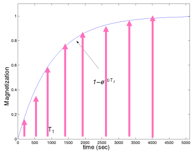
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2014
MRI Lecture 5

Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$

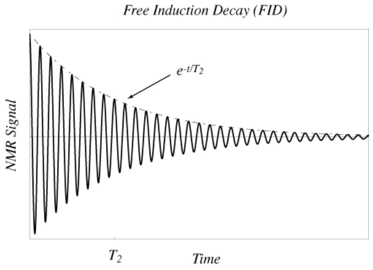


After a 90 degree pulse $M_z(t) = M_0(1 - e^{-t/T_1})$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that T_1 increases with **B**.

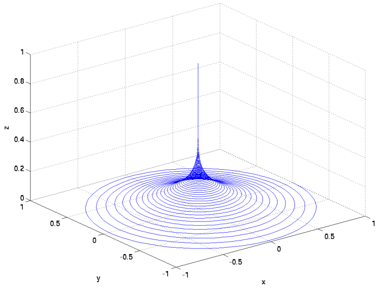
T2 Relaxation



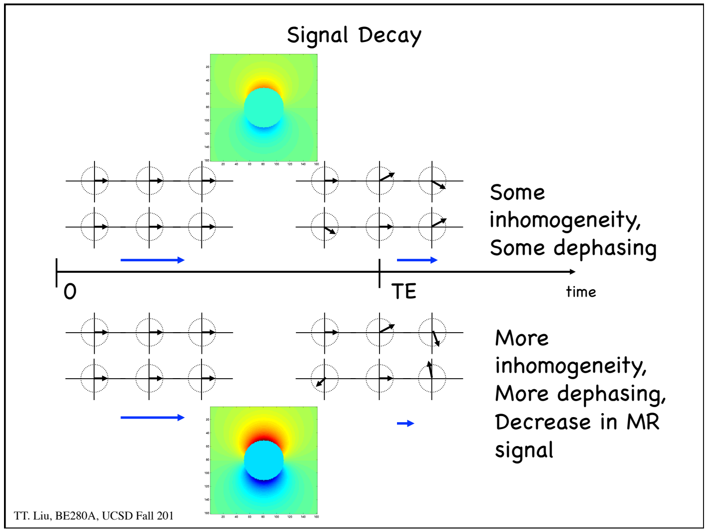
After a 90 degree excitation $M_{xy}(t) = M_0 e^{-t/T_2}$

Summary

1) Longitudinal component recovers exponentially.
2) Transverse component precesses and decays exponentially.



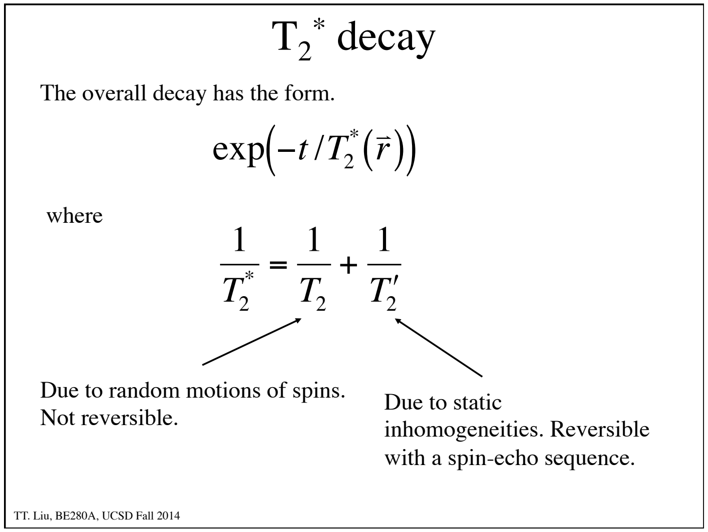
Fact: Can show that $T_2 < T_1$ in order for $|M(t)| \le M_0$
Physically, the mechanisms that give rise to T_1 relaxation also contribute to transverse T_2 relaxation.



Signal Decay

Some inhomogeneity, Some dephasing

More inhomogeneity, More dephasing, Decrease in MR signal



T_2^* decay

The overall decay has the form.

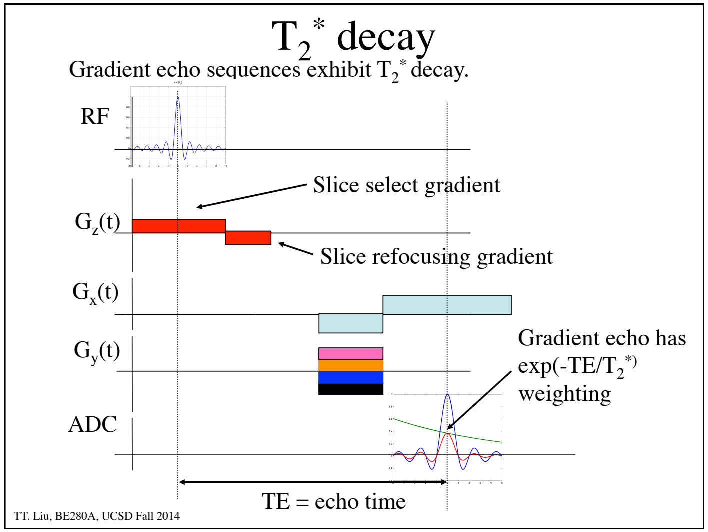
$$\exp\left(-t/T_2^*(\vec{r})\right)$$

where

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Due to random motions of spins. Not reversible.

Due to static inhomogeneities. Reversible with a spin-echo sequence.



T_2^* decay

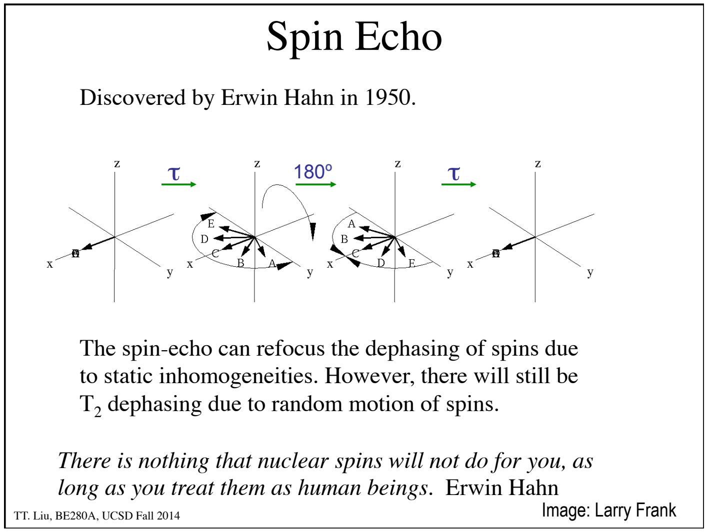
Gradient echo sequences exhibit T_2^* decay.

