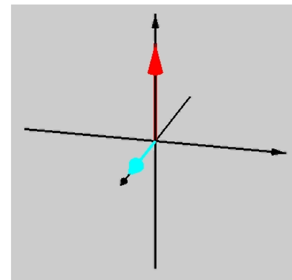
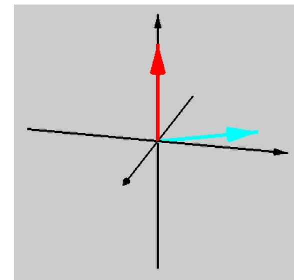
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2014
MRI Lecture 6



# RF Excitation





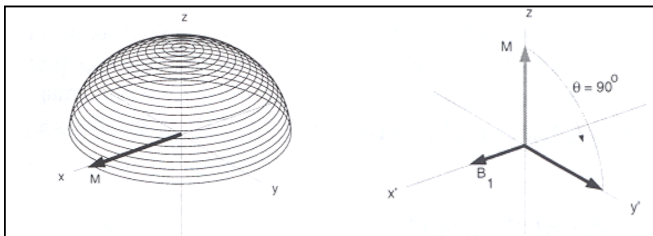
http://www.eecs.umich.edu/%7Ednoll/BME516/



# Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

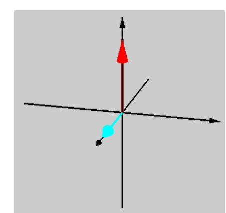





a) Laboratory frame behavior of **M** b) Rotating frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3

Note that in the rotating frame, there is no precession about the z-axis. From the point of view of the precession of the spins, this is equivalent to the z-component of the field being set equal to zero.





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## Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

Note: we use the RF frequency to define the rotating frame. If this RF frequency is on-resonance, then the main B0 field doesn't cause any precession in the rotating frame. However, if the RF frequency is off-resonance, then there will be a net precession in the rotating frame that is give by the difference between the RF frequency and the local Larmor frequency.



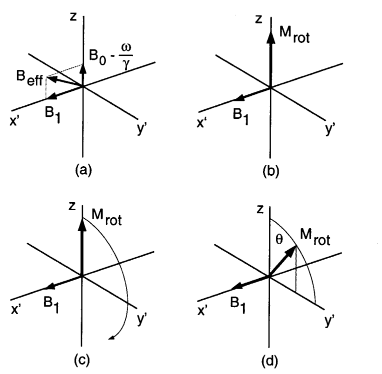
$$Let \ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$
$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

$$\text{If } \omega = \omega_0 = \gamma B_0$$

$$\text{Then } \mathbf{B}_{eff} = B_1(t)\mathbf{i}$$






Flip angle

$$\theta = \int_0^{\tau} \omega_1(s)ds$$

*where*

$$\omega_1(t) = \gamma B_1(t)$$

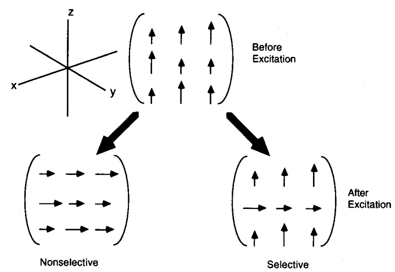


Nishimura 1996

Example

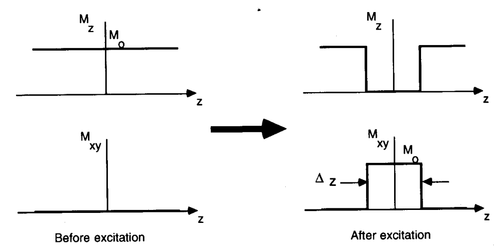
$\tau = 400 \ \mu\text{sec}; \ \theta=\pi/2$

$$B_1 = \frac{\theta}{\gamma\tau} = \frac{\pi/2}{2\pi(4257Hz/G)(400e-6)} = 0.1468 \ G$$






