

Basic Properties

Linearity

$$F[g(x,y) + bh(x,y)] = aG(k_x, k_y) + bH(k_x, k_y)$$

Scaling

$$F\{g(ax)\} = \frac{1}{|a|} G\left(\frac{k_x}{a}\right) \quad F[g(ax, by)] = \frac{1}{|ab|} G\left(\frac{k_x}{a}, \frac{k_y}{b}\right)$$

Separability

$$F[g(x)h(y)] = G(k_x)H(k_y)$$

Duality

$$F\{G(x)\} = g(-k_x)$$

Shift

$$F[g(x-a, y-b)] = G(k_x, k_y) e^{-j2\pi(k_x a + k_y b)}$$

Convolution

$$F[g(x,y) \ast h(x,y)] = G(k_x, k_y) H(k_x, k_y)$$

Multiplication

$$F[g(x,y)h(x,y)] = G(k_x, k_y) \ast H(k_x, k_y)$$

Modulation

$$F[g(x,y)e^{j2\pi(xa+yb)}] = G(k_x - a, k_y - b)$$

Transform Pairs

$$\begin{aligned} \delta(x) &\leftrightarrow 1 \\ \delta(x - x_0) &\leftrightarrow e^{-j2\pi k_x x_0} \\ 1 &\leftrightarrow \delta(k_x) \\ rect(x) &\leftrightarrow \text{sinc}(k_x) \\ \text{sinc}(x) &\leftrightarrow rect(k_x) \\ e^{j2\pi k_0 x} &\leftrightarrow \delta(k_x - k_0) \\ \cos 2\pi k_0 x &\leftrightarrow \frac{1}{2}(\delta(k_x - k_0) + \delta(k_x + k_0)) \\ \sin 2\pi k_0 x &\leftrightarrow \frac{1}{2j}(\delta(k_x - k_0) - \delta(k_x + k_0)) \\ \Pi(x)\Pi(y) &\leftrightarrow \text{sinc}(k_x)\text{sinc}(k_y) \\ \Lambda(x) &\leftrightarrow \text{sinc}^2(k_x) \\ comb(x) &\leftrightarrow comb(k_x) \end{aligned}$$

Useful facts and definitions

$$\begin{aligned} \text{sinc}(k_x) &= \frac{\sin(\pi k_x)}{\pi k_x} \\ \delta(x, y) &= \delta(x)\delta(y) \\ \text{sinc}(k_x, k_y) &= \text{sinc}(k_x)\text{sinc}(k_y) \\ rect(x, y) &= rect(x)rect(y) \\ \Lambda(x) &= \begin{cases} 1 - |x| & |x| < 1 \\ 0 & otherwise \end{cases} \\ \delta(ax) &= \frac{1}{|a|}\delta(x) \end{aligned}$$