Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2014
MRI Lecture 4

K-space

At each point in time, the received signal is the Fourier transform of the object

$$s(t) = M\left(k_x(t), k_y(t)\right) = F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}$$

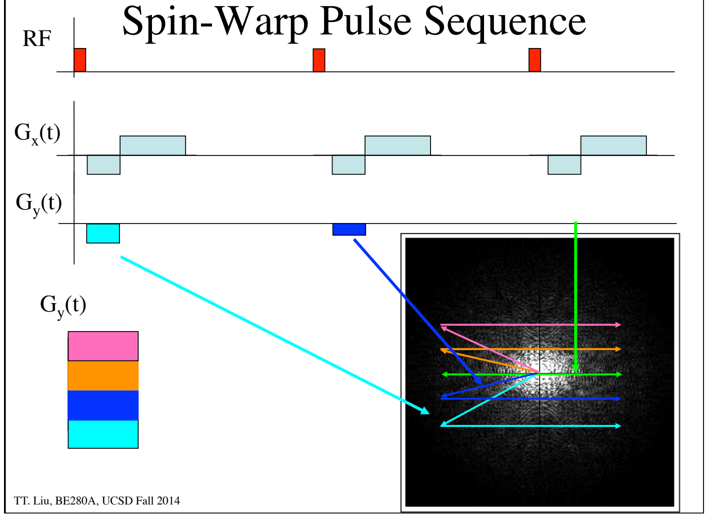
evaluated at the spatial frequencies:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

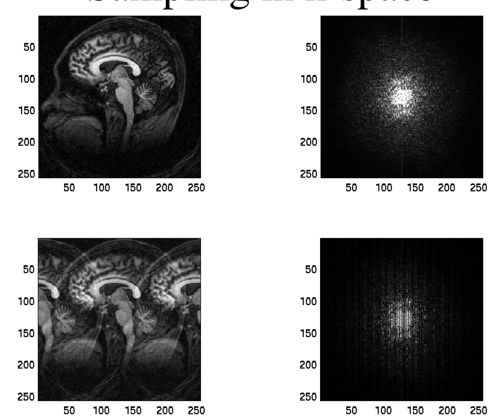
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Thus, the gradients control our position in k-space. The design of an MRI pulse sequence requires us to efficiently cover enough of k-space to form our image.

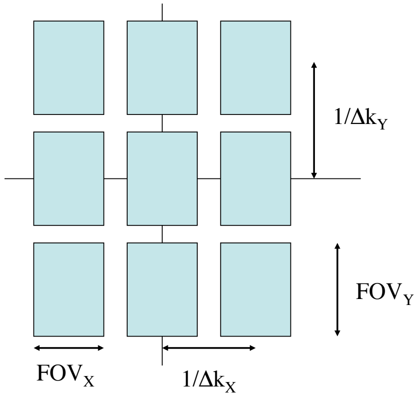
Spin-Warp Pulse Sequence



Sampling in k-space



Nyquist Conditions



$1/\Delta k_Y > FOV_Y$

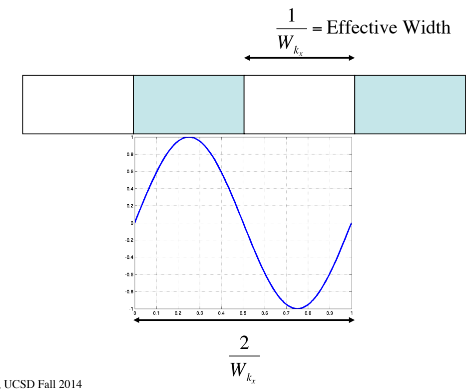
$1/\Delta k_X > FOV_X$

Resolution and spatial frequency

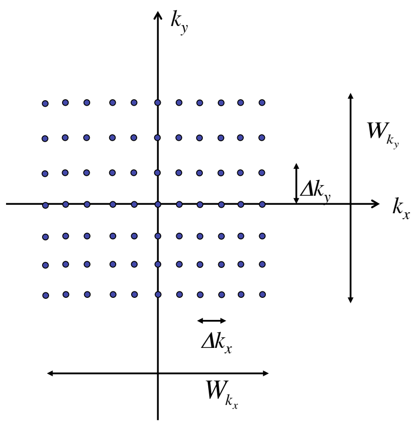
With a window of width W_{k_x} the highest spatial frequency is $W_{k_x}/2$.

This corresponds to a spatial period of $2/W_{k_x}$.

