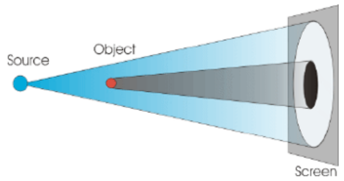
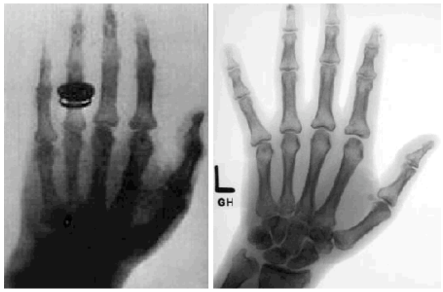
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2014
X-Rays Lecture 1





EM spectrum

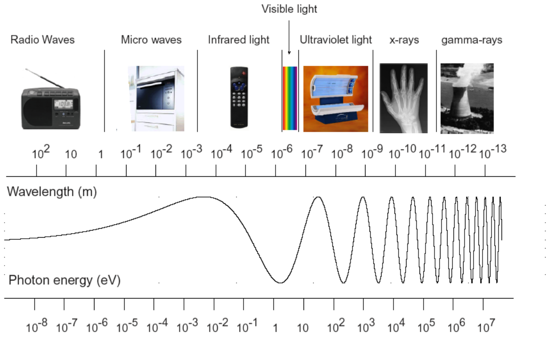


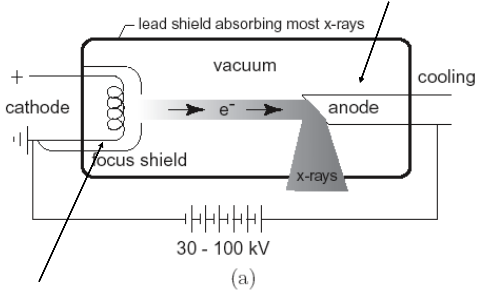
Figure 4.1: *The electromagnetic spectrum.*

Suetens 2002

http://www.youtube.com/watch?v=wbbsbE2mQuA

X-Ray Tube

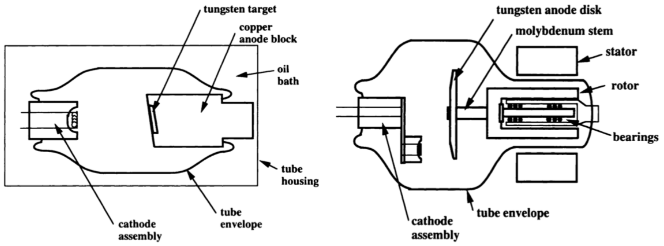
Usually tungsten is used for anode
Molybdenum for mammography



Tungsten filament heated to about 2200 C leading to thermionic emission of electrons.