Nishimura 1996






Nishimura 1996

$$Let\ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z\right)\mathbf{k}$$

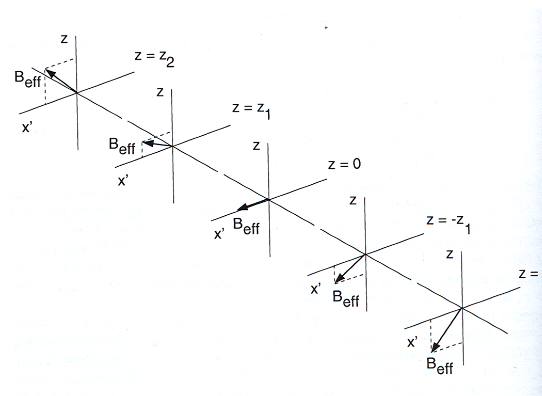
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

If $\omega = \omega_0$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + \left(\gamma G_z z\right)\mathbf{k}$$

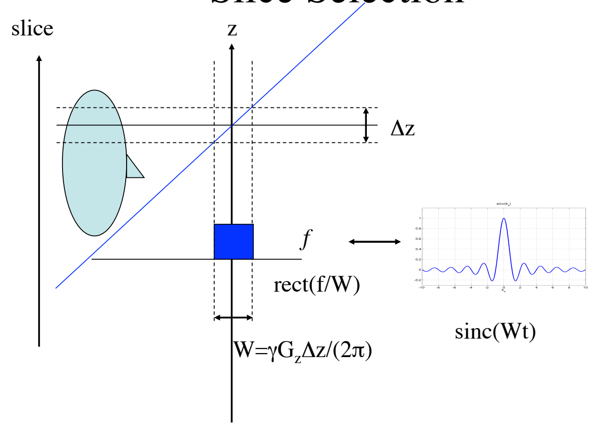







Nishimura 1996

# Slice Selection

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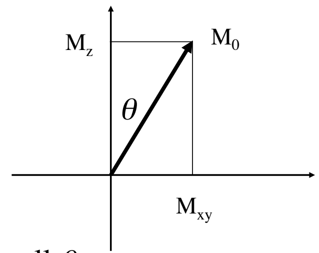
## Small Tip Angle Approximation




*For* small $\theta$

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$

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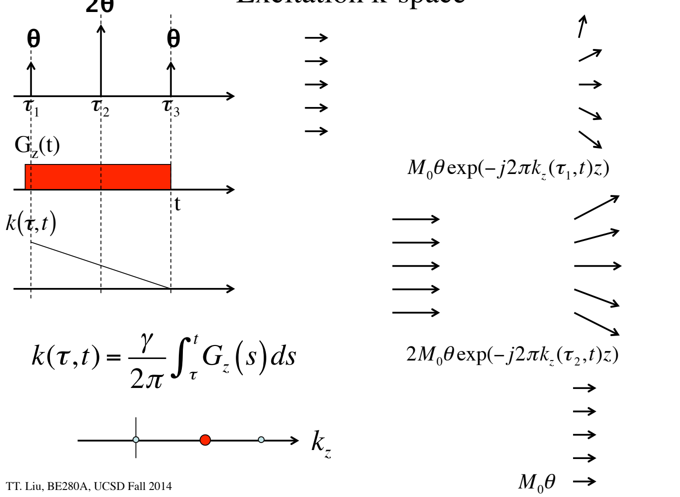
## Excitation k-space




$$k(\tau,t) = \frac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$$

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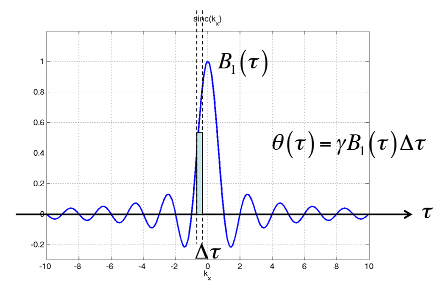
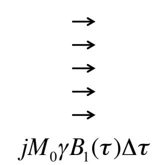
# Excitation k-space

At each time increment of width $\Delta\tau$, the excitation $B_1(\tau)$ produces an increment in magnetization of the form $\Delta M_{xy} \approx jM_0\theta(\tau) = jM_0\gamma B_1(\tau)\Delta\tau$ (small tip angle approximation)