$\frac{1}{W_{k_x}}$ = Effective Width



Sampling and Windowing

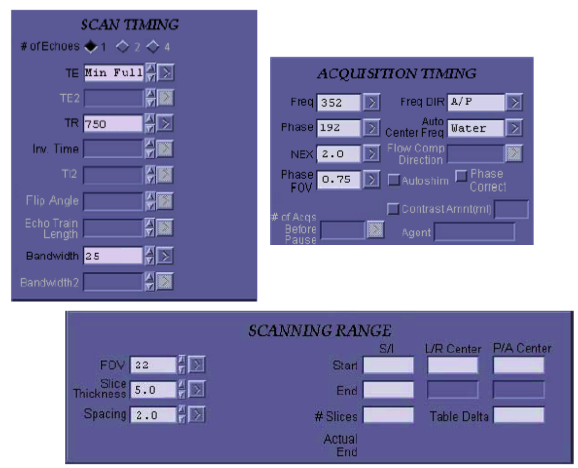


$\Delta k_Y < 1/(FOV_Y)$

$\Delta k_X < 1/(FOV_X)$

$W_{k_x} > 1/\delta_x$

$W_{k_y} > 1/\delta_y$



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Example

Goal :

$FOV_x = FOV_y = 25.6$ cm

$\delta_x = \delta_y = 0.1$ cm

Readout Gradient:

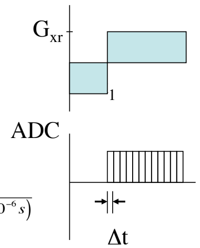
$$FOV_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t}$$

Assume $\Delta t = 32\ \mu\text{sec}$

$$G_{xr} = \frac{1}{FOV_x \frac{\gamma}{2\pi} \Delta t} = \frac{1}{(25.6cm)(42.57\times 10^6 T^{-1}s^{-1})(32\times 10^{-6} s)}$$

$$= 2.8675 \times 10^{-5}\ \text{T/cm}$$
$$= .28675\ \text{G/cm}$$

1 Gauss = 1×10^{-4} Tesla



Example

Readout Gradient :

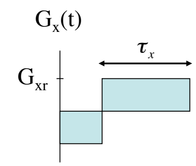
$$\delta_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$

$$\tau_x = \frac{1}{\delta_x \frac{\gamma}{2\pi} G_{xr}} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(0.28675\ \text{G/cm})}$$

$$= 8.192\ \text{ms}$$
$$= N_{read} \Delta t$$

where

$$N_{read} = \frac{FOV_x}{\delta_x} = 256$$



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Example

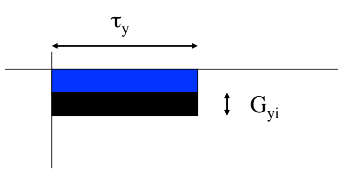
Phase-Encode Gradient:

$$FOV_y = \frac{1}{\frac{\gamma}{2\pi} G_{yi} \tau_y}$$

Assume $\tau_y = 4.096$ *m*sec

$$G_{yi} = \frac{1}{FOV_y \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(25.6cm)(42.57\times 10^6 T^{-1}s^{-1})(4.096\times 10^{-3} s)}$$

$$= 2.2402\times 10^{-7}\ \text{T/cm}$$
$$= .00224\ \text{G/cm}$$



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Example

Phase - Encode Gradient :

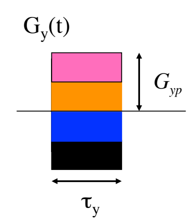
$$\delta_y = \frac{1}{\frac{\gamma}{2\pi} 2 G_{yp} \tau_y}$$

$$G_{yp} = \frac{1}{\delta_y 2 \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(4.096\times 10^{-3} s)}$$

$$= 0.2868\ \text{G/cm}$$
$$= \frac{N_p}{2} G_{yi}$$

where

$$N_p = \frac{FOV_y}{\delta_y} = 256$$



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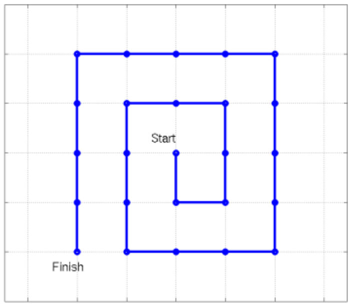
Example

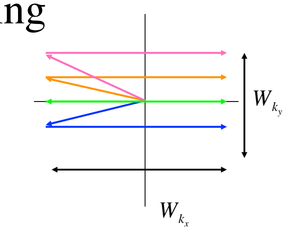
Consider the k-space trajectory shown below. ADC samples are acquired at the points shown with $\Delta t = 10\ \mu\text{sec}$. The desired FOV (both x and y) is 10 cm and the desired resolution (both x and y) is 2.5 cm. Draw the gradient waveforms required to achieve the k-space trajectory. Label the waveform with the gradient amplitudes required to achieve the desired FOV and resolution. Also, make sure to label the time axis correctly.

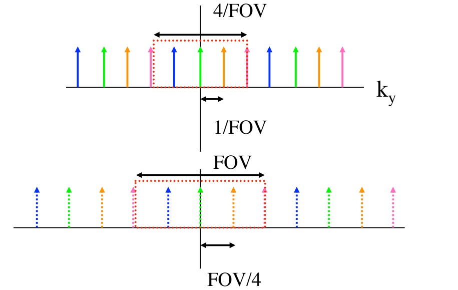
$$Assume\ \gamma/(2\pi) = 4000 Hz/G$$



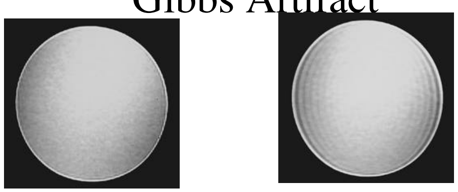
Sampling

In practice, an even number (typically power of 2) sample is usually taken in each direction to take advantage of the Fast Fourier Transform (FFT) for reconstruction.