Slice select gradient

Slice refocusing gradient

Gradient echo has $\exp(-TE/T_2^*)$ weighting

TE = echo time



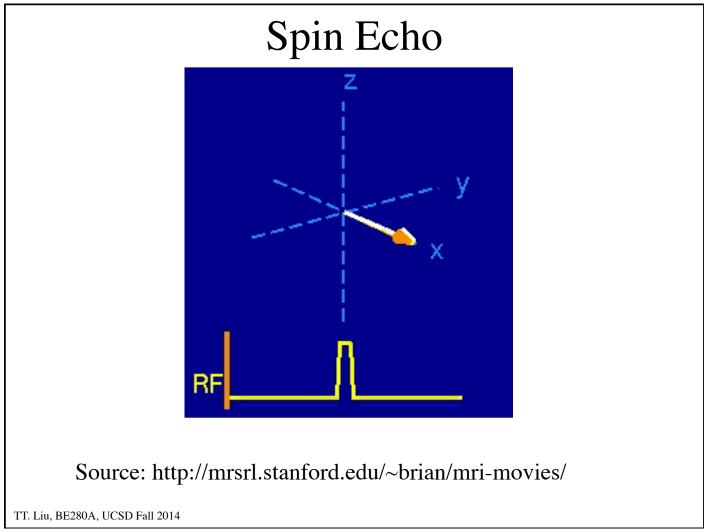
Spin Echo

Discovered by Erwin Hahn in 1950.

The spin-echo can refocus the dephasing of spins due to static inhomogeneities. However, there will still be T_2 dephasing due to random motion of spins.

There is nothing that nuclear spins will not do for you, as long as you treat them as human beings. Erwin Hahn

Image: Larry Frank



Spin Echo

Source: http://mrsrl.stanford.edu/~brian/mri-movies/

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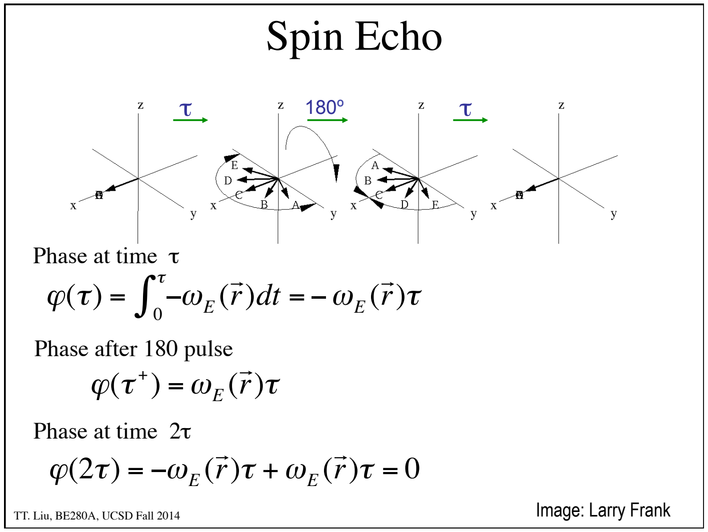

Spin Echo

Phase at time τ

$$\varphi(\tau) = \int_0^\tau -\omega_E(\vec{r})dt = -\,\omega_E(\vec{r})\tau$$

Phase after 180 pulse

$$\varphi(\tau^+) = \omega_E(\vec{r})\tau$$

Phase at time 2τ

$$\varphi(2\tau) = -\omega_E(\vec{r})\tau + \omega_E(\vec{r})\tau = 0$$

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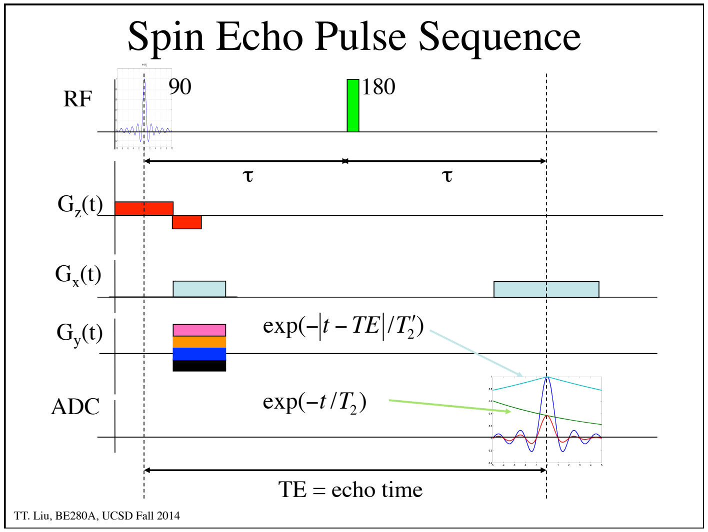
Image: Larry Frank



Spin Echo Pulse Sequence

$\exp(-|t-TE|/T_2')$

$\exp(-t/T_2)$

TE = echo time

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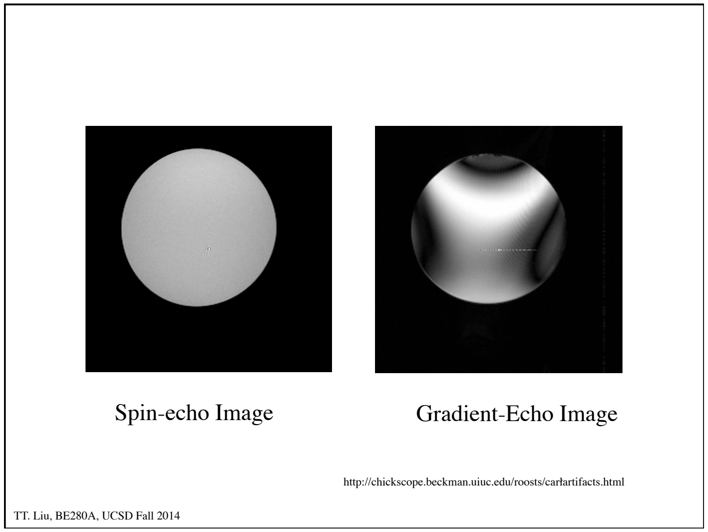

