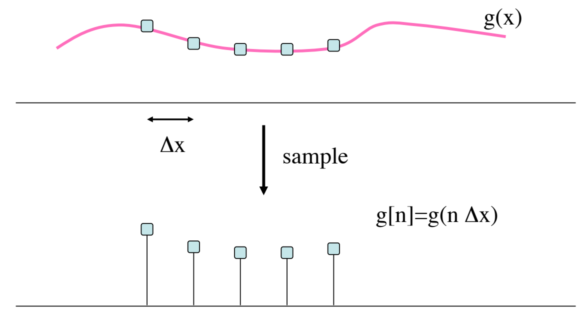
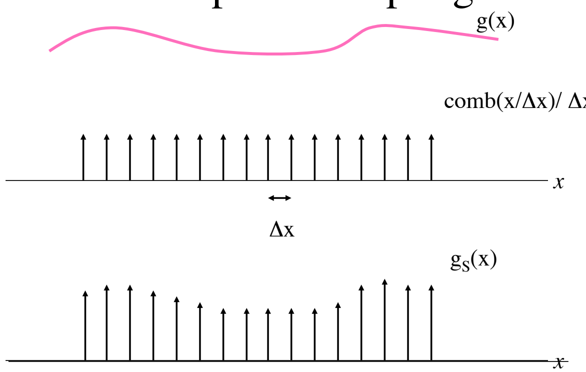
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2015
CT/Fourier Lecture 6

The Process of Sampling



1D spatial sampling

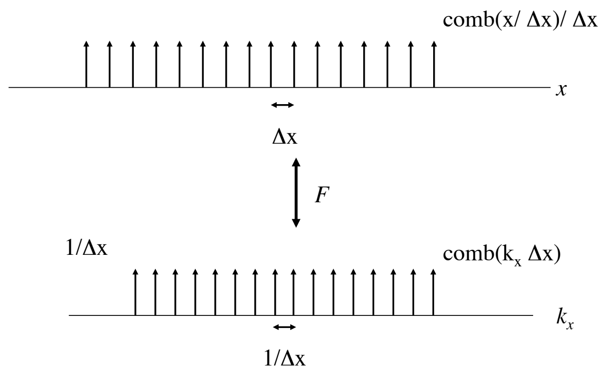


Fourier Transform of comb(x)

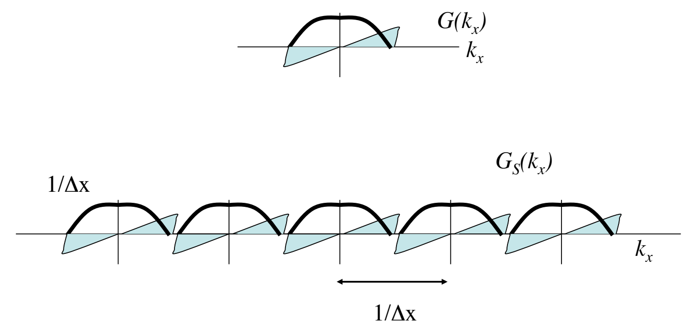
$$F\left[comb(x)\right] = comb(k_x)$$
$$= \sum_{n=-\infty}^{\infty} \delta(k_x - n)$$

$$F\left[\frac{1}{\Delta x} comb(\frac{x}{\Delta x})\right] = \frac{1}{\Delta x} \Delta x comb(k_x \Delta x)$$
$$= \sum_{n=-\infty}^{\infty} \delta(k_x \Delta x - n)$$
$$= \frac{1}{\Delta x} \sum_{n=-\infty}^{\infty} \delta(k_x - \frac{n}{\Delta x})$$

Fourier Transform of comb(x/ Δx)