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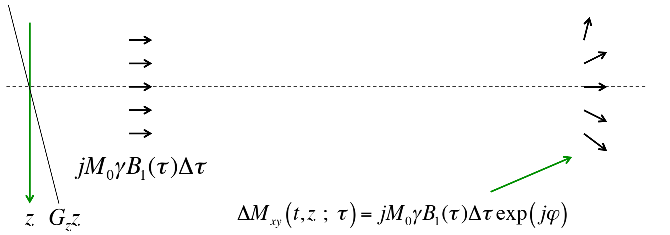
# Excitation k-space

In the presence of a gradient, this will accumulate phase of the form

$\varphi = -\gamma \int_{\tau}^{t} z G_z(s)\,ds$, such that the incremental magnetization at time t is

$$\Delta M_{xy}(t,z\,;\,\tau) = jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t} zG_z(s)\,ds\right)\Delta\tau$$




$$\Delta M_{xy}(t,z\,;\,\tau) = jM_0\gamma B_1(\tau)\Delta\tau\exp(j\varphi)$$
$$= jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t} zG_z(s)\,ds\right)\Delta\tau$$



# Excitation k-space

Integrating over all time increments, we obtain

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t} zG_z(s)\,ds\right)d\tau$$
$$= jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp(-j2\pi k(\tau,t)z)\,d\tau$$

where $k(\tau,t) = \frac{\gamma}{2\pi}\int_{\tau}^{t} G_z(s)\,ds$

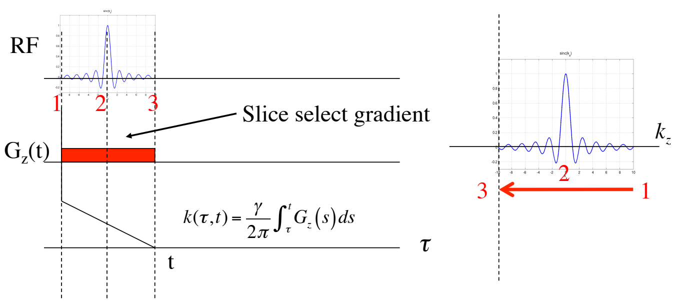
This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.



# Excitation k-space

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp(-j2\pi k(\tau,t)z)\,d\tau$$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.

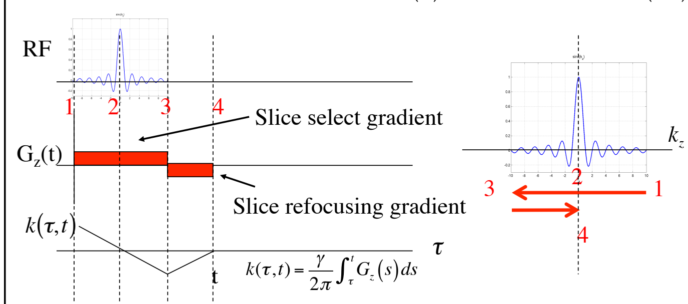




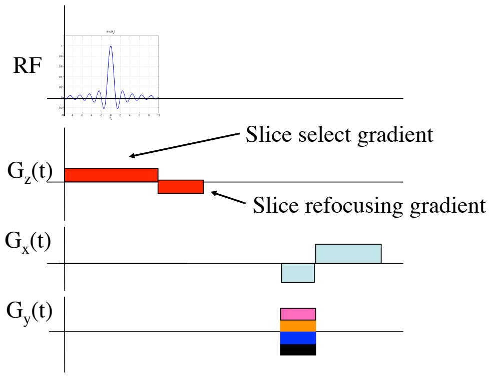

# Refocusing

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp(-j2\pi k(\tau,t)z)\,d\tau$$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.