Spin-echo Image

Gradient-Echo Image

http://chickscope.beckman.uiuc.edu/roosts/carl/artifacts.html

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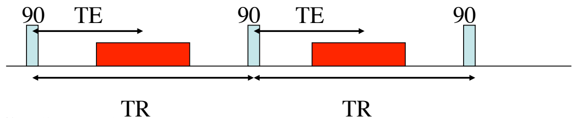
Image Contrast

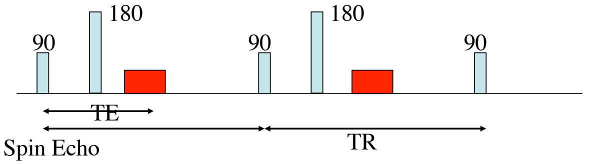
Different tissues exhibit different relaxation rates, T_1, T_2, and T_2^*. In addition different tissues can have different densities of protons. By adjusting the pulse sequence, we can create contrast between the tissues. The most basic way of creating contrast is adjusting the two sequence parameters: TE (echo time) and TR (repetition time).

Saturation Recovery Sequence



Gradient Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right] e^{-TE/T_2^*(x,y)}$$



Spin Echo

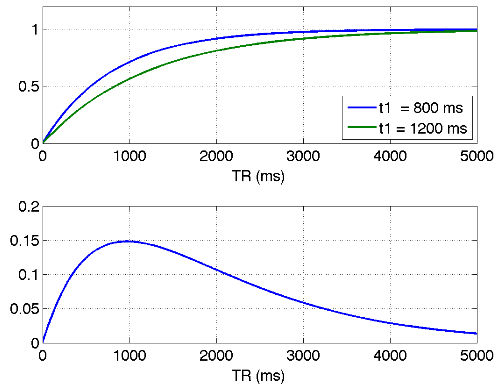
$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right] e^{-TE/T_2(x,y)}$$

T1-Weighted Scans

Make TE very short compared to either T_2 or T_2^*. The resultant image has both proton and T_1 weighting.

$$I(x,y) \approx \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]$$

T1-Weighted Scans

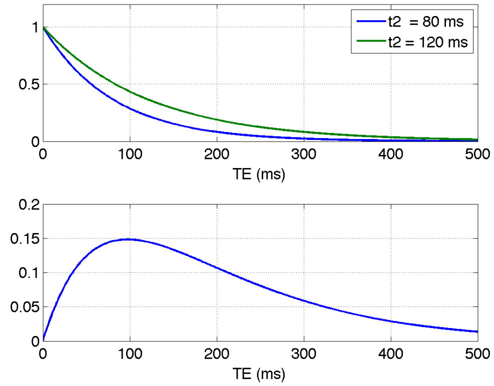


T2-Weighted Scans

Make TR very long compared to T_1 and use a spin-echo pulse sequence. The resultant image has both proton and T_2 weighting.

$$I(x,y) \approx \rho(x,y)e^{-TE/T_2}$$

T2-Weighted Scans

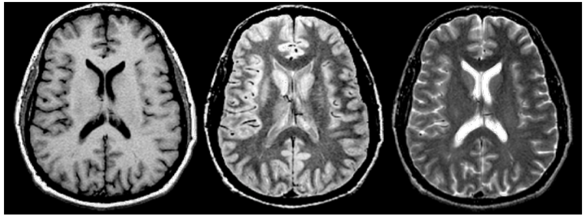


Proton Density Weighted Scans

Make TR very long compared to T_1 and use a very short TE. The resultant image is proton density weighted.

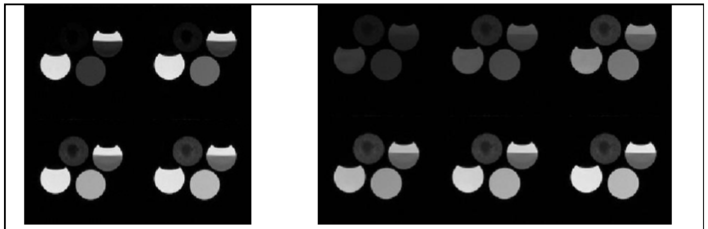
$$I(x,y) \approx \rho(x,y)$$

Example



T_1-weighted Density-weighted T_2-weighted

Tissue	Proton Density	T1 (ms)	T2 (ms)
Csf	1.0	4000	2000
Gray	0.85	1350	110
White	0.7	850	80

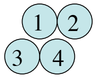


(a) Four images, all obtained with a common TR=5 seconds and TE=90, 50, 20, 15 ms (shown in reading order).

(b) Six images obtained with a common TE=15 ms and TR=500, 1000, 2000, 3000, 4000, 5000 ms (shown in reading order).

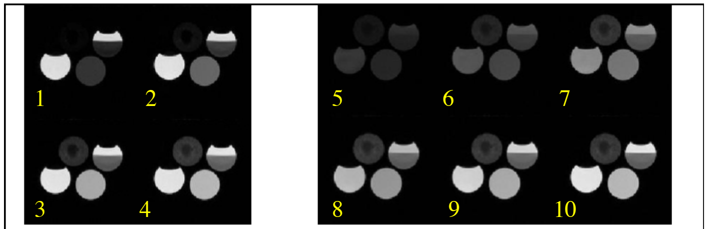
Figure 8: Phantom data which illustrates signal intensity and contrast for bottles filled with jello af varying consistency. Where is T_1 long/short? How long, how short? The same for T_2? Which bottles might be pure water? Which jello is most firm? What pictures are the most T_1-, T_2- and PD-weighted?

Hanson 2009



a) Which has the longest T1?
b) Which has the shortest T1?
c) Which has the longest T2?
d) Which has the shortest T2?
e) Which might be pure water?
f) Which has the most firm jello?

