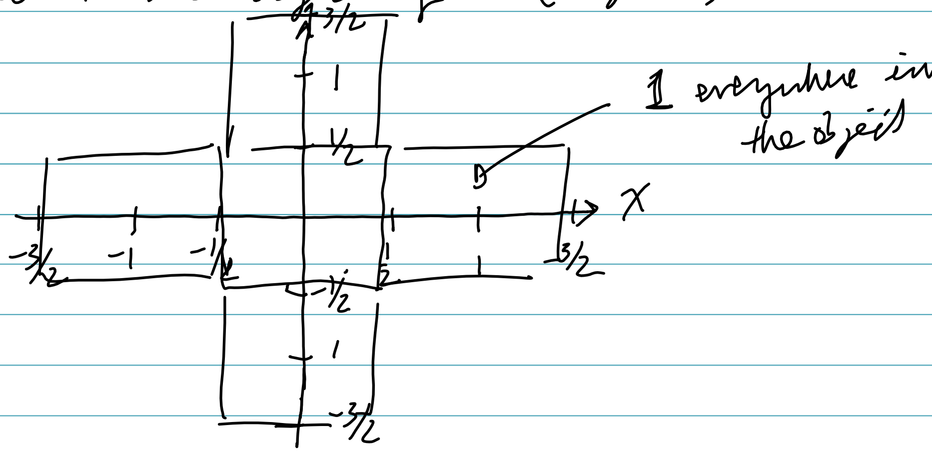
BE 280A Exam 11/12/13

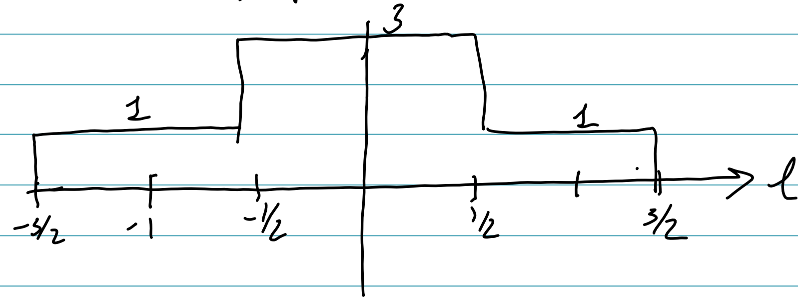
Problem 1

Define $\mu(x,y) = rect(x,y) * [\delta(x,y) + \delta(x-1,y) + \delta(x+1,y) + \delta(x,y-1) + \delta(x,y+1)]$

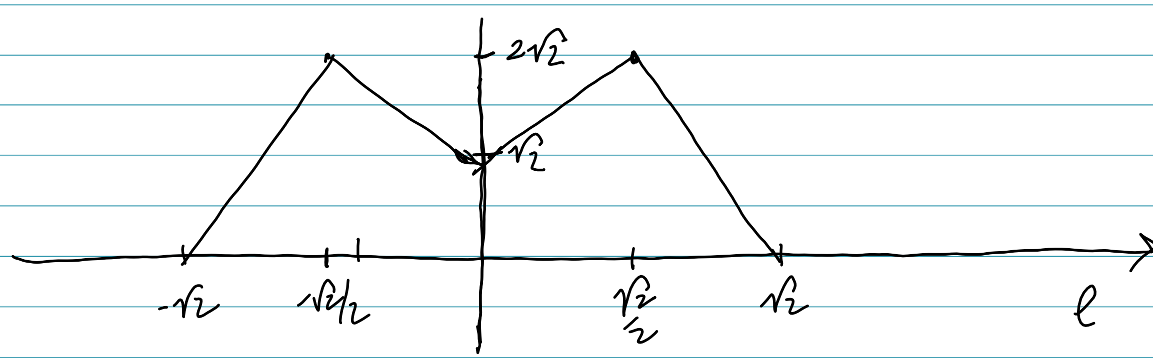
a) sketch the object (5 pts.)



b) sketch the projection at $\theta = 0°$ (5 pts.)



c) sketch the projection at $\theta = 45°$ (10 pts.)