Suetens 2002

X-Ray Tube

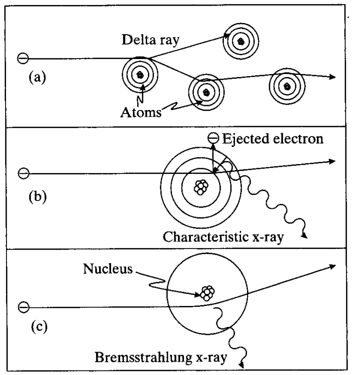


Zink, Radiographics, 1997

X-Ray Production

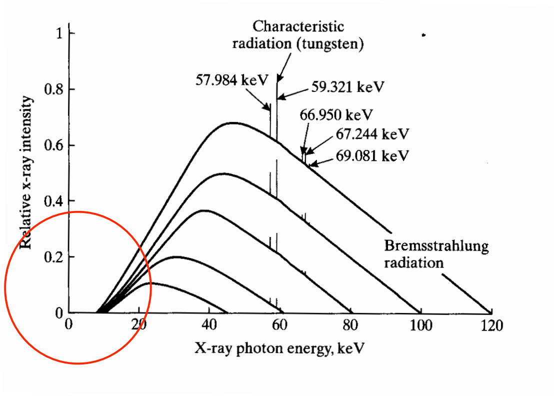
Collisional transfers

Radiative transfers



Prince and Links 2005

X-Ray Spectrum

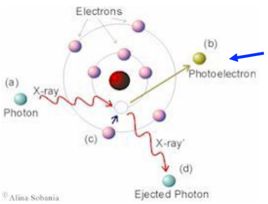


Lower energy photons are absorbed by anode, tube, and other filters

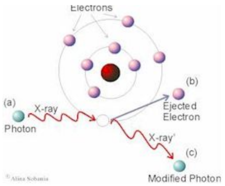
Prince and Links 2005

Interaction with Matter

Typical energy range for diagnostic x-rays is below 200 keV. The two most important types of interaction are photoelectric absorption and Compton scattering.



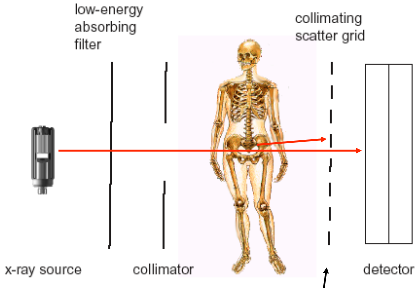
Photoelectric effect dominates at low x-ray energies and high atomic numbers. $E_{e^-} = \hbar\nu - E_B$



Compton scattering dominates at high x-ray energies and low atomic numbers, not much contrast

http://www.eee.ntu.ac.uk/research/vision/asobania

X-Ray Imaging Chain

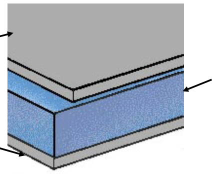


Reduces effects of Compton scattering

Suetens 2002

X-ray film

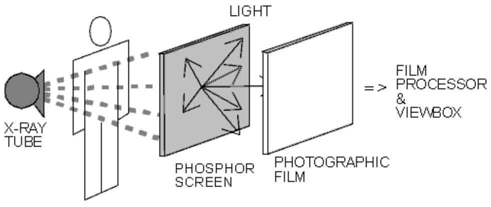
Emulsion with silver halide crystals
Each layer
~ 10 µm

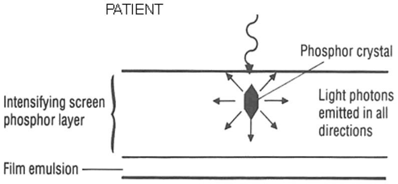


Flexible base
~ 150 µm

Silver halide crystals absorb optical energy. After development, crystals that have absorbed enough energy are converted to metallic silver and look dark on the screen. Thus, parts in the object that attenuate the x-rays will look brighter.

Intensifying Screen





http://learntech.uwe.ac.uk/radiography/RScience/imaging_principles_d/diagimage11.htm
http://www.sunnybrook.utoronto.ca:8080/~selenium/xray.html#Film

Digital Radiography

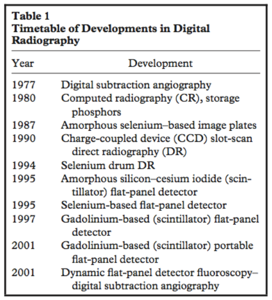
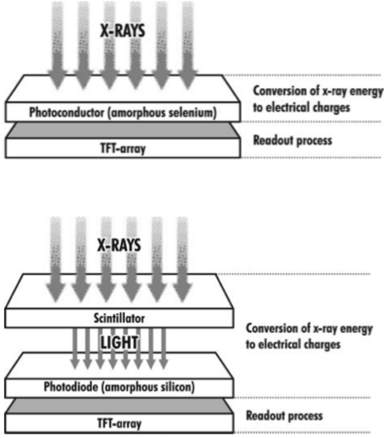


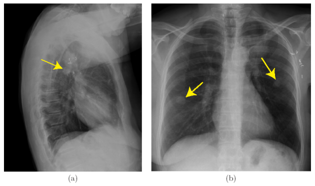
Table 1
Timetable of Developments in Digital Radiography