PollEv.com/be280a



(a) Four images, all obtained with a common TR=5 seconds and TE=90, 50, 20, 15 ms (shown in reading order).

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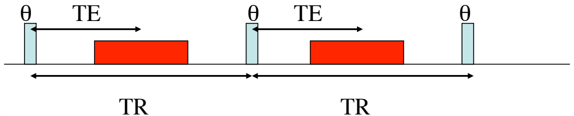
Figure 8: Phantom data which illustrates signal intensity and contrast for bottles filled with jello af varying consistency. Where is T_1 long/short? How long, how short? The same for T_2? Which bottles might be pure water? Which jello is most firm? What pictures are the most T_1-, T_2- and PD-weighted?

Hanson 2009

a) Which is the most T1 weighted?
b) Which is the most T2 weighted?
c) Which is the most PD weighted?

PollEv.com/be280a

FLASH sequence



Gradient Echo

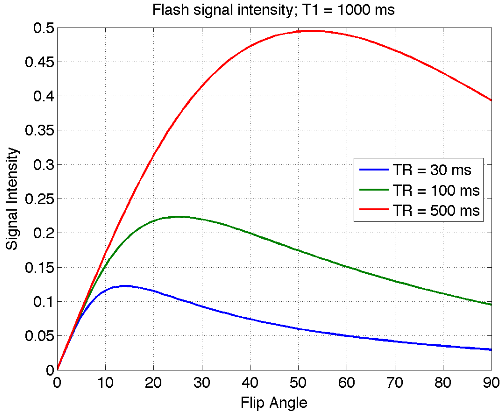
$$I(x,y)=\rho(x,y)\frac{\left[1-e^{-TR/T_1(x,y)}\right]\sin\theta}{\left[1-e^{-TR/T_1(x,y)}\cos\theta\right]}\exp(-TE/T_2^*)$$

Signal intensity is maximized at the Ernst Angle

$$\theta_E=\cos^{-1}\left(\exp(-TR/T_1)\right)$$

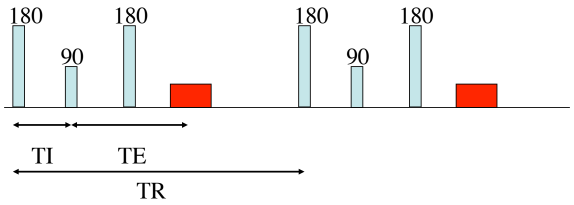
FLASH equation assumes no coherence from shot to shot. In practice this is achieved with RF spoiling.

FLASH sequence



$$\theta_E=\cos^{-1}\left(\exp(-TR/T_1)\right)$$

Inversion Recovery

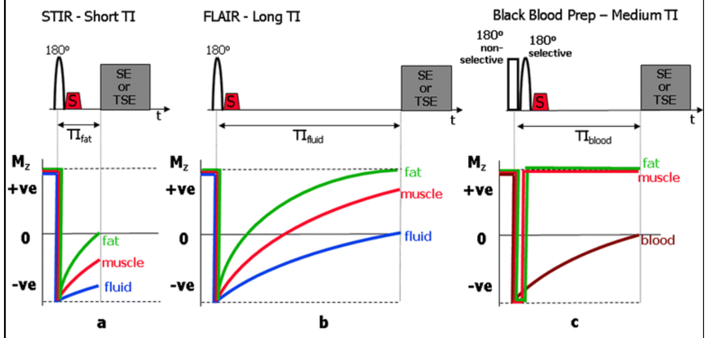


$$I(x,y)=\rho(x,y)\left[1-2e^{-TI/T_1(x,y)}+e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

Intensity is zero when inversion time is

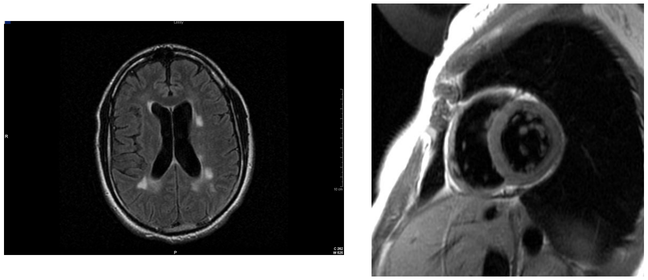
$$TI=-T_1\ln\left[\frac{1+\exp(-TR/T_1)}{2}\right]$$

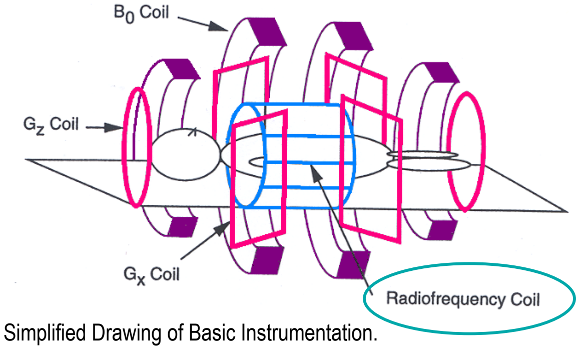
Inversion Recovery



Biglands *et al*. *Journal of Cardiovascular Magnetic Resonance* 2012 **14**:66 doi:10.1186/1532-429X-14-66