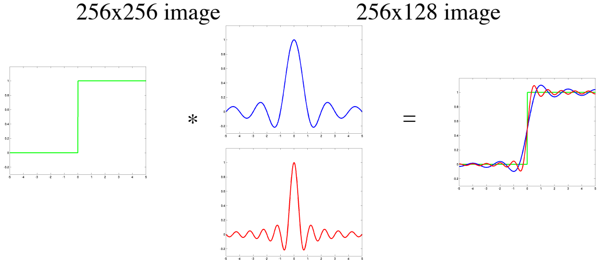


Gibbs Artifact



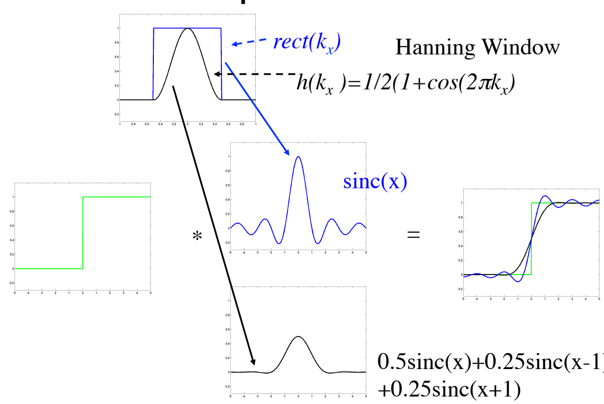
256x256 image

256x128 image



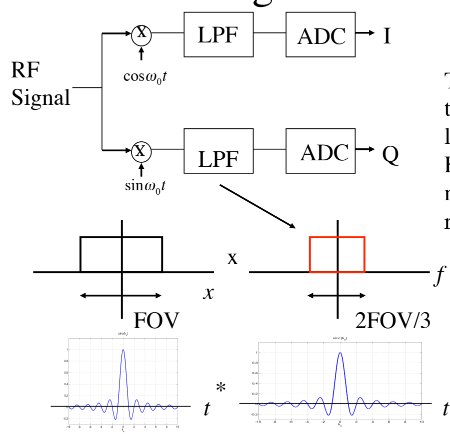
Images from http://www.mritutor.org/mritutor/gibbs.htm

Apodization



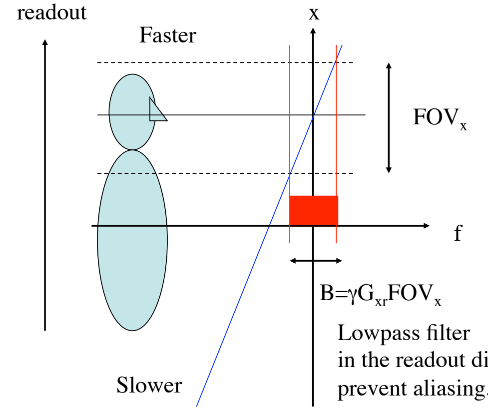
Images from http://www.mritutor.org/mritutor/gibbs.htm

Aliasing and Bandwidth



Temporal filtering in the readout direction limits the readout FOV. So there should never be aliasing in the readout direction.

Aliasing and Bandwidth



Lowpass filter
in the readout direction to
prevent aliasing.

Figure 7-31 Default Axial Directions

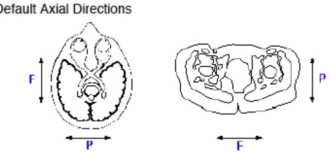
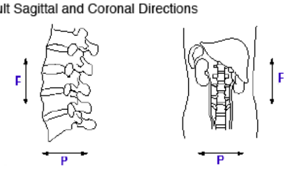


Figure 7-32 Default Sagittal and Coronal Directions



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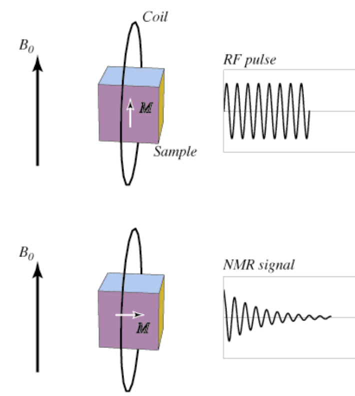
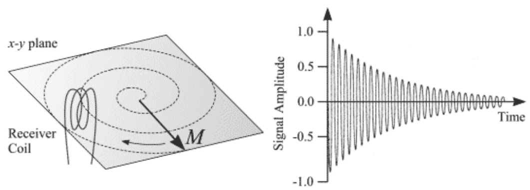
RF Excitation



From Buxton 2002

Free Induction Decay (FID)



http://www.easymeasure.co.uk/principlesmri.aspx

Relaxation

An excitation pulse rotates the magnetization vector away from its equilibrium state (purely longitudinal). The resulting vector has both longitudinal $\mathbf{M_z}$ and tranverse $\mathbf{M_{xy}}$ components.