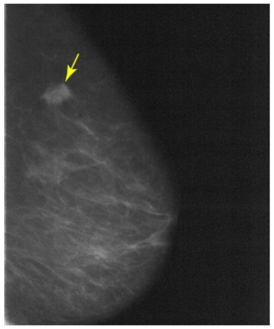
Year	Development
1977	Digital subtraction angiography
1980	Computed radiography (CR), storage phosphors
1987	Amorphous selenium–based image plates
1990	Charge-coupled device (CCD) slot-scan direct radiography (DR)
1994	Selenium drum DR
1995	Amorphous silicon–cesium iodide (scintillator) flat-panel detector
1995	Selenium-based flat-panel detector
1997	Gadolinium-based (scintillator) flat-panel detector
2001	Gadolinium-based (scintillator) portable flat-panel detector
2001	Dynamic flat-panel detector fluoroscopy–digital subtraction angiography



Korner et al, 2007

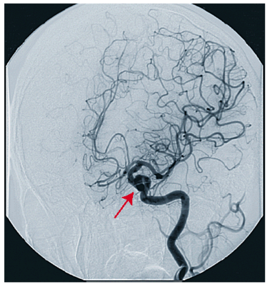
X-Ray Examples


(a) (b)

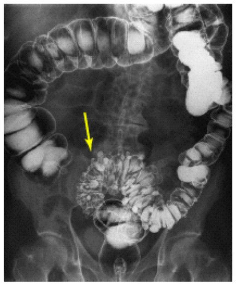


Suetens 2002

X-Ray w/ Contrast Agents



Angiogram using an iodine-based contrast agent.
K-edge of iodine is 33.2 keV



Barium Sulfate
K-edge of Barium is 37.4 keV

Suetens 2002

Intensity

$I = E\phi$

Energy

Photon flux rate

$$\phi = \frac{N}{A\Delta t}$$

Number of photons

Unit Area

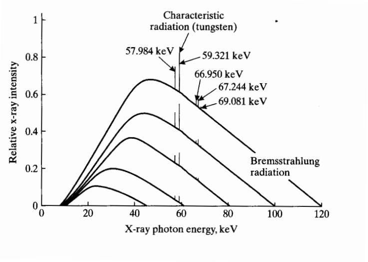
Unit Time

Intensity

$$\phi = \int_0^\infty S(E')dE'$$

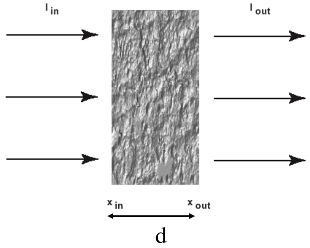
$\uparrow$

X-ray spectrum



$$I = \int_0^\infty S(E')E'dE'$$

Attenuation



For single-energy x-rays passing through a homogenous object:

$$I_{out} = I_{in}\exp(-\underset{\uparrow}{\mu} d)$$

Linear attenuation coefficient

Attenuation

$n = \mu N \Delta x$ photons lost per unit length

$\mu = \dfrac{n/N}{\Delta x}$ fraction of photons lost per unit length

$$\Delta N = -n \longrightarrow \frac{dN}{dx} = -\mu N \longrightarrow N(x) = N_0 e^{-\mu x}$$

For mono-energetic case, intensity is

$$I(\Delta x) = I_0 e^{-\mu \Delta x}$$

Attenuation

Inhomogeneous Slab

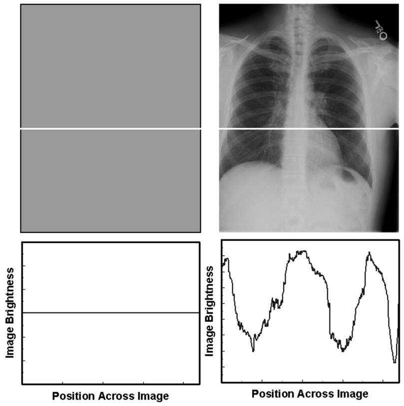
$$\frac{dN}{dx} = -\mu(x)N \longrightarrow N(x) = N_0 \exp\left(-\int_0^x \mu(x')dx'\right)$$