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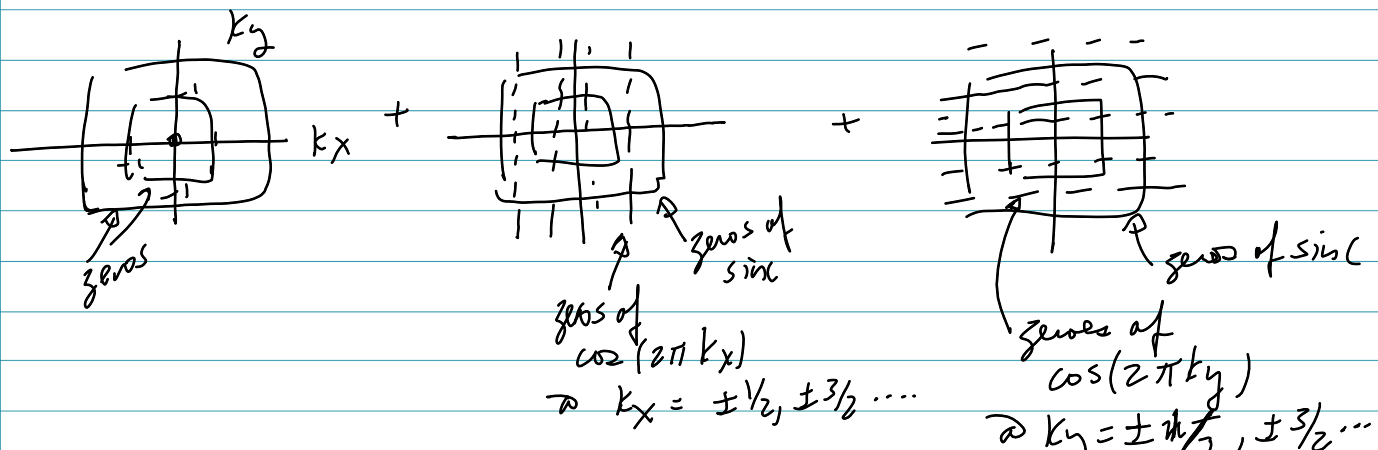
d] Derive + Sketch the Fourier Transform (10 pts.)

$$M(k_x, k_y) = \text{sinc}(k_x, k_y) \cdot \left[1 + 2\cos(2\pi k_x) + 2\cos(2\pi k_y) \right]$$

$$= \text{sinc}(k_x, k_y) + 2\,\text{sinc}(k_x, k_y)\cos(2\pi k_x) + 2\,\text{sinc}(k_x, k_y)\cos(2\pi k_y)$$



e] Show that the projection slice theorem holds for $\theta = 0°$ (10 pts.)

$$M(k_x, 0) = \text{sinc}(k_x) \cdot \left[1 + 2\cos(2\pi k_x) + 2 \right]$$

$$= 3\,\text{sinc}(k_x) + 2\,\text{sinc}(k_x)\cos(2\pi k_x)$$

$$\mathcal{F}^{-1}(M(k_x, 0)) = 3\,\text{rect}(x) + \text{rect}(x) * \left[\delta(x-1) + \delta(x+1) \right]$$

$$= 3\,\text{rect}(x) + \text{rect}(x-1) + \text{rect}(x+1)$$

f] Show that the projection slice theorem holds at $\theta = 45°$ (15 pts.)

$$M\left(\tfrac{\sqrt{2}}{2}k, \tfrac{\sqrt{2}}{2}k\right) = \text{sinc}^2\left(\tfrac{\sqrt{2}}{2}k\right) \cdot \left[1 + 4\cos\left(2\pi \tfrac{\sqrt{2}}{2} k\right) \right]$$

$$\mathcal{F}^{-1}\left(M\left(\tfrac{\sqrt{2}}{2}k, \tfrac{\sqrt{2}}{2}k\right)\right) = \frac{2}{\sqrt{2}} \Lambda\left(\frac{2}{\sqrt{2}} \ell\right) * \left[\delta(\ell) + 2\left(\delta\left(\ell - \tfrac{\sqrt{2}}{2}\right) + \delta\left(\ell + \tfrac{\sqrt{2}}{2}\right)\right) \right]$$

