Problem 1

Define \( m(x, y) = \text{rect}(x, y) * \left[ \delta(x) + \delta(x-1, y) + \delta(x+1, y) \ight] + \delta(x, y-1) + \delta(x, y+1) \]

a) Sketch the object \( y \) (5 pts.)

b) Sketch the projection at \( \theta = 0^\circ \) (5 pts.)

c) Sketch the projection at \( \theta = 45^\circ \) (10 pts.)
\[ \text{d)} \quad \text{Derive and sketch the Fourier Transform \hspace{1cm} (10 \text{ pts})} \]

\[ M(k_x, k_y) = \text{sinc}(k_x, k_y) \cdot \left[ 1 + 2 \cos(2\pi k_x) + 2 \cos(2\pi k_y) \right] \]

\[ = \text{sinc}(k_y, k_y) + 2\text{sinc}(k_x, k_y) \cos(2\pi k_x) + 2\text{sinc}(k_x, k_y) \cos(2\pi k_y) \]

\[ \text{e)} \quad \text{Show that the projection slice theorem holds for } \theta = 0^\circ \hspace{1cm} (10 \text{ pts}) \]

\[ M(k_x, 0) = \text{sinc}(k_x) \cdot \left[ 1 + 2 \cos(2\pi k_x) \right] \]

\[ = 3\text{sinc}(k_x) + 2\text{sinc}(k_x) \cos(2\pi k_x) \]

\[ \Theta^{-1}(M(k_x, 0)) = 3\text{rect}(k_x) + \text{rect}(k_x) \ast \left[ \delta(x-1) + \delta(x+1) \right] \]

\[ = 3\text{rect}(k_x) \]

\[ + \text{rect}(k_x-1) + \text{rect}(k_x+1) \]

\[ \text{f)} \quad \text{Show that the projection slice theorem holds at } \theta = 45^\circ \hspace{1cm} (15 \text{ pts}) \]

\[ M\left(\frac{\sqrt{2}}{2} k_x, \frac{\sqrt{2}}{2} k_y\right) = \text{sinc}^2\left(\frac{\sqrt{2}}{2} k_y\right) \cdot \left[ 1 + 4 \cos \left(2\pi \frac{\sqrt{2}}{2} k_x\right) \right] \]

\[ \Theta^{-1}(M\left(\frac{\sqrt{2}}{2} k_x, \frac{\sqrt{2}}{2} k_y\right)) = \frac{\sqrt{2}}{2} \Delta\left(\frac{\sqrt{2}}{2} \theta\right) \ast \left[ \delta(l) + 2 \delta(l - \frac{\sqrt{2}}{2}) + \delta(l + \frac{\sqrt{2}}{2}) \right] \]

\[ = \sqrt{2} \Delta(\sqrt{2} l) + 2\Delta(\sqrt{2}(l - \frac{\sqrt{2}}{2})) + 2\Delta(\sqrt{2}(l + \frac{\sqrt{2}}{2})) \]
Problem 2

\[ m(x,y) = \cos(2\pi(x+y)) \text{rect}(\frac{x}{4}) \text{rect}(\frac{y}{4}) \]

Assume units of cm

a) Sketch the object (5 pts)

b) Compute and sketch the Fourier Transform (5 pts.)

\[ M(k_x, k_y) = \frac{1}{2} \left[ 8(k_x - 1, k_y - 1) + 1 (k_x + 1, k_y + 1) \right] \]

\[ \ast 16 \text{ sinc}(4k_x, 4k_y) \]

\[ = 8 \cdot \text{sinc}(4(k_x - 1), 4(k_y - 1)) + 8 \text{sinc}(4(k_x + 1), 4(k_y + 1)) \]
Determine $\Delta k_x$ and $\Delta k_y$ (5 pts)

$F_{ou\_x} = F_{ou\_y} = 4$ cm

$\Delta k_x = \Delta k_y = \frac{1}{4}$ cm$^{-1} = 0.25$ cm$^{-1}$

a) Assume resolution of $\delta x = \delta y = 0.5$ cm (5 pts)

$W_{k_x} = W_{k_y} = \frac{1}{\delta x} = \frac{1}{\delta y} = 2$ cm$^{-1}$

$k_{x,\max} = k_{y,\max} = \frac{W_{k_x}}{2} = \frac{W_{k_y}}{2} = 1$ cm$^{-1}$

c) object at $y = 0$; $k_y = \phi$ (15 pts)

![Diagram with arrows and labels for $k_x$ and $x$ values]

Multiply by $-1$
\[
\text{BE280A}
\]
\[
\square = 1
\]
\[
\overline{\square} = -1
\]

Consider the spins at \( x = 1 \), \( (y = 1 \); or \( x = -1 \)
\( y = -1 \)

\[
\text{F.T. maximized at } x = 1, \quad \{y = 1 \text{ or } y = -1 \}
\]

\[
\text{sum} = 64
\]

G) Consider the spins at \( x = -2 \) and \( x = 0 \)

(10 pts, in part 1c) \( \Rightarrow \) if we slip every other \( x \) line then the spin orientations are identical at these \( x \) locations.