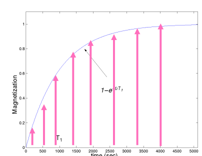
Due to thermal interactions, the magnetization will return to its equilibrium state with characteristic time constants.

T_1 spin-lattice time constant, return to equilibrium of $\mathbf{M_z}$

T_2 spin-spin time constant, return to equilibrium of $\mathbf{M_{xy}}$

Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$

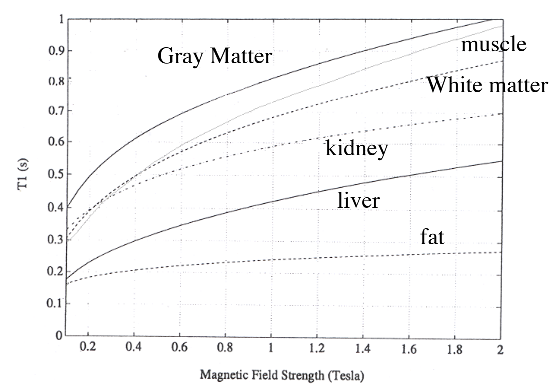


After a 90 degree pulse $M_z(t) = M_0(1-e^{-t/T_1})$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that T_1 increases with **B**.

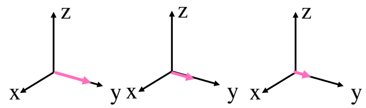
T1 Values



Image, caption: Nishimura, Fig. 4.2

Transverse Relaxation

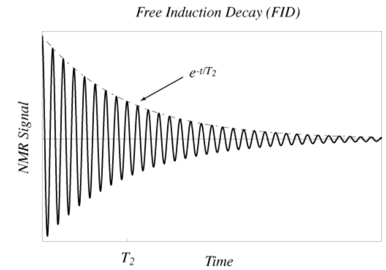
$$\frac{d\mathbf{M}_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$



Each spin's local field is affected by the z-component of the field due to other spins. Thus, the Larmor frequency of each spin will be slightly different. This leads to a dephasing of the transverse magnetization, which is characterized by an exponential decay.

T_2 is largely independent of field. T_2 is short for low frequency fluctuations, such as those associated with slowly tumbling macromolecules.

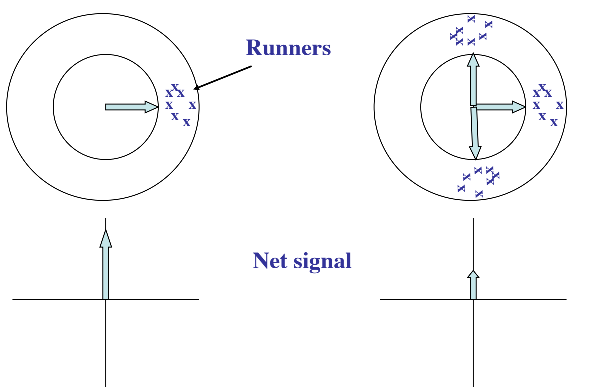
T2 Relaxation



After a 90 degree excitation

$$M_{xy}(t) = M_0 e^{-t/T_2}$$

T2 Relaxation



Credit: Larry Frank

T2 Values

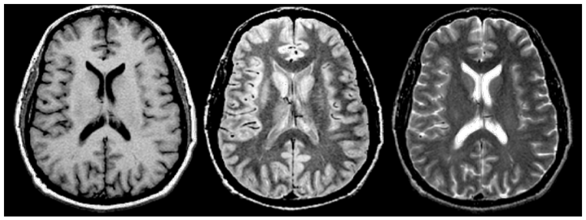
Tissue	T_2 (ms)
gray matter	100
white matter	92
muscle	47
fat	85
kidney	58
liver	43
CSF	4000

Table: adapted from Nishimura, Table 4.2

Solids exhibit very short T_2 relaxation times because there are many low frequency interactions between the immobile spins.

On the other hand, liquids show relatively long T_2 values, because the spins are highly mobile and net fields average out.

Example



T_1-weighted Density-weighted T_2-weighted

Questions: How can one achieve T2 weighting? What are the relative T2's of the various tissues?

Bloch Equation

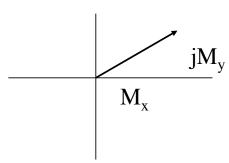
$$\frac{d\mathbf{M}}{dt} = \underbrace{\mathbf{M} \times \gamma \mathbf{B}}_{\text{Precession}} - \underbrace{\frac{M_x \mathbf{i} + M_y \mathbf{j}}{T_2}}_{\substack{\text{Transverse} \\ \text{Relaxation}}} - \underbrace{\frac{(M_z - M_0)\mathbf{k}}{T_1}}_{\substack{\text{Longitudinal} \\ \text{Relaxation}}}$$

i, j, k are unit vectors in the x,y,z directions.