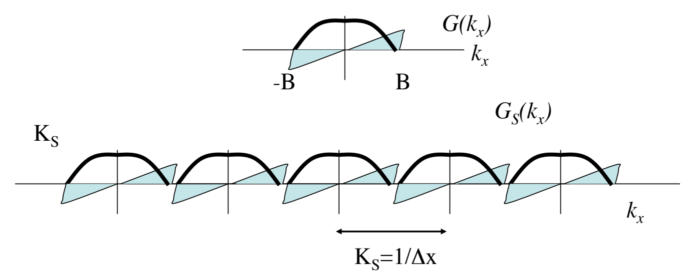
Fourier Transform of $g_S(x)$



Fourier Transform of $g_S(x)$

$$
\begin{aligned}
F\left[g_S(x)\right] &= F\left[g(x)\frac{1}{\Delta x}comb\left(\frac{x}{\Delta x}\right)\right] \\
&= G(k_x) * F\left[\frac{1}{\Delta x}comb\left(\frac{x}{\Delta x}\right)\right] \\
&= G(k_x) * \frac{1}{\Delta x}\sum_{n=-\infty}^{\infty}\delta\left(k_x - \frac{n}{\Delta x}\right) \\
&= \frac{1}{\Delta x}\sum_{n=-\infty}^{\infty} G(k_x) * \delta\left(k_x - \frac{n}{\Delta x}\right) \\
&= \frac{1}{\Delta x}\sum_{n=-\infty}^{\infty} G\left(k_x - \frac{n}{\Delta x}\right)
\end{aligned}
$$

Nyquist Condition



To avoid overlap, we require that $1/\Delta x > 2B$ or
$K_S > 2B$ where $K_S = 1/\Delta x$ is the sampling frequency

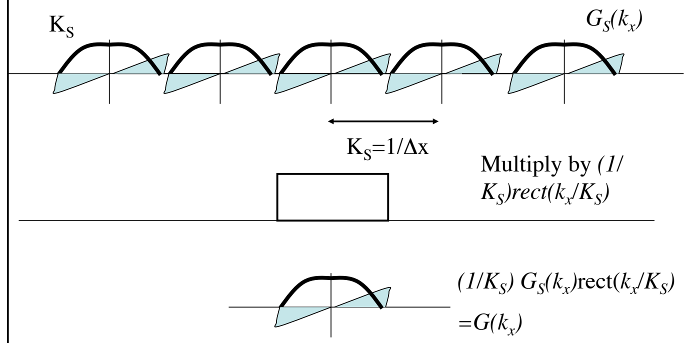
Example

Assume that the highest spatial frequency in an object is $B = 2\ \text{cm}^{-1}$.
Thus, smallest spatial period is 0.5 cm.

Nyquist theorem says we need to sample with
$\Delta x < 1/2B = 0.25$ cm

This corresponds to 2 samples per spatial period.

Reconstruction from Samples



Reconstruction from Samples

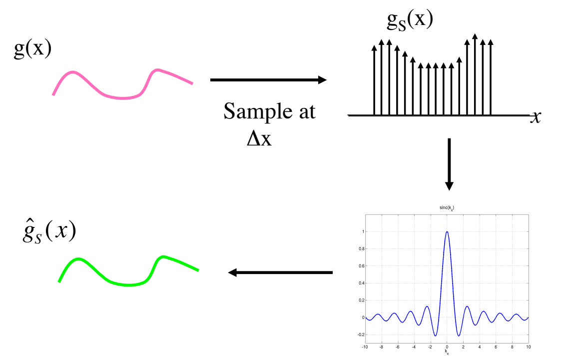
If the Nyquist condition is met, then

$$\hat{G}_S(k_x) = \frac{1}{K_S} G_S(k_x) rect(k_x / K_S) = G(k_x)$$

And the signal can be reconstructed by convolving the sample with a sinc function

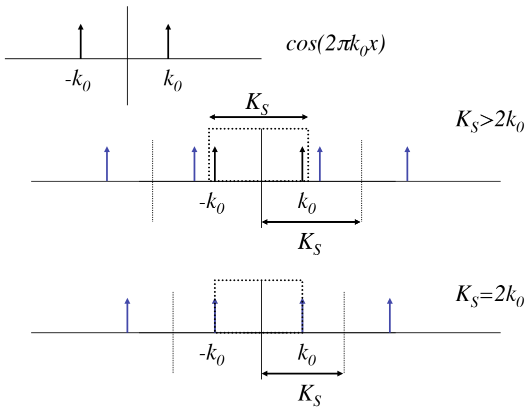
$$\begin{aligned}\hat{g}_S(x) &= g_S(x) * \text{sinc}(K_s x) \\ &= \left(\sum_{n=-\infty}^{\infty} g(n\Delta X)\delta(x - n\Delta X)\right) * \text{sinc}(K_s x) \\ &= \sum_{n=-\infty}^{\infty} g(n\Delta X)\,\text{sinc}(K_s(x - n\Delta x))\end{aligned}$$