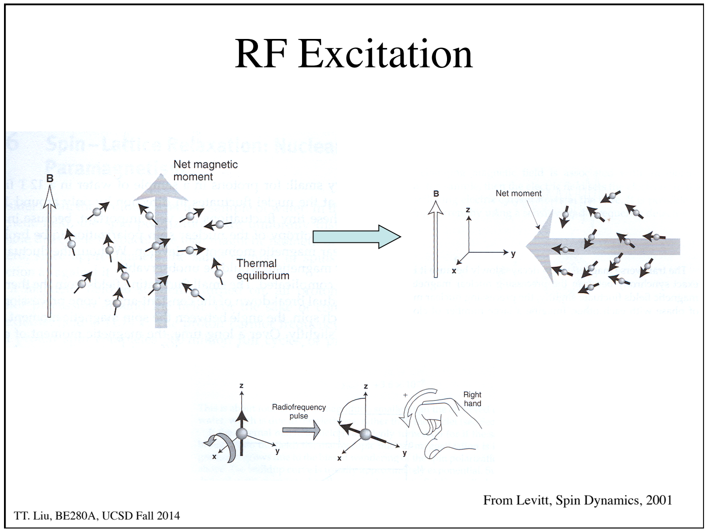
Inversion Recovery





Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15



RF Excitation

From Levitt, Spin Dynamics, 2001

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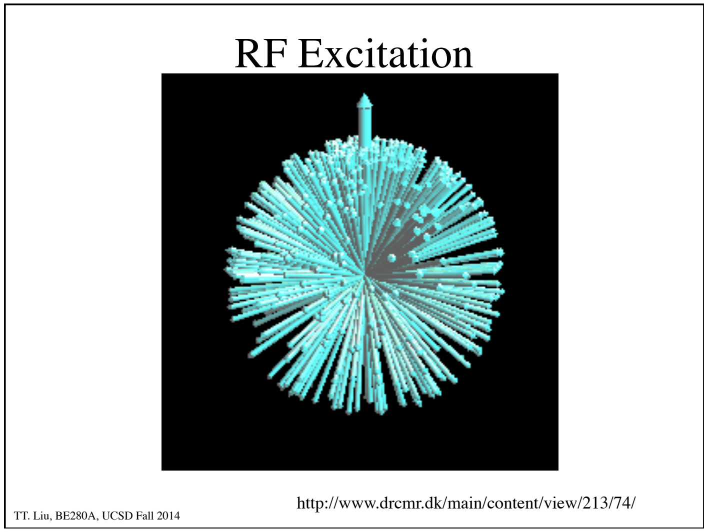

RF Excitation

http://www.drcmr.dk/main/content/view/213/74/

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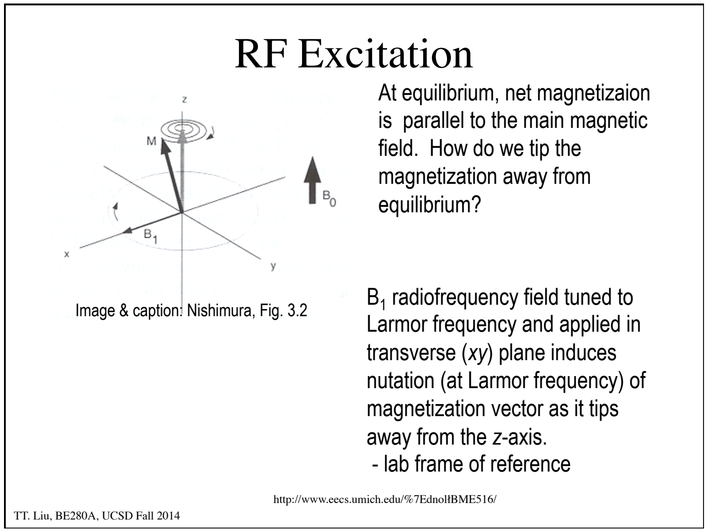

RF Excitation

Image & caption: Nishimura, Fig. 3.2

At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
 - lab frame of reference

http://www.eecs.umich.edu/%7Ednoll/BME516/

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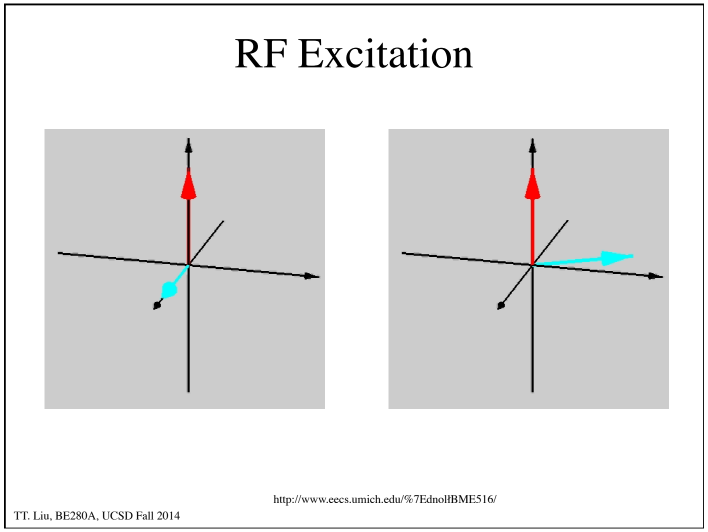

RF Excitation

http://www.eecs.umich.edu/%7Ednoll/BME516/

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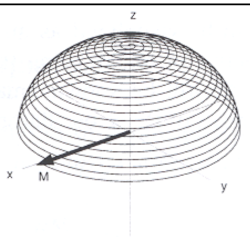
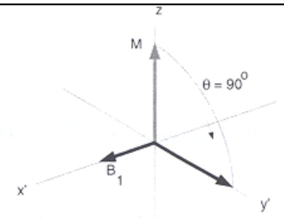
Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

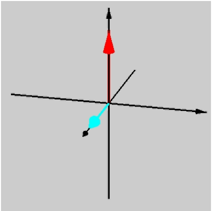


a) Laboratory frame behavior of **M**

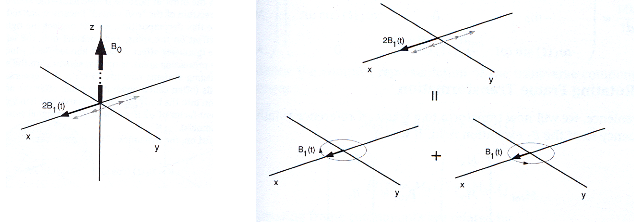
Images & caption: Nishimura, Fig. 3.3



b) Rotating frame behavior of **M**



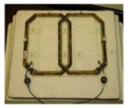
http://www.eecs.umich.edu/%7EdnollBME516/



$$\mathbf{B_1}(t) = 2B_1(t)\cos(\omega t)\mathbf{i}$$

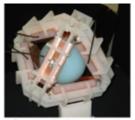
$$= B_1(t)\left(\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}\right) + B_1(t)\left(\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\right)$$

Nishimura 1996



2-channel phased array for cardio MRI (Rx)

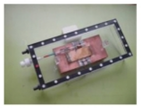
Breast coil with detached coupling box



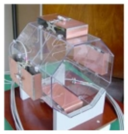
Octahedral head coil with 12 TxRx channels

Cable head coil (TxRx) for 129Xe (34.66 MHz)

"X coil", 125 MHz (Rx)