Precession

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} 0 & B_0 & 0 \\ -B_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Useful to define $M \equiv M_x + jM_y$



$$dM/dt = d/dt(M_x + iM_y)$$
$$= -j\gamma B_0 M$$

Solution is a time-varying phasor

$$M(t) = M(0)e^{-j\gamma B_0 t} = M(0)e^{-j\omega_0 t}$$

Question: which way does this rotate with time?

Matrix Form with B=B_0

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \begin{bmatrix} -1/T_2 & \gamma B_0 & 0 \\ -\gamma B_0 & 1/T_2 & 0 \\ 0 & 0 & -1/T_1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

Z-component solution

$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}$$

Saturation Recovery

$$\text{If } M_z(0) = 0 \text{ then } M_z(t) = M_0(1 - e^{-t/T_1})$$

Inversion Recovery

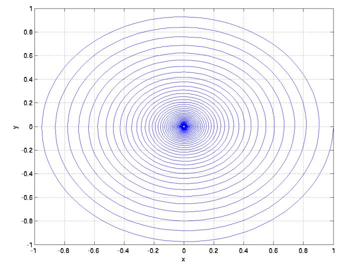
$$\text{If } M_z(0) = -M_0 \text{ then } M_z(t) = M_0(1 - 2e^{-t/T_1})$$

Transverse Component

$$M \equiv M_x + jM_y$$

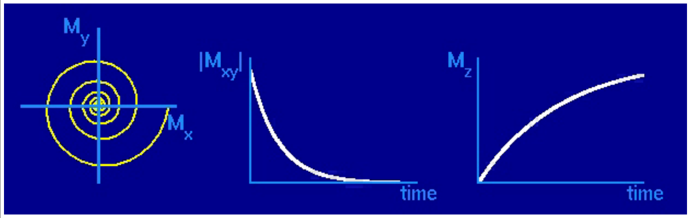
$$dM/dt = d/dt(M_x + iM_y) = -j(\omega_0 + 1/T_2)M$$

$$M(t) = M(0)e^{-j\omega_0 t}e^{-t/T_2}$$



Summary

1) Longitudinal component recovers exponentially.
2) Transverse component precesses and decays exponentially.



Source: http://mrsrl.stanford.edu/~brian/mri-movies/

Summary

1) Longitudinal component recovers exponentially.
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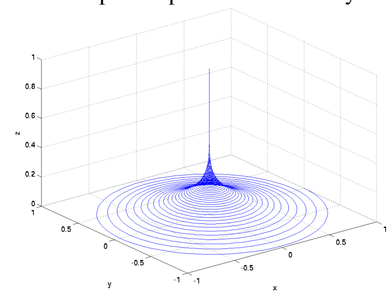


Fact: Can show that $T_2 < T_1$ in order for $|M(t)| \leq M_0$

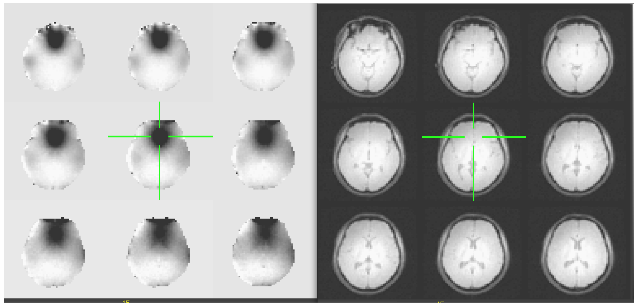
Physically, the mechanisms that give rise to T_1 relaxation also contribute to transverse T_2 relaxation.

Static Inhomogeneities

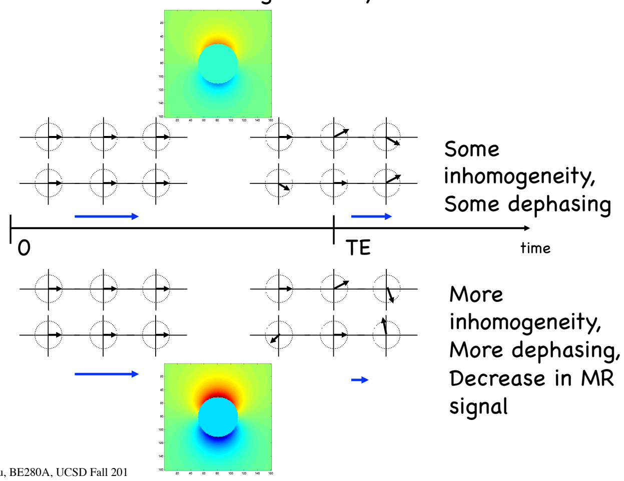
In the ideal situation, the static magnetic field is totally uniform and the reconstructed object is determined solely by the applied gradient fields. In reality, the magnet is not perfect and will not be totally uniform. Part of this can be addressed by additional coils called "shim" coils, and the process of making the field more uniform is called "shimming". In the old days this was done manually, but modern magnets can do this automatically.

In addition to magnet imperfections, most biological samples are inhomogeneous and this will lead to inhomogeneity in the field. This is because, each tissue has different magnetic properties and will distort the field.

