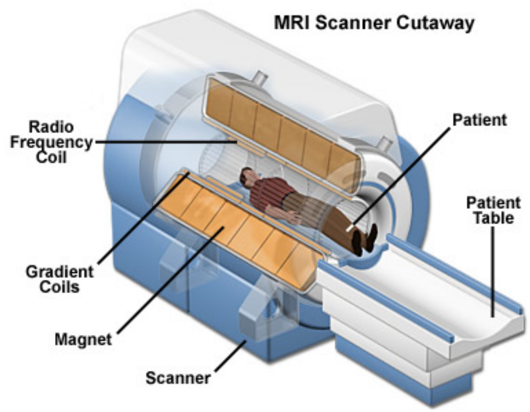
The 11.7 T MR magnet with members of the MRI team at Agilent's Magnet Technology Center in Yarnton, UK (from left to right): Darren Houlden, Andrew Winter, Paul Johnson, Nigel Haynes and Alistair Courtney.

11.7T NIH

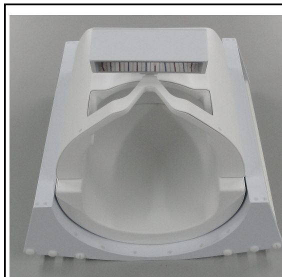


11.7T NeuroSpin France

MRI System



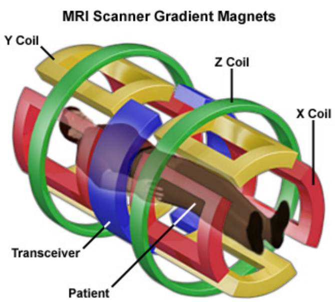
http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/fullarticle.html



Nova 32 channel

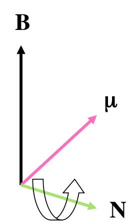
Siemens 32 channel

MRI Gradients



http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/fullarticle.html
http://gulfnews.com/business/economy/german-factories-race-to-meet-big-surge-in-orders-1.638066

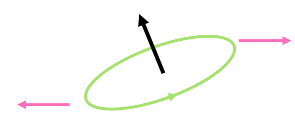
Torque



For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)

$\mathbf{N} = \boldsymbol{\mu} \times \mathbf{B}$

Torque



Precession

Torque

$$\mathbf{N} = \mu \times \mathbf{B}$$

$$\frac{d\mathbf{S}}{dt} = \mathbf{N}$$

Change in Angular momentum

$$\frac{d\mathbf{S}}{dt} = \mu \times \mathbf{B}$$

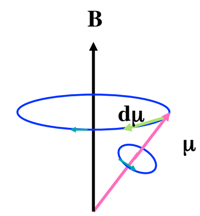
$$\mu = \gamma \, \mathbf{S}$$

Relation between magnetic moment and angular momentum

$$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

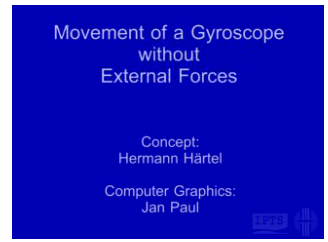


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma \, B$$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertelmpg_e/gyros_free.mpg

Magnetization Vector

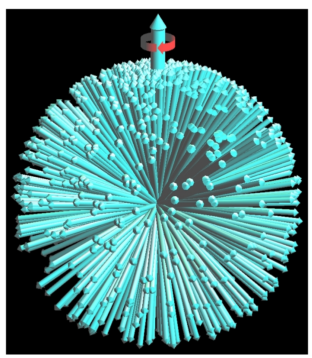
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

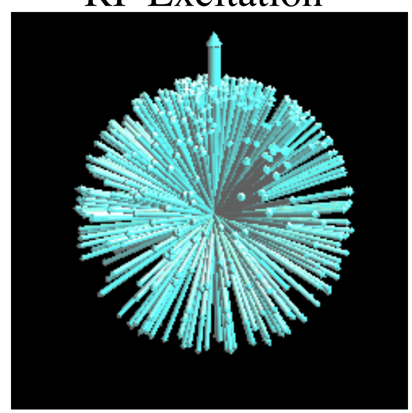
$$\mathbf{M} = \frac{1}{V} \sum_{\substack{\text{protons} \\ \text{in V}}} \mu_i$$