# Slice Selection

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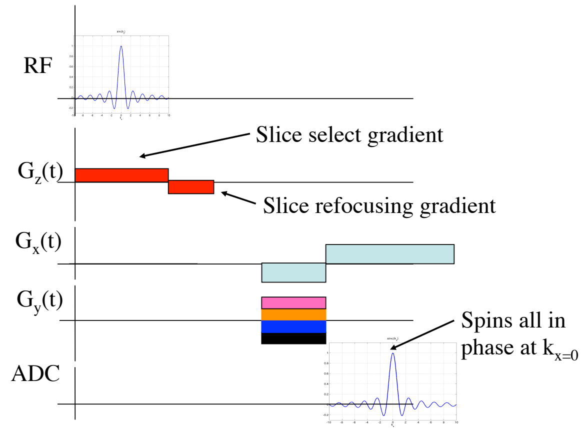
# Gradient Echo

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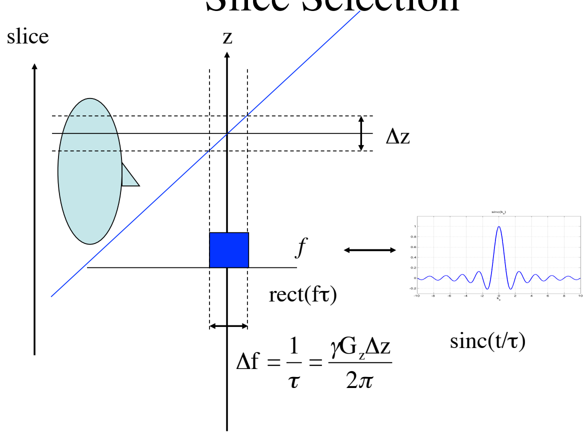
# Slice Selection




$$\Delta f = \frac{1}{\tau} = \frac{\gamma G_z \Delta z}{2\pi}$$

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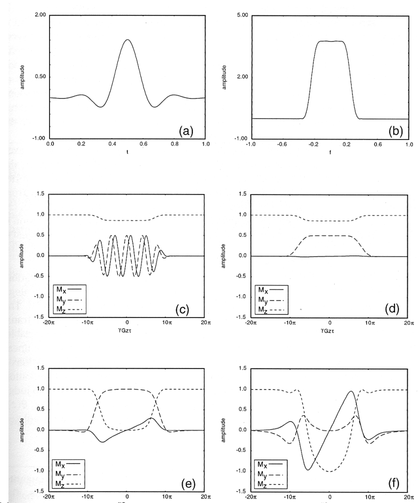


Nishimura 1996

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Example

$M_{xy}(x) = M_0 \cos(4\pi x)$

$F\left(M_{xy}(x)\right) = \frac{M_0}{2}\left(\delta(k_x - 2) + \delta(k_x + 2)\right)$

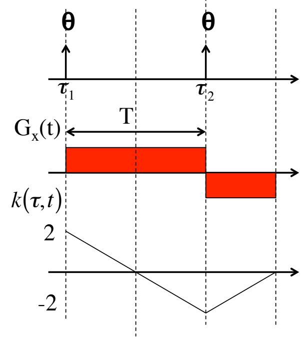
$g_{\max} = 4\ G/cm$

$\frac{\gamma}{2\pi} g_{\max} T = 4\ \text{cm}^{-1};\ \text{T} = 235\ \mu\text{sec}$

with small tip angle approximation --> $\theta = \frac{1}{2}$

Compare with $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6} = 0.5236$

*Question*: Should we use $\theta = \frac{\pi}{4}$ instead?




Exercise: Sketch the quiver diagrams corresponding to the contributions of the two RF pulses and show that they produce the desired pattern.



# Multi-dimensional Excitation k-space

$$M_{xy}(t,\mathbf{r}) = jM_0 \int_{-\infty}^{t} \omega_1(\tau) \exp\left(-j\gamma \int_{\tau}^{t} \mathbf{G}(s) \cdot \mathbf{r}\, ds\right) d\tau$$