$$I(x) = I_0 \exp\left(-\int_0^x \mu(x')dx'\right)$$

Attenuation depends on energy, so also need to integrate over energies

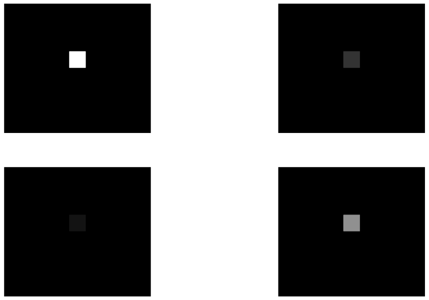
$$I(x) = \int_0^\infty S_0(E')E'\exp\left(-\int_0^x \mu(x';E')dx'\right)dE'$$

Contrast

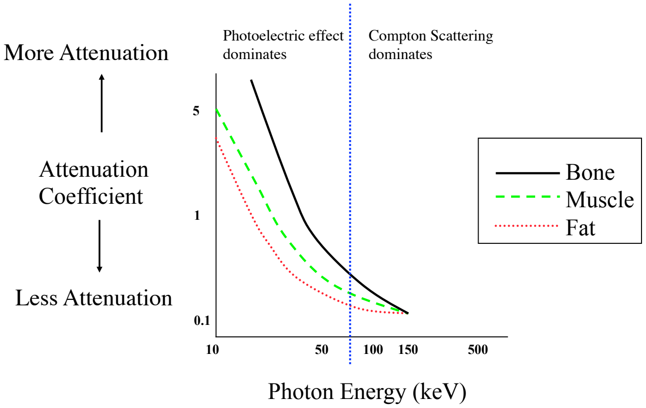


Bushberg et al 2001

Contrast



Attenuation



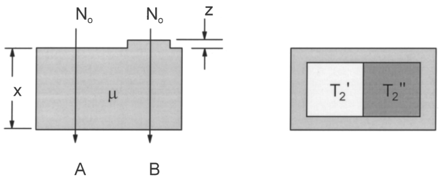
Adapted from www.cis.rit.edu/class/simg215/xrays.ppt

Half Value Layer

X-ray energy (keV)	HVL, muscle (cm)	HVL Bone (cm)
30	1.8	0.4
50	3.0	1.2
100	3.9	2.3
150	4.5	2.8

In chest radiography, about 90% of x-rays are absorbed by body. Average energy from a tungsten source is 68 keV. However, many lower energy beams are absorbed by tissue, so average energy is higher. This is referred to as beam-hardening, and reduces the contrast.

Values from Webb 2003



(A) X-ray Imaging (B) MR Imaging

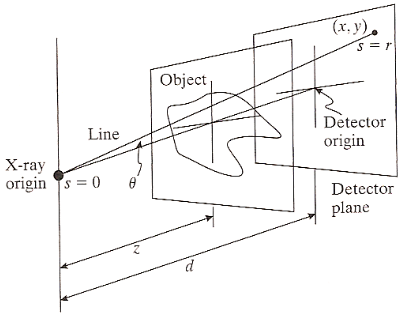
Bushberg et al 2001

$$A = N_0 \exp(-\mu x)$$
$$B = N_0 \exp(-\mu(x+z))$$

Subject Contrast

$$C_S = \frac{A-B}{A}$$
$$= \frac{N_0 \exp(-\mu x) - N_0 \exp(-\mu(x+z))}{N_0 \exp(-\mu x)}$$
$$= 1 - \exp(-\mu z)$$

X-Ray Imaging Geometry

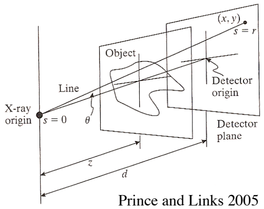


Prince and Links 2005

Inverse Square Law

Inverse Square Law

$$I_0 = \frac{I_S}{4\pi d^2}$$
$$I_d(x,y) = \frac{I_S}{4\pi r^2} \quad \text{where } r^2 = x^2 + y^2 + d^2$$
$$= \frac{I_0 d^2}{r^2} = I_0 \cos^2\theta$$