Field Inhomogeneities



Signal Decay



Static Inhomogeneities

The spatial nonuniformity in the field can be modeled by adding an additional term to our signal equation.

$$s_r(t) = \int_V M(\vec{r},t)dV$$
$$= \int_x\int_y\int_z M(x,y,z,0)e^{-t/T_2(\vec{r})}e^{-j\omega_0 t}e^{-j\omega_E(\vec{r})t}\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdydz$$

The effect of this nonuniformity is to cause the spins to dephase with time and thus for the signal to decrease more rapidly. To first order this can be modeled as an additional decay term of the form

$$s_r(t) = \int_x\int_y\int_z M(x,y,z,0)e^{-t/T_2(\vec{r})}e^{-t/T_2'(\vec{r})}e^{-j\omega_0 t}\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdydz$$

T_2^* decay

The overall decay has the form.

$$\exp\left(-t/T_2^*(\vec{r})\right)$$

where

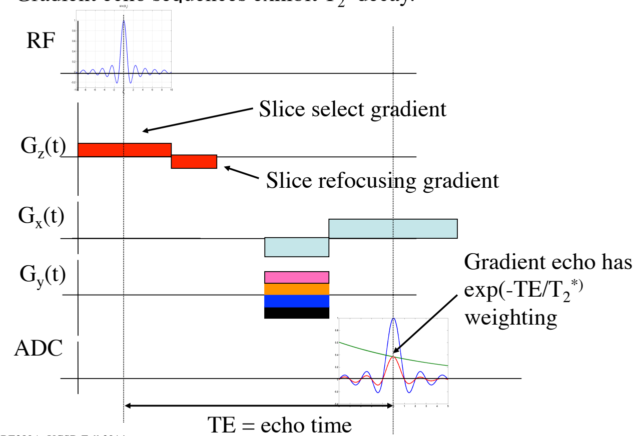
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Due to random motions of spins. Not reversible.

Due to static inhomogeneities. Reversible with a spin-echo sequence.

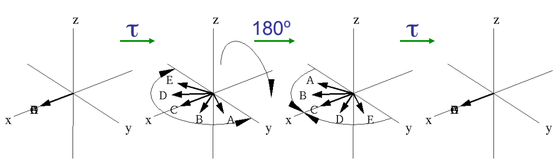
T_2^* decay

Gradient echo sequences exhibit T_2^* decay.



Spin Echo

Discovered by Erwin Hahn in 1950.

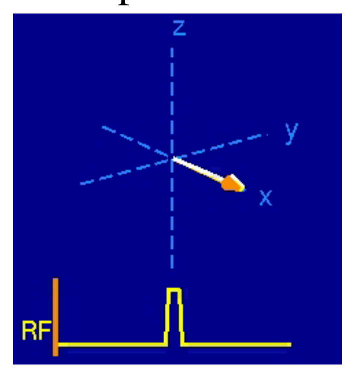


The spin-echo can refocus the dephasing of spins due to static inhomogeneities. However, there will still be T_2 dephasing due to random motion of spins.

There is nothing that nuclear spins will not do for you, as long as you treat them as human beings. Erwin Hahn

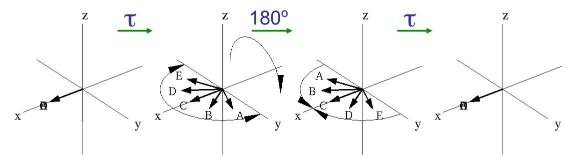
Image: Larry Frank

Spin Echo



Source: http://mrsrl.stanford.edu/~brian/mri-movies/

Spin Echo



Phase at time τ

$$\varphi(\tau) = \int_0^{\tau} -\omega_E(\vec{r})dt = -\omega_E(\vec{r})\tau$$

Phase after 180 pulse

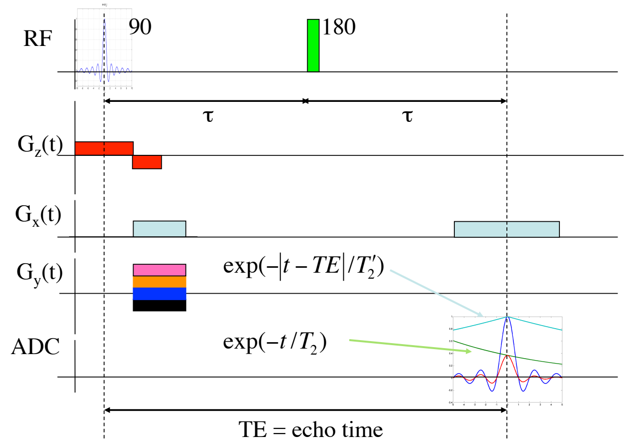
$$\varphi(\tau^+) = \omega_E(\vec{r})\tau$$