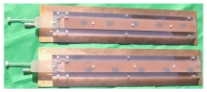
Tunable CSA hyperthermia module (97 MHz, Tx)



4-channel CSA array (TxRx)

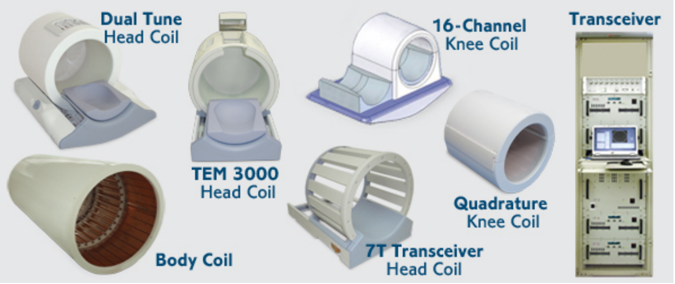
125-MHz CSA

Lung coil (TxRx) with differential transmitter for 129Xe (34.66 MHz)



2-channel strip-line array (TxRx)

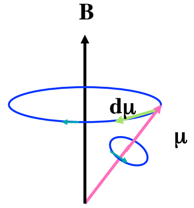
http://www.berlin.ptb.de/en/org/8/81/811/ResearchTopics/CoilDevelopment.html



http://www.mrinstruments.com/

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma \mathbf{B}$$

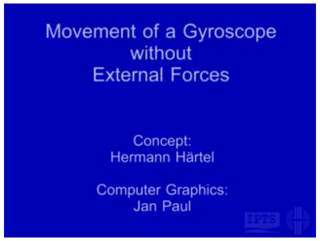


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma B$$

This is known as the **Larmor** frequency.



Movement of a Gyroscope
without
External Forces

Concept:
Hermann Härtel

Computer Graphics:
Jan Paul

http://www.astrophysik.uni-kiel.de/~hhaertełmpg_e/gyros_free.mpg

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Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

Note: we use the RF frequency to define the rotating frame. If this RF frequency is on-resonance, then the main B0 field doesn't cause any precession in the rotating frame. However, if the RF frequency is off-resonance, then there will be a net precession in the rotating frame that is give by the difference between the RF frequency and the local Larmor frequency.

$$Let \ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

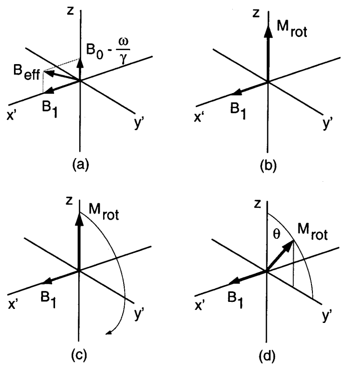
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

$$\text{If } \omega = \omega_0$$

$$= \gamma B_0$$

$$\text{Then } \mathbf{B}_{eff} = B_1(t)\mathbf{i}$$



Flip angle

$$\theta = \int_0^{\tau} \omega_1(s)\,ds$$

where

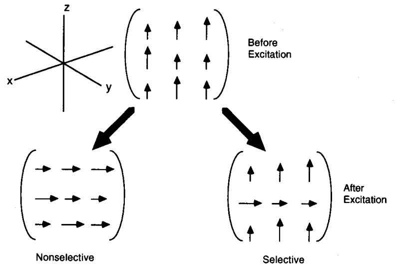
$$\omega_1(t) = \gamma B_1(t)$$

Nishimura 1996

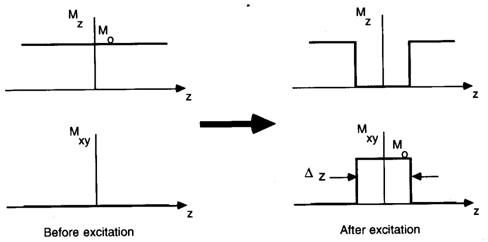
Example

$\tau = 400\ \mu\text{sec};\ \theta=\pi/2$

$$B_1 = \frac{\theta}{\gamma\tau} = \frac{\pi/2}{2\pi(4257 Hz/G)(400e-6)} = 0.1468\ G$$



Nishimura 1996



Nishimura 1996

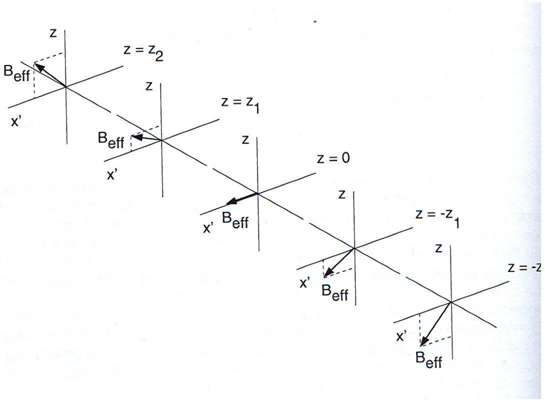
$$\text{Let } \mathbf{B}_{rot} = B_1(t)\mathbf{i} + (B_0 + \gamma G_z z)\mathbf{k}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

$$\text{If } \omega = \omega_0$$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + (\gamma G_z z)\mathbf{k}$$



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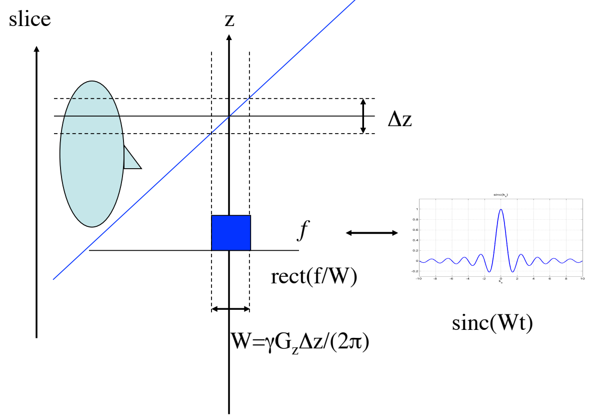
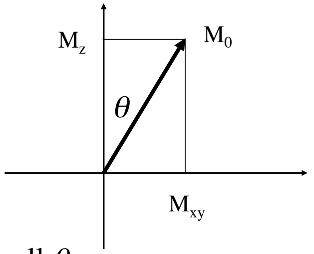
Slice Selection