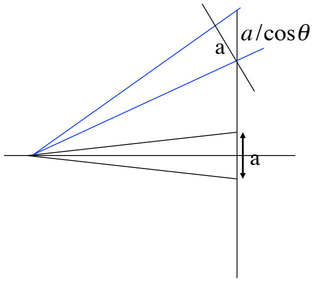


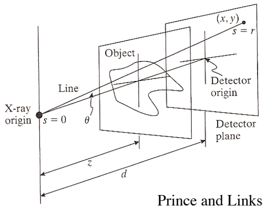
Prince and Links 2005

Obliquity Factor

Obliquity Factor

$$I_d(x,y) = I_0 \cos\theta$$





Prince and Links 2005

X-Ray Imaging Geometry

Beam Divergence and Flat Panel

$I_r = I_0 \cos^3 \theta$

Example: Chest x-ray at 2 yards with 14x17 inch film.

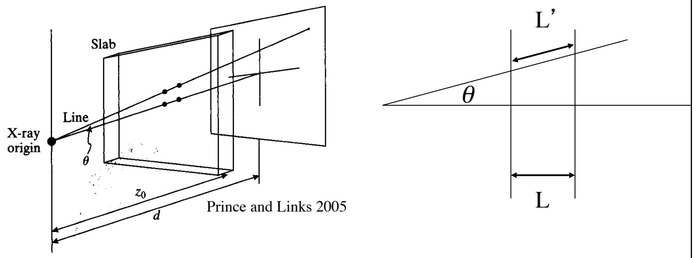
Question: What is the smallest ratio I_r / I_0 across the film?

$$r_d = \sqrt{7^2 + 8.5^2} = 11$$

$$\cos\theta = \frac{d}{\sqrt{r_d^2 + d^2}} = 0.989$$

$$\frac{I_r}{I_0} = \cos^3\theta = 0.966$$

Path Length

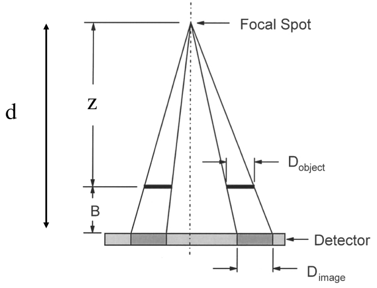


Prince and Links 2005

$$L' = L / \cos\theta$$

$$I_d(x,y) = I_0 \cos^3\theta \exp(-\mu L / \cos\theta)$$

Magnification of Object

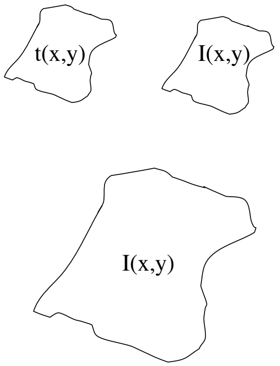


$$M(z) = \frac{d}{z}$$

$$= \frac{\text{Source to Image Distance (SID)}}{\text{Source to Object Distance (SOD)}}$$

Bushberg et al 2001

Magnification of Object



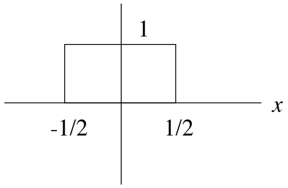
M = 1: I(x,y) = t(x,y)

M = 2: I(x,y) = t(x/2,y/2)

In general, I(x,y) = t(x/M(z),y/M(z))