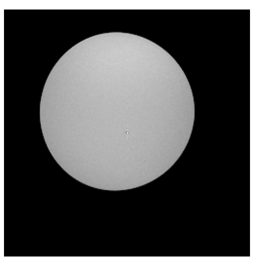
Phase at time 2τ

$$\varphi(2\tau) = -\omega_E(\vec{r})\tau + \omega_E(\vec{r})\tau = 0$$

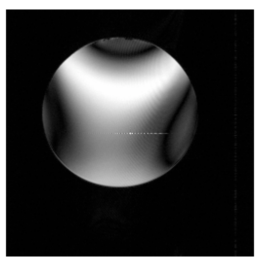
Image: Larry Frank

Spin Echo Pulse Sequence





Spin-echo Image



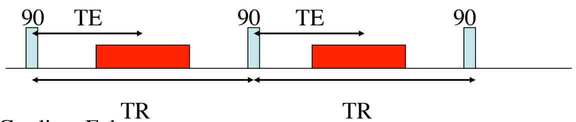
Gradient-Echo Image

http://chickscope.beckman.uiuc.edu/roosts/carl/artifacts.html

Image Contrast

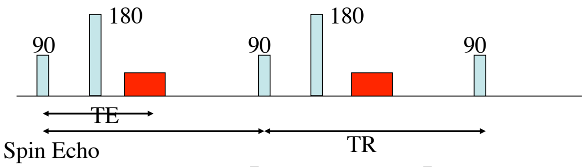
Different tissues exhibit different relaxation rates, T_1, T_2, and T_2^*. In addition different tissues can have different densities of protons. By adjusting the pulse sequence, we can create contrast between the tissues. The most basic way of creating contrast is adjusting the two sequence parameters: TE (echo time) and TR (repetition time).

Saturation Recovery Sequence



Gradient Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2^*(x,y)}$$



Spin Echo

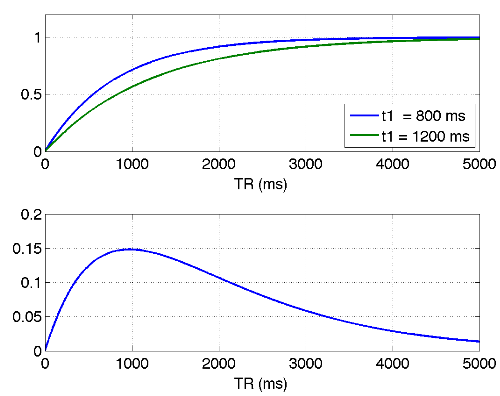
$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

T1-Weighted Scans

Make TE very short compared to either T_2 or T_2^*. The resultant image has both proton and T_1 weighting.

$$I(x,y) \approx \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]$$

T1-Weighted Scans

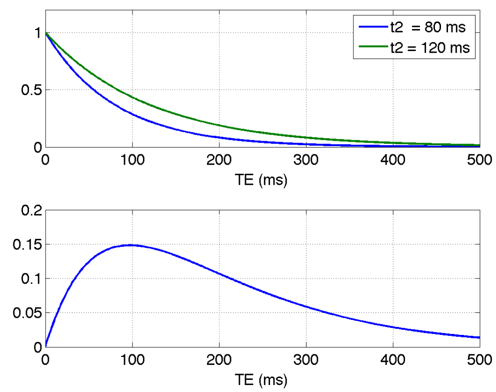


T2-Weighted Scans

Make TR very long compared to T_1 and use a spin-echo pulse sequence. The resultant image has both proton and T_2 weighting.

$$I(x,y) \approx \rho(x,y)e^{-TE/T_2}$$

T2-Weighted Scans

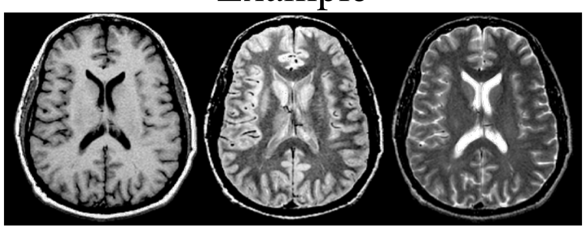


Proton Density Weighted Scans

Make TR very long compared to T_1 and use a very short TE. The resultant image is proton density weighted.

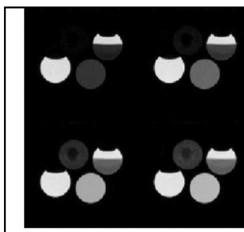
$$I(x,y) \approx \rho(x,y)$$

Example

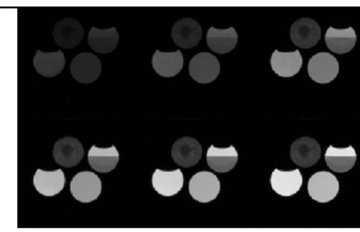


T_1-weighted Density-weighted T_2-weighted

Tissue	Proton Density	T1 (ms)	T2 (ms)
Csf	1.0	4000	2000
Gray	0.85	1350	110
White	0.7	850	80



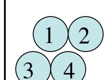
(a) Four images, all obtained with a common TR=5 seconds and TE=90, 50, 20, 15 ms (shown in reading order).