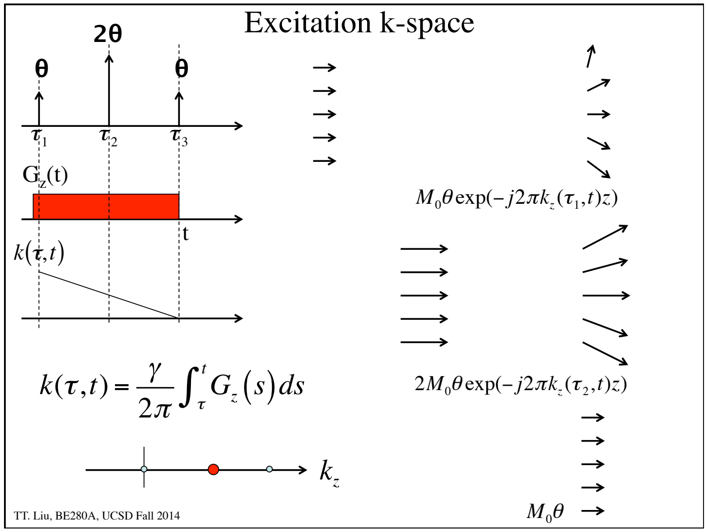
Small Tip Angle Approximation



For small θ

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$



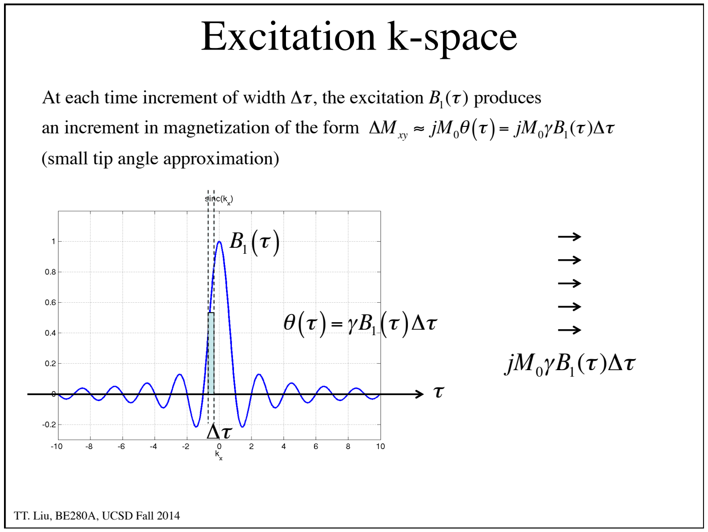
Excitation k-space

$M_0\theta\exp(-j2\pi k_z(\tau_1,t)z)$

$2M_0\theta\exp(-j2\pi k_z(\tau_2,t)z)$

$M_0\theta$

$$k(\tau,t)=\frac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$$

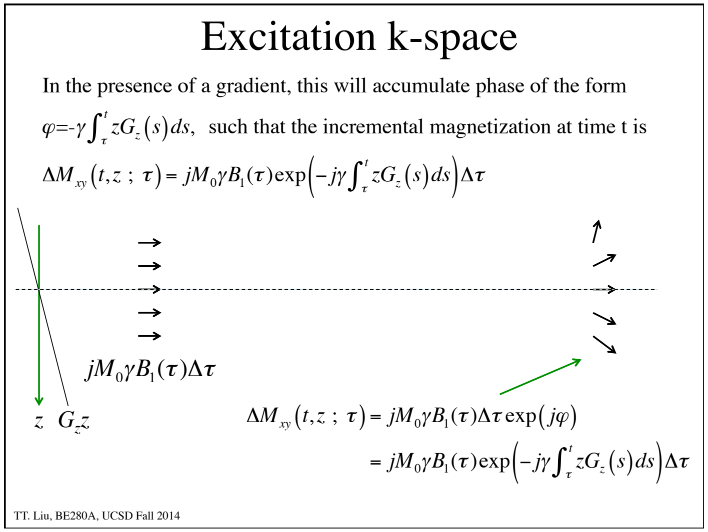


Excitation k-space

At each time increment of width $\Delta\tau$, the excitation $B_1(\tau)$ produces an increment in magnetization of the form $\Delta M_{xy}\approx jM_0\theta(\tau)=jM_0\gamma B_1(\tau)\Delta\tau$ (small tip angle approximation)

$\theta(\tau)=\gamma B_1(\tau)\Delta\tau$

$jM_0\gamma B_1(\tau)\Delta\tau$



Excitation k-space

In the presence of a gradient, this will accumulate phase of the form $\varphi = -\gamma\int_\tau^t zG_z(s)\,ds$, such that the incremental magnetization at time t is

$$\Delta M_{xy}(t,z\ ;\ \tau)=jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)\,ds\right)\Delta\tau$$

$jM_0\gamma B_1(\tau)\Delta\tau$

$$\begin{aligned}\Delta M_{xy}(t,z\ ;\ \tau)&=jM_0\gamma B_1(\tau)\Delta\tau\exp(j\varphi)\\&=jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)\,ds\right)\Delta\tau\end{aligned}$$

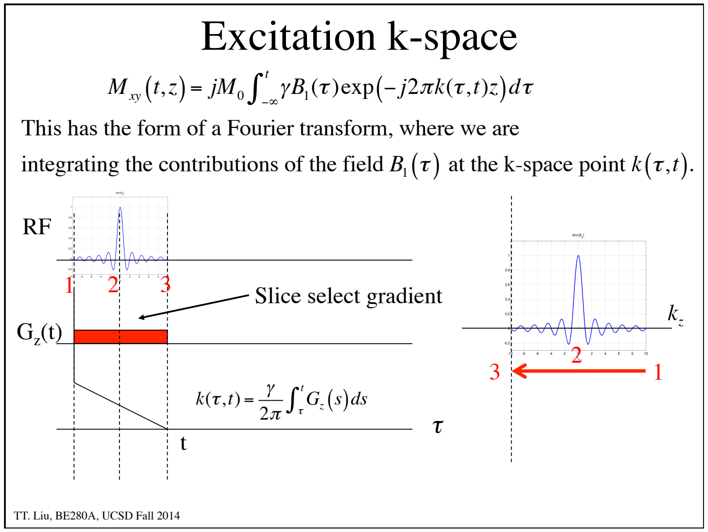
Excitation k-space

Integrating over all time increments, we obtain

$$\begin{aligned}M_{xy}(t,z)&=jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j\gamma\int_\tau^t zG_z(s)\,ds\right)d\tau\\&=jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau\end{aligned}$$

where $k(\tau,t)=\dfrac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.