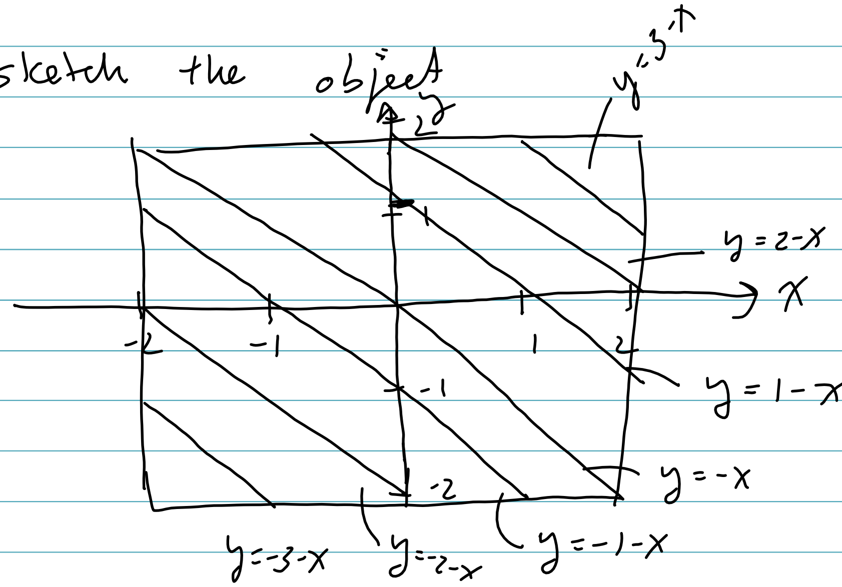
$$= \sqrt{2}\,\Lambda(\sqrt{2}\,\ell) + 2\Lambda\left(\sqrt{2}\left(\ell - \tfrac{\sqrt{2}}{2}\right)\right) + 2\Lambda\left(\sqrt{2}\left(\ell + \tfrac{\sqrt{2}}{2}\right)\right)$$

Problem 2

$$m(x,y) = \cos(2\pi(x+y))\, \mathrm{rect}(x/4)\, \mathrm{rect}(y/4)$$

Assume units of cm

a) sketch the object (5 pts)

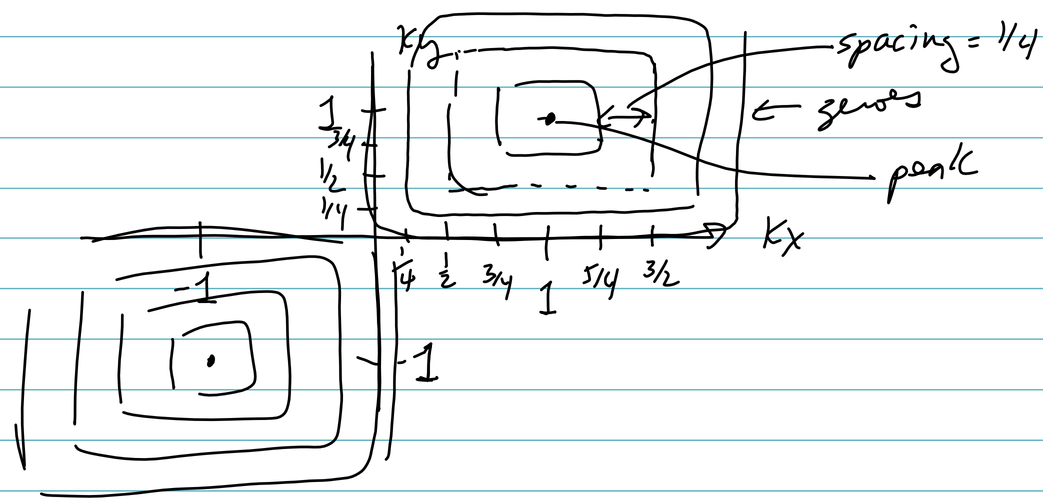


peaks at
$x+y=n$
$y=n-x$

b) Compute + sketch the Fourier transform (5 pts.)