(b) Six images obtained with a common TE=15 ms and TR=500, 1000, 2000, 3000, 4000, 5000 ms (shown in reading order).

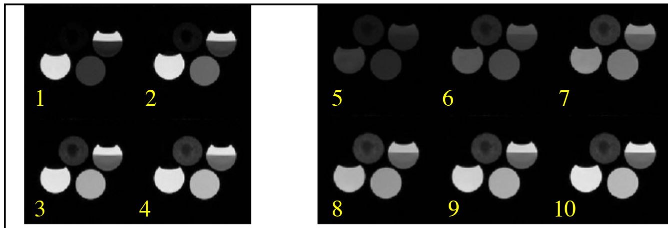
Figure 8: Phantom data which illustrates signal intensity and contrast for bottles filled with jello af varying consistency. Where is T_1 long/short? How long, how short? The same for T_2? Which bottles might be pure water? Which jello is most firm? What pictures are the most T_1-, T_2- and PD-weighted?

Hanson 2009



PollEv.com/be280a

a) Which has the longest T1?
b) Which has the shortest T1?
c) Which has the longest T2?
d) Which has the shortest T2?
e) Which might be pure water?
f) Which has the most firm jello?



(a) Four images, all obtained with a common TR=5 seconds and TE=90, 50, 20, 15 ms (shown in reading order).

(b) Six images obtained with a common TE=15 ms and TR=500, 1000, 2000, 3000, 4000, 5000 ms (shown in reading order).

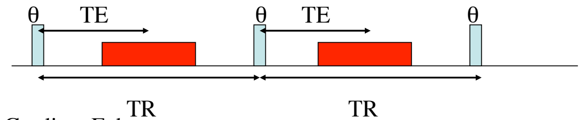
Figure 8: Phantom data which illustrates signal intensity and contrast for bottles filled with jello af varying consistency. Where is T_1 long/short? How long, how short? The same for T_2? Which bottles might be pure water? Which jello is most firm? What pictures are the most T_1-, T_2- and PD-weighted?

Hanson 2009

a) Which is the most T1 weighted?
b) Which is the most T2 weighted?
c) Which is the most PD weighted?

PollEv.com/be280a

FLASH sequence



Gradient Echo

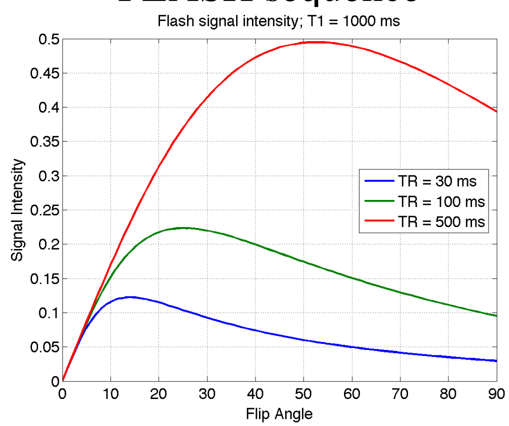
$$I(x,y) = \rho(x,y)\frac{\left[1-e^{-TR/T_1(x,y)}\right]\sin\theta}{\left[1-e^{-TR/T_1(x,y)}\cos\theta\right]}\exp(-TE/T_2^*)$$

Signal intensity is maximized at the Ernst Angle

$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

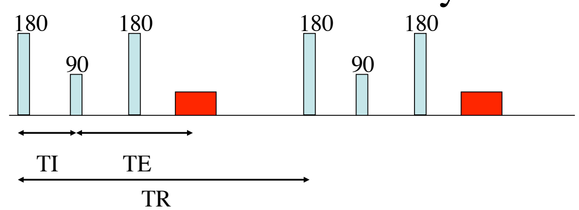
FLASH equation assumes no coherence from shot to shot. In practice this is achieved with RF spoiling.

FLASH sequence



$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

Inversion Recovery

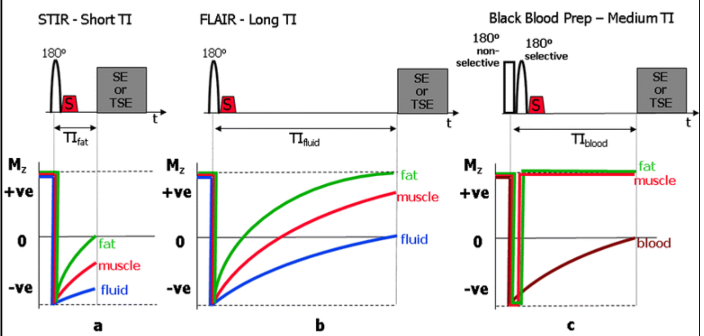


$$I(x,y) = \rho(x,y)\left[1-2e^{-TI/T_1(x,y)}+e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

Intensity is zero when inversion time is

$$TI = -T_1\ln\left[\frac{1+\exp(-TR/T_1)}{2}\right]$$

Inversion Recovery



Biglands *et al*. *Journal of Cardiovascular Magnetic Resonance* 2012 **14**:66 doi:10.1186/1532-429X-14-66

Inversion Recovery