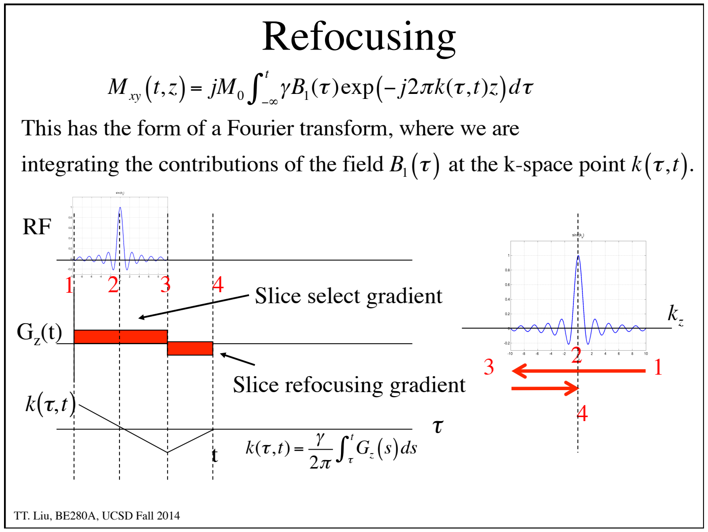


Excitation k-space

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.

$$k(\tau,t)=\frac{\gamma}{2\pi}\int_{\tau}^{t}G_z(s)ds$$

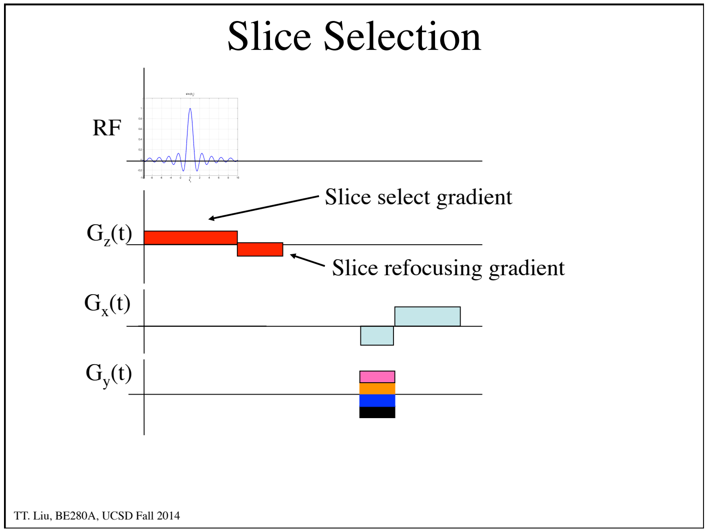


Refocusing

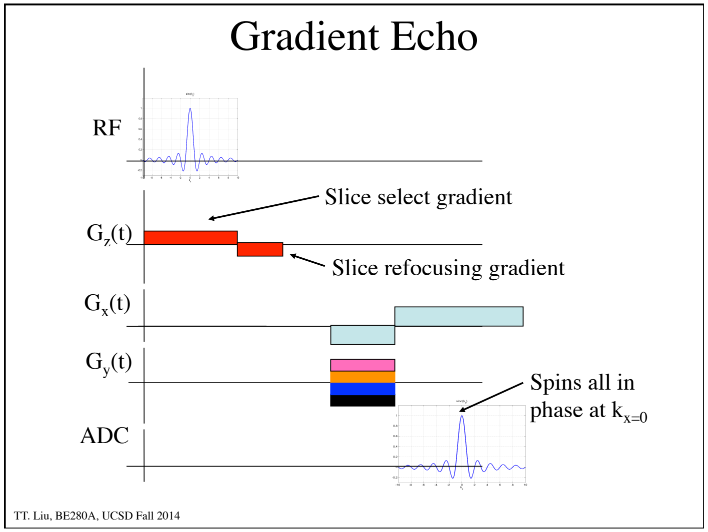
$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j2\pi k(\tau,t)z\right)d\tau$$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.

$$k(\tau,t)=\frac{\gamma}{2\pi}\int_{\tau}^{t}G_z(s)ds$$

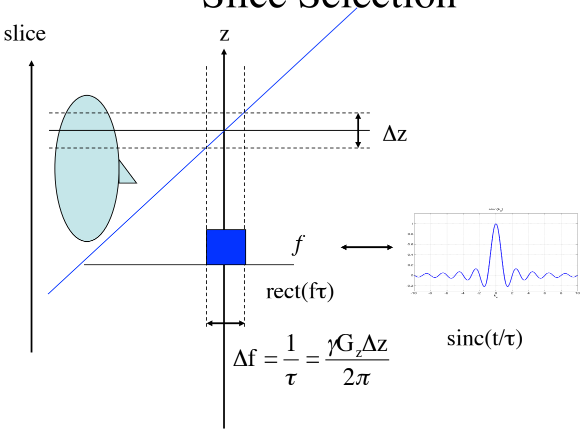


Slice Selection

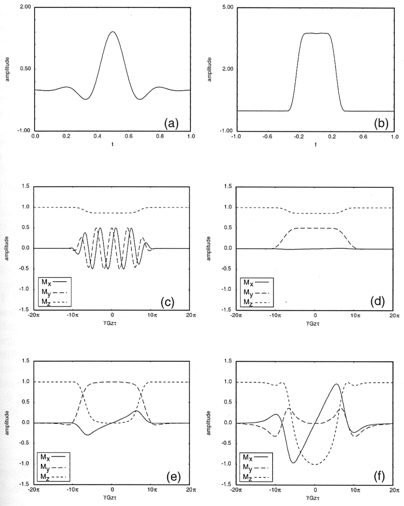


Gradient Echo

Slice Selection



$$\Delta f = \frac{1}{\tau} = \frac{\gamma G_z \Delta z}{2\pi}$$



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