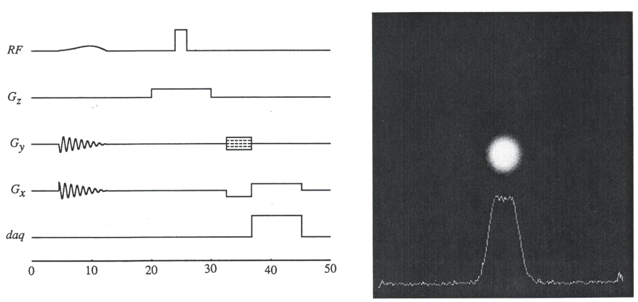
$$= jM_0 \int_{-\infty}^{t} \omega_1(\tau) \exp\left(j2\pi \mathbf{k}(\tau) \cdot \mathbf{r}\right) d\tau$$

where $\mathbf{k}(\tau) = -\frac{\gamma}{2\pi} \int_{\tau}^{t} \mathbf{G}(t')\, dt'$

Pauly et al 1989



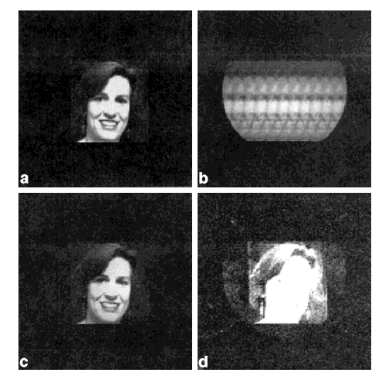
# Excitation k-space




Pauly et al 1989

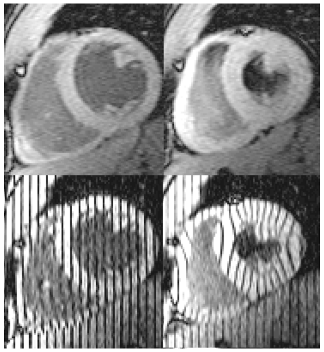


# Excitation k-space




Panych MRM 1999

# Cardiac Tagging





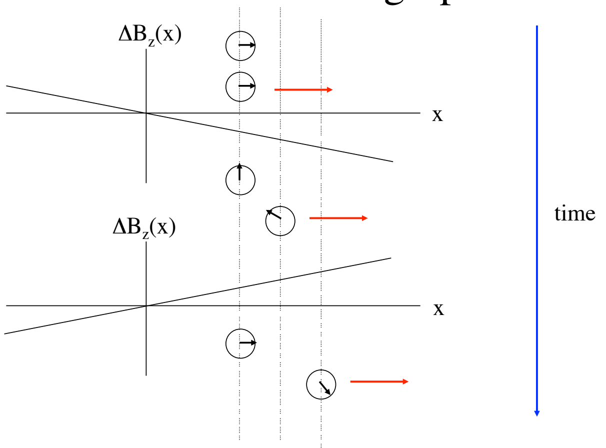
# Moving Spins (preview)

So far we have assumed that the spins are not moving (aside from thermal motion giving rise to relaxation), and contrast has been based upon $T_1$, $T_2$, and proton density. We were able to achieve different contrasts by adjusting the appropriate pulse sequence parameters.

Biological samples are filled with moving spins, and we can also use MRI to image the movement. Examples: blood flow, diffusion of water in the white matter tracts. In addition, we can also sometimes induce motion into the object to image its mechanical properties, e.g. imaging of stress and strain with MR elastography.



# Phase of Moving Spin






# Phase of a Moving Spin

$$
\begin{aligned}
\varphi(t) &= -\int_0^t \Delta\omega(\tau)d\tau \\
&= -\int_0^t \gamma \Delta B(\tau)d\tau \\
&= -\int_0^t \gamma \vec{G}(\tau)\cdot \vec{r}(\tau)d\tau \\
&= -\gamma \int_0^t \left[G_x(\tau)x(\tau) + G_y(\tau)y(\tau) + G_z(\tau)z(\tau)\right]d\tau
\end{aligned}
$$

# Phase of Moving Spin

Consider motion along the x-axis

$$x(t) = x_0 + vt + \frac{1}{2}at^2$$

$$\begin{aligned}
\varphi(t) &= -\gamma \int_0^t G_x(\tau)x(\tau)d\tau \\
&= -\gamma \int_0^t G_x(\tau)\left[x_0 + v\tau + \frac{1}{2}a\tau^2\right]d\tau \\
&= -\gamma\left[x_0 \int_0^t G_x(\tau)d\tau + v\int_0^t G_x(\tau)\tau d\tau + \frac{a}{2}\int_0^t G_x(\tau)\tau^2 d\tau\right] \\
&= -\gamma\left[x_0 M_0 + vM_1 + \frac{a}{2}M_2\right]
\end{aligned}$$