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$



Hansen 2009

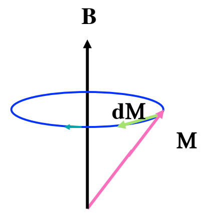
RF Excitation



http://www.drcmr.dk/main/content/view/213/74/

Free precession about static field

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B}$$
$$= \gamma \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{vmatrix}$$
$$= \gamma \begin{pmatrix} \hat{i}\left(B_z M_y - B_y M_z\right) \\ -\hat{j}\left(B_z M_x - B_x M_z\right) \\ \hat{k}\left(B_y M_x - B_x M_y\right) \end{pmatrix}$$



Free precession about static field

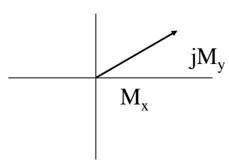
$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} B_z M_y - B_y M_z \\ B_x M_z - B_z M_x \\ B_y M_x - B_x M_y \end{bmatrix}$$
$$= \gamma \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Precession

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} 0 & B_0 & 0 \\ -B_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Useful to define $M \equiv M_x + jM_y$

$$dM/dt = d/dt\left(M_x + iM_y\right)$$
$$= -j\gamma B_0 M$$



Solution is a time-varying phasor

$$M(t) = M(0)e^{-j\gamma B_0 t} = M(0)e^{-j\omega_0 t}$$

Question: which way does this rotate with time?

Gyromagnetic Ratios

Nucleus	Spin	Magnetic Moment	$\gamma/(2\pi)$ (MHz/Tesla)	Abundance
^{1}H	1/2	2.793	42.58	88 M
^{23}Na	3/2	2.216	11.27	80 mM
^{31}P	1/2	1.131	17.25	75 mM

Source: Haacke et al., p. 27

Larmor Frequency

$\omega = \gamma B$ Angular frequency in rad/sec

$f = \gamma B / (2\pi)$ Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Notation and Units