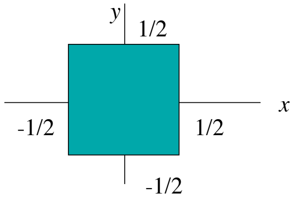
Rectangle Function

$$\Pi(x) = \begin{cases} 0 & |x| > 1/2 \\ 1 & |x| \le 1/2 \end{cases}$$



Also called rect(x)

$$\Pi(x,y) = \Pi(x)\Pi(y)$$



X-Ray Imaging Equation

At z = d there is no magnification, so

$$I_d(x,y) = I_0 \cos^3\theta \cdot \exp\left(-\int_{L_{x,y}} \mu(s)ds \Big/ \cos\theta \right)$$

$$= I_0 \cos^3\theta \cdot t_d(x,y)$$

where $t_z(x,y)$ is the transmittivity of the object at distance z

In general, with magnification

$$I_d(x,y) = I_0 \cos^3\theta \cdot t_z(x/M(z), y/M(z))$$

Prince and Links 2005

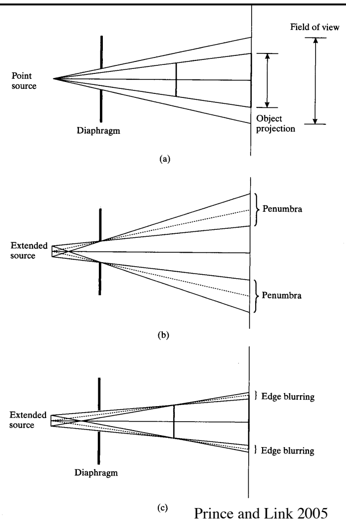
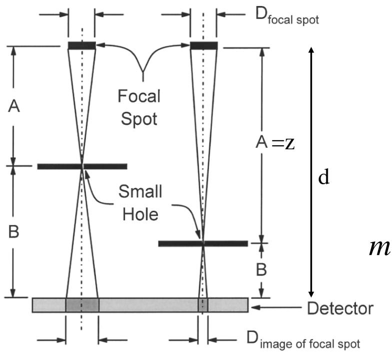


Figure 5.18
Effects of extended source. (a) Ideal field of view and object projection (with magnification). (b) Penumbra at edges of field of view due to extended source. (c) Blurred object edges due to extended source.

Prince and Link 2005

Source magnification

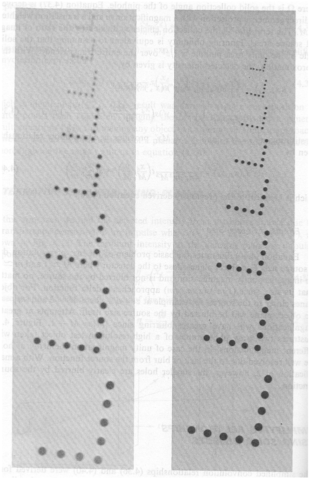


Bushberg et al 2001

$$\frac{D_{image}}{D_{focal}} = \frac{d-z}{z}$$

$$m(z) = -\frac{d-z}{z} = -\frac{B}{A}$$
$$= 1 - M(z)$$

M=2
m=-1



M=1
m=0

Macovski 1983

Dirac Delta Function

Notation:

$\delta(x)$ - 1D Dirac Delta Function

$\delta(x,y)$ or ${}^2\delta(x,y)$ - 2D Dirac Delta Function

$\delta(x,y,z)$ or ${}^3\delta(x,y,z)$ - 3D Dirac Delta Function

$\delta(\vec{r})$ - N Dimensional Dirac Delta Function

1D Dirac Delta Function

$\delta(x) \; = \; 0$ when $x \neq 0$ and $\int_{-\infty}^{\infty} \delta(x)dx = 1$

Can interpret the integral as a limit of the integral of an ordinary function that is shrinking in width and growing in height, while maintaining a constant area. For example, we can use a shrinking rectangle function such that $\int_{-\infty}^{\infty} \delta(x)dx = \lim_{\tau \to 0} \int_{-\infty}^{\infty} \tau^{-1}\Pi(x/\tau)dx.$