$$M(k_x,k_y) = \tfrac{1}{2}\left[\delta(k_x-1, k_y-1) + \delta(k_x+1, k_y+1)\right] * 16\,\mathrm{sinc}(4k_x, 4k_y)]$$

$$= 8 \cdot \mathrm{sinc}(4(k_x-1), 4(k_y-1)) + 8\,\mathrm{sinc}(4(k_x+1), 4(k_y+1))]$$



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c) Determine Δk_x and Δk_y (5 pts)

$$FOV_x = FOV_y = 4 \text{ cm}$$

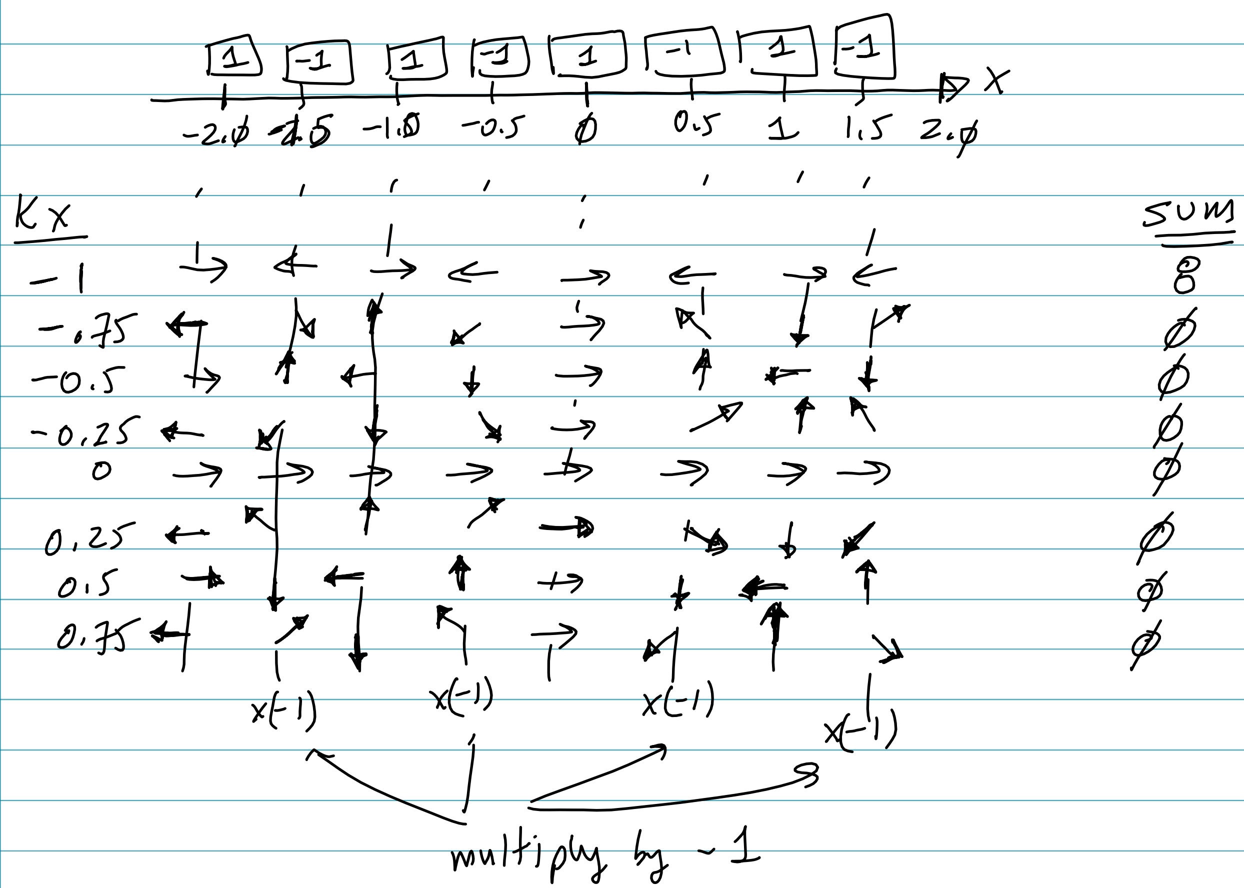
$$\Delta k_x = \Delta k_y = \frac{1}{4}\text{ cm}^{-1} = 0.25 \text{ cm}^{-1}$$

