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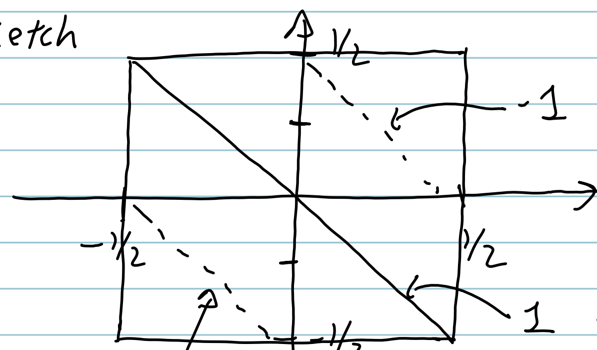
$\mu(x,y) = \text{rect}(x,y)\cos(2\pi(x+y))$

sketch



$-1 : y = \frac{1}{2} - x$

$1 : y = -x$

$-1 : y = -\frac{1}{2} - x$

peaks of $\cos(2\pi(x+y))$ occur when $y = -1 - x$

$x + y = n \rightarrow y = n - x \Rightarrow y = -x$

$y = 1 - x$

valleys of $\cos(2\pi(x+y))$ occur when

$x + y = \frac{(2n+1)}{2} \rightarrow y = \frac{(2n+1)}{2} - x \Rightarrow y = \frac{1}{2} - x$

$y = -\frac{1}{2} - x$

Projection at $\theta = 0°$

$$g(\ell, \theta=0) = \int_{-1/2}^{1/2} \cos(2\pi(\ell + y))\,dy$$

$$= \frac{1}{2\pi}\sin(2\pi(\ell+y))\Big|_{-1/2}^{1/2}$$

$$= \frac{1}{2\pi}\left[\sin(2\pi(\ell+\tfrac{1}{2})) - \sin(2\pi(\ell - \tfrac{1}{2}))\right]$$

$$= \phi$$

Note: integral of cosine over one period $= \phi$.

Projection at $\theta = 90°$

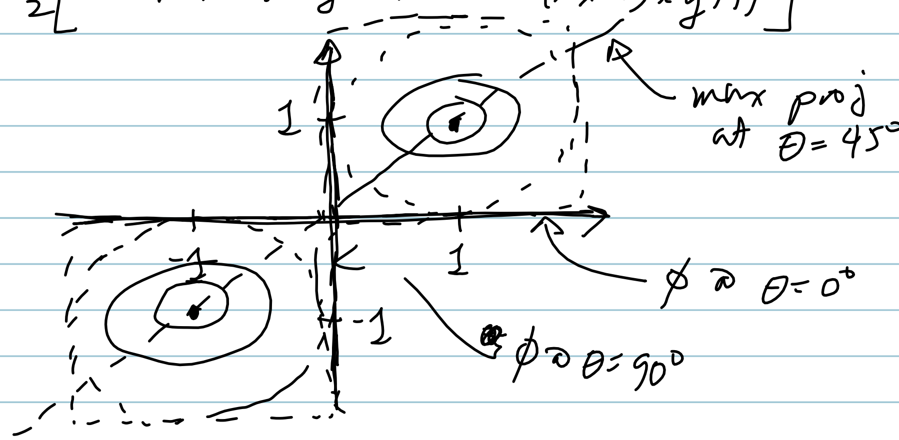
$$g(l, \theta = 90°) = \int_{-1/2}^{1/2} \cos(2\pi(x+l))\,dx$$

$$= 0$$

Maximum projection? Consider the F.T.

$$\mathcal{F}(\mu(x,y)) = \text{sinc}(k_x, k_y) * \frac{1}{2}\left[\delta(k_x - 1, k_y - 1) + \delta(k_x+1, k_y+1)\right]$$

$$U(k_x, k_y) = \frac{1}{2}\left[\text{sinc}(k_x - 1, k_y - 1) + \text{sinc}(k_x+1, k_y+1)\right]$$



at $\theta = 0°$; consider F.T. along k_x, $k_y = \phi$

$$U(k_x, k_y = 0) = \frac{1}{2}\left[\text{sinc}(k_x - 1)\,\text{sinc}(-1) + \text{sinc}(k_x + 1)\,\text{sinc}(1)\right] = \phi$$

at $\theta = 90°$

$$U(k_x = 0, k_y) = \frac{1}{2}\left[\text{sinc}(-1)\,\text{sinc}(k_y - 1) + \text{sinc}(1)\,\text{sinc}(k_y + 1)\right] = \phi$$

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@ $\theta = 45^\circ$ $\quad k_x = k\cos\theta = \sqrt{2}/2 \cdot k$

$\qquad\qquad\qquad k_y = \sqrt{2}/2\, k$

$$\mho(k\cos\theta, k\sin\theta) = \cancel{\mathrm{sinc}\left(k\tfrac{\sqrt{2}}{2}\right)\ \ldots\sqrt{2}}$$

$$= \frac{1}{2}\mathrm{sinc}\left(\frac{\sqrt{2}}{2}k - 1, \frac{\sqrt{2}}{2}k - 1\right)$$

$$+ \frac{1}{2}\mathrm{sinc}\left(\frac{\sqrt{2}}{2}k + 1, \frac{\sqrt{2}}{2}k + 1\right)$$

$$= \frac{1}{2}\mathrm{sinc}^2\left(\frac{\sqrt{2}}{2}(k - \sqrt{2})\right) + \frac{1}{2}\mathrm{sinc}^2\left(\frac{\sqrt{2}}{2}(k + \sqrt{2})\right)$$

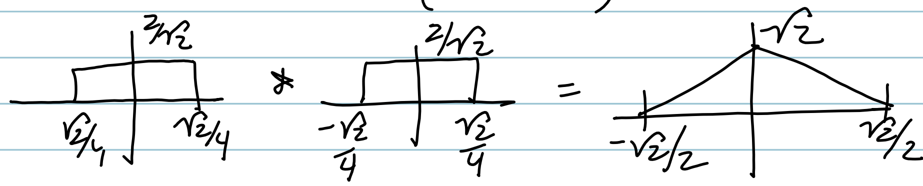
$$= \mathrm{sinc}^2\left(\frac{\sqrt{2}}{2}k\right) * \frac{1}{2}\left[\delta(k - \sqrt{2}) + \delta(k + \sqrt{2})\right]$$

$\Downarrow$ Projection slice theorem.

$$g(l)\Big|_{\theta=45} = \mathcal{F}^{-1}\left(\mho(k\cos\theta, k\sin\theta)\right)$$

$$= \left[\frac{2}{\sqrt{2}}\mathrm{rect}\left(\frac{2}{\sqrt{2}}l\right) * \frac{2}{\sqrt{2}}\mathrm{rect}\left(\frac{2}{\sqrt{2}}l\right)\right]$$

$$\cdot \cos(2\pi\sqrt{2}\, l)$$



$$g(l)\Big|_{\theta=45} = \sqrt{2}\,\Lambda\left(\frac{2l}{\sqrt{2}}\right)\cos(2\pi\sqrt{2}\, l)$$