# Phase of Moving Spin

$$\varphi(t) = -\gamma\left[x_0 M_0 + vM_1 + \frac{a}{2}M_2\right]$$

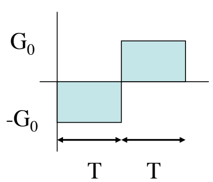
$M_0 = \int_0^t G_x(\tau)d\tau$ Zeroth order moment

$M_1 = \int_0^t G_x(\tau)\tau d\tau$ First order moment

$M_2 = \int_0^t G_x(\tau)\tau^2 d\tau$ Second order moment



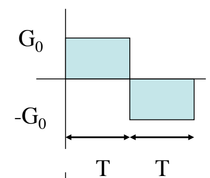
# Flow Moment Example



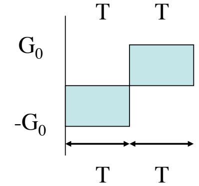

$$\begin{aligned}
M_0 &= \int_0^t G_x(\tau)d\tau = 0 \\
M_1 &= \int_0^t G_x(\tau)\tau d\tau \\
&= -\int_0^T G_0\tau d\tau + \int_T^{2T} G_0\tau d\tau \\
&= G_0\left[-\frac{\tau^2}{2}\bigg|_0^T + \frac{\tau^2}{2}\bigg|_T^{2T}\right] \\
&= G_0\left[-\frac{T^2}{2} + \frac{4T^2}{2} - \frac{T^2}{2}\right] = G_0T^2
\end{aligned}$$



Phase Contrast Angiography (PCA)




$$\varphi_1 = -\gamma v_x M_1 = \gamma v_x G_0 T^2$$

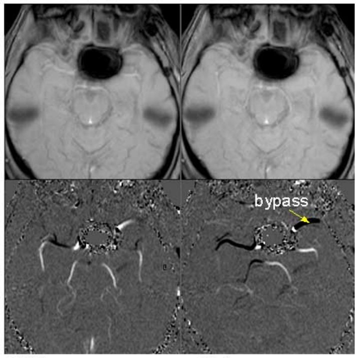



$$\varphi_2 = -\gamma v_x M_1 = -\gamma v_x G_0 T^2$$

$$\Delta\varphi = \varphi_1 - \varphi_2 = 2\gamma v_x G_0 T^2$$

$$v_x = \frac{\Delta\varphi}{2G_0T^2}$$

# PCA example



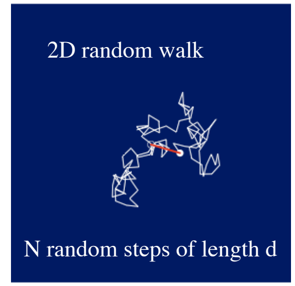

White = Flow direction AP (↓)

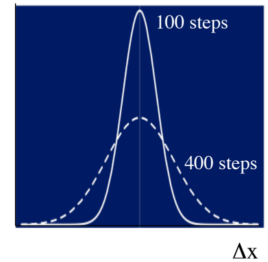
White = Flow direction RL (→)



http://www.medical.philips.com/main/products/mri/assets/images/case_of_week/cotw_51_s5.jpg

# Diffusion




$<\Delta x^2> = Nd^2 = 2DT$

D = diffusivity

In brain:

$D \cong 0.001\ mm^2/s$

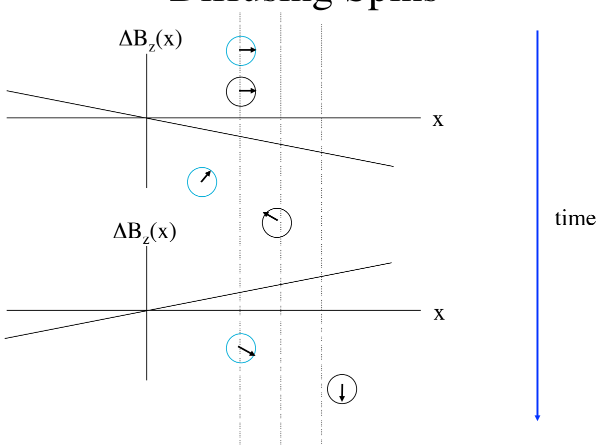
For T=100 msec,

$\Delta x \cong 15\ \mu$

Credit: Larry Frank

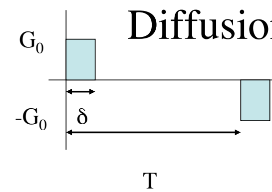


# Diffusing Spins






# Diffusion Weighting



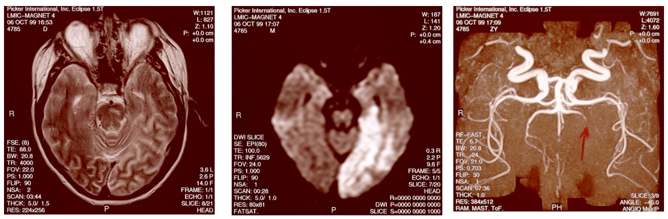

Signal

$$S \propto e^{-\gamma^2 G_0^2 \delta^2 DT} = e^{-bD} \quad \text{where } b = \gamma^2 G_0^2 \delta^2 (T - \delta/3)$$

Diffusivity

# Diffusion Weighted Images

T2 weighted — Diffusion Weighted — Angiogram



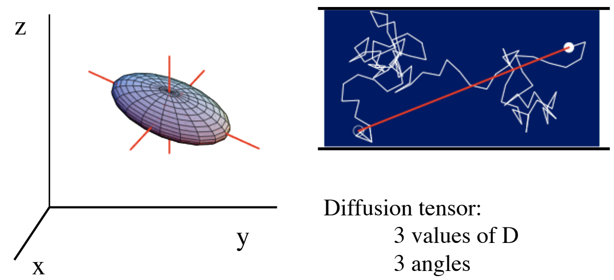
After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

http://lehighmri.com/cases/dwi/patient-b.html



# Restricted Diffusion

D depends on direction

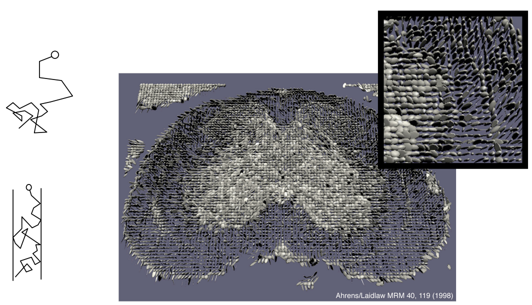



Diffusion tensor:
- 3 values of D
- 3 angles

Credit: Larry Frank

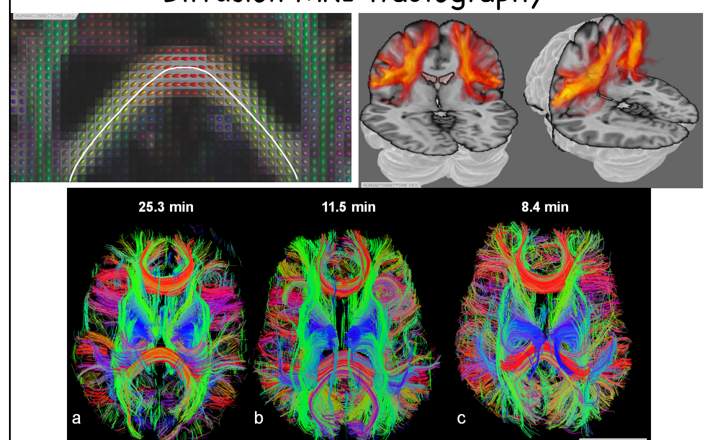


# Diffusion Imaging Example






# Diffusion MRI Tractography




from the Human Connectome Project