d) Assume resolution of $\delta_x = \delta_y = 0.5$ cm (5 pts.)

$$W_{kx} = W_{ky} = \frac{1}{\delta x} = \frac{1}{\delta y} = 2 \text{ cm}^{-1}$$

$$k_{x,max} = k_{y,max} = \frac{W_{kx}}{2} = \frac{W_{ky}}{2} = 1 \text{ cm}^{-1}$$

e) object at $y=0$; $k_y = \phi$ (15 pts.)



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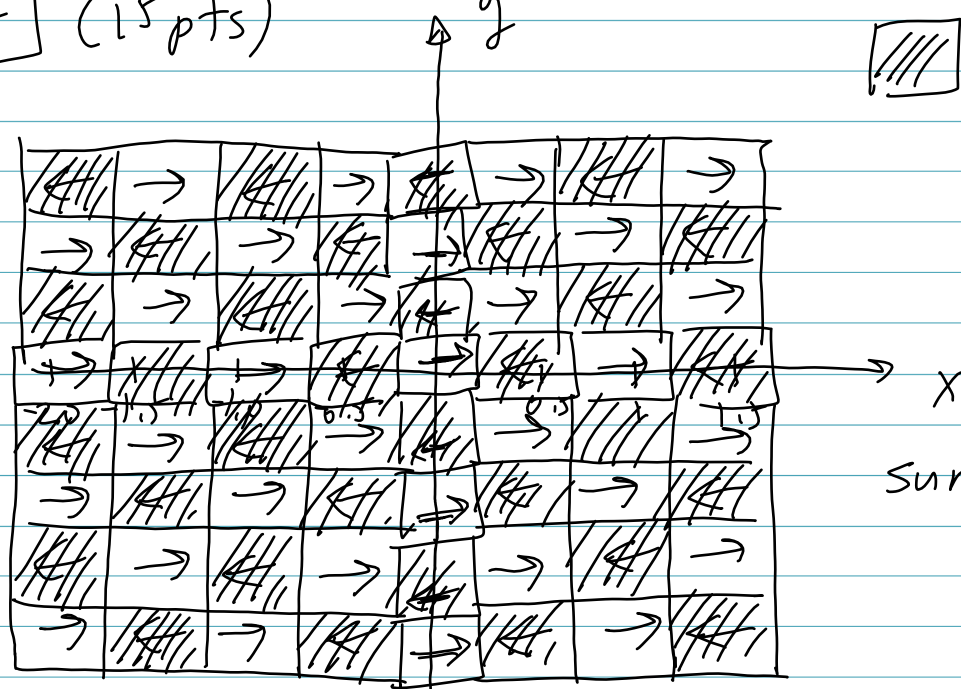
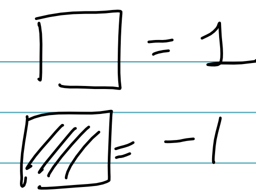
f] (15pts)





sum = 64

F.T. maximized at $k_x=1, k_y=1$ or $k_x=-1$, $k_y=-1$

g] (10pts.) Consider the spins at $x=-2$ and $x=0$ in part (e) → if we skip every other k_x line then the spin orientations are identical at these x locations.