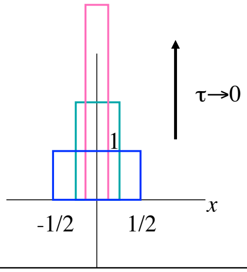


Image of a point object

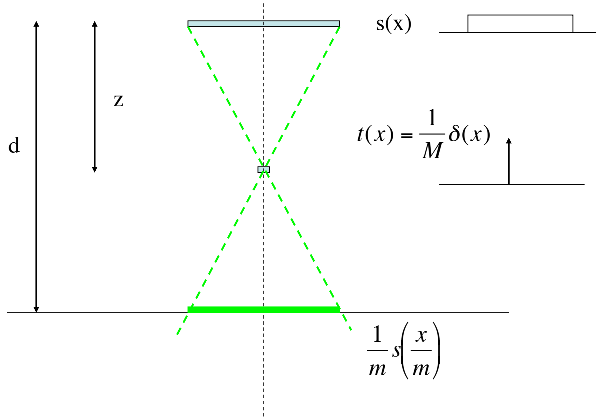
$$I_d(x,y) = ks(x/m, y/m)$$

$$\int\int ks\big(x/m(z), y/m(z)\big)dxdy = \text{ constant}$$

$$\Rightarrow k = \frac{1}{m^2(z)}$$

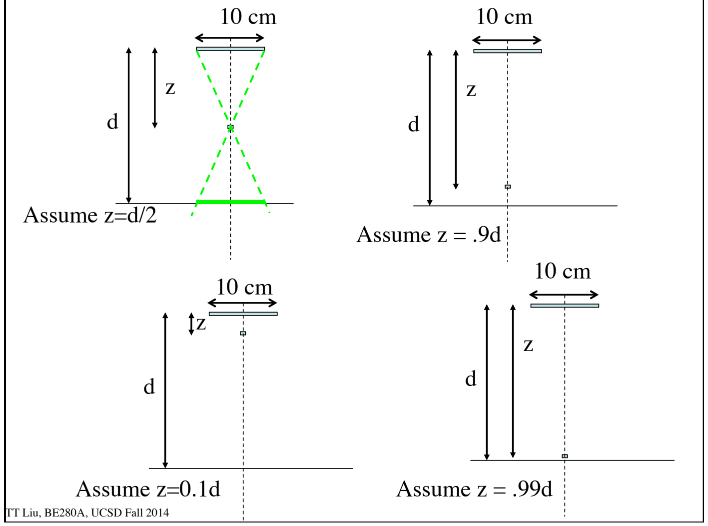
$$I_d(x,y) = \lim_{m \to 0} \frac{s(x/m, y/m)}{m^2}$$
$$= \delta(x,y)$$

X-Ray Imaging



$t(x) = \frac{1}{M}\delta(x)$

$\frac{1}{m}s\left(\frac{x}{m}\right)$

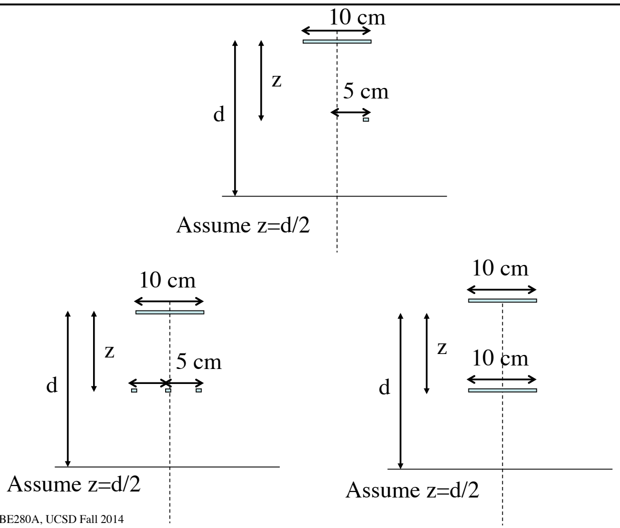


Assume z=d/2

Assume z = .9d

Assume z=0.1d

Assume z = .99d



Assume z=d/2

Assume z=d/2

Assume z=d/2

Convolution