1 Tesla = 10,000 Gauss

Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

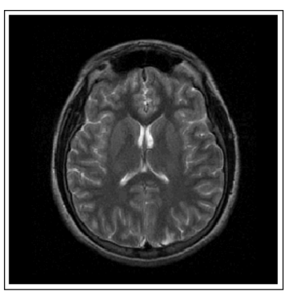
$\gamma = 26752$ radians/second/Gauss

$\bar{\gamma} = \gamma / 2\pi = 4258$ Hz/Gauss

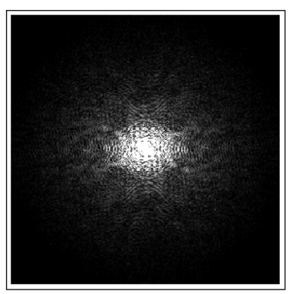
$= 42.58$ MHz/Tesla

k-space

Image space



k-space



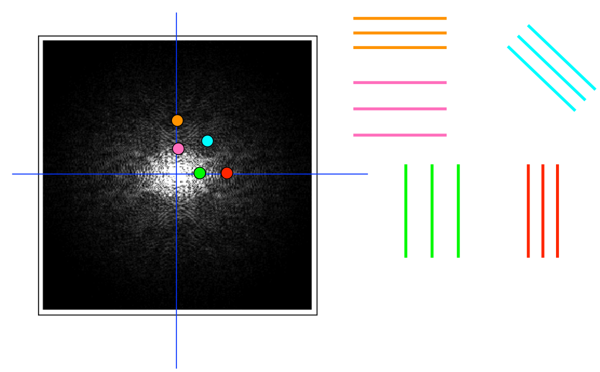
y

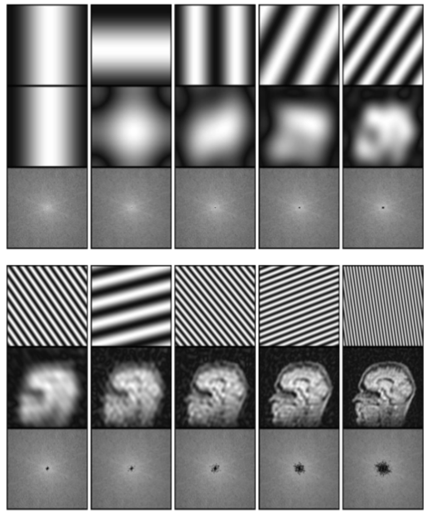
x

k_y

k_x

Fourier Transform





Hanson 2009

2D Fourier Transform

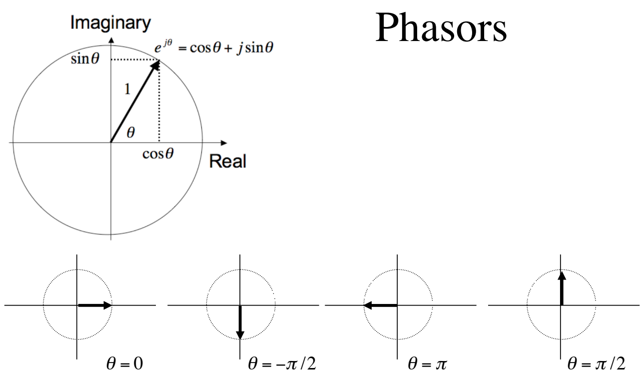
Fourier Transform

$$G(k_x,k_y) = F\left[g(x,y)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi\left(k_x x + k_y y\right)}dxdy$$

Inverse Fourier Transform

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(k_x,k_y)e^{j2\pi\left(k_x x + k_y y\right)}dk_x dk_y$$

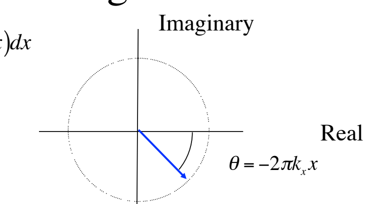
Phasors



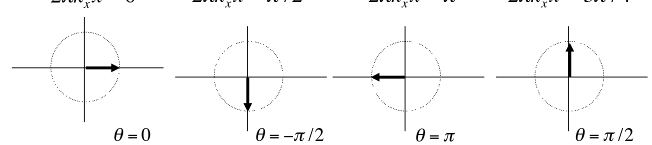
Phasor Diagram

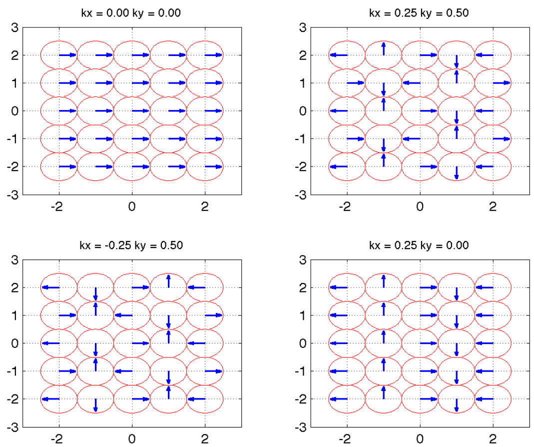
$$G(k_x) = \int_{-\infty}^{\infty} g(x)\exp(-j2\pi k_x x)dx$$

$\theta = -2\pi k_x x$



$k_x = 1;\ x = 0$	$x = 1/4$	$x = 1/2$	$x = 3/2$
$2\pi k_x x = 0$	$2\pi k_x x = \pi/2$	$2\pi k_x x = \pi$	$2\pi k_x x = 3\pi/4$




kx = 0.00 ky = 0.00
kx = 0.25 ky = 0.50
kx = -0.25 ky = 0.50
kx = 0.25 ky = 0.00