Reconstruction from Samples

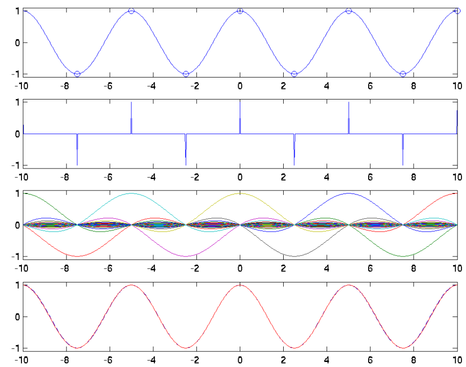


$\text{sinc}(K_s x) = \text{sinc}(x/\Delta x)$

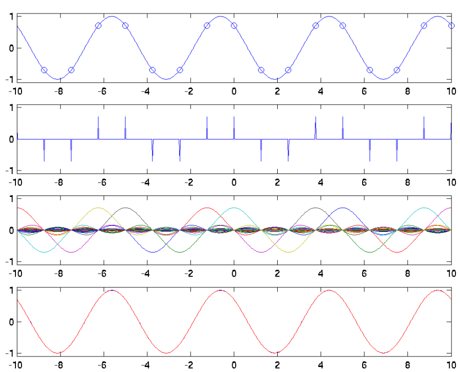
Example Cosine Reconstruction



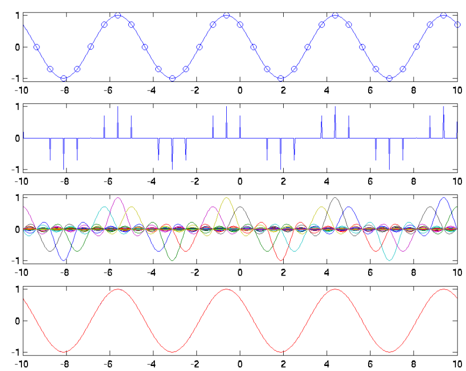
Cosine Example with $K_S=2k_0$



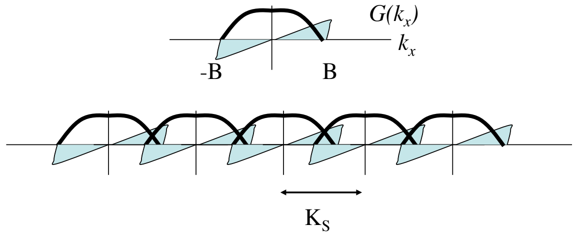
Example with $K_s=4k_0$



Example with $K_s=8k_0$

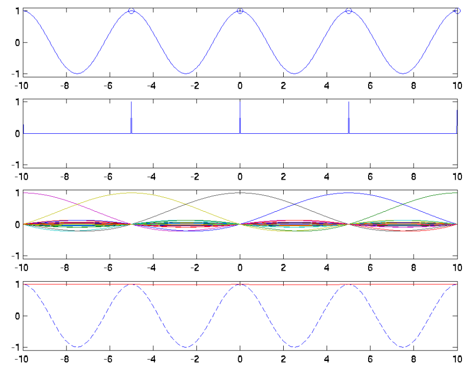


Aliasing

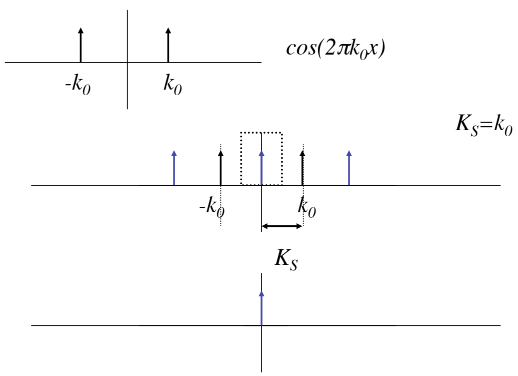


Aliasing occurs when the Nyquist condition is not satisfied.
This occurs for $K_S \leq 2B$

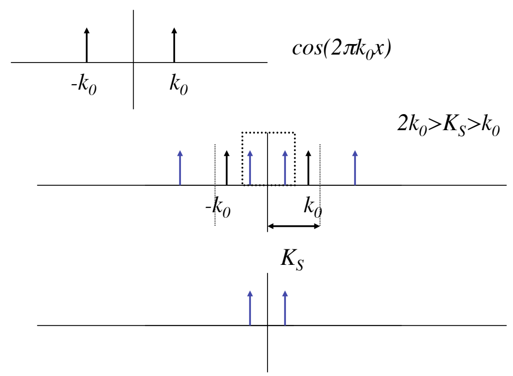
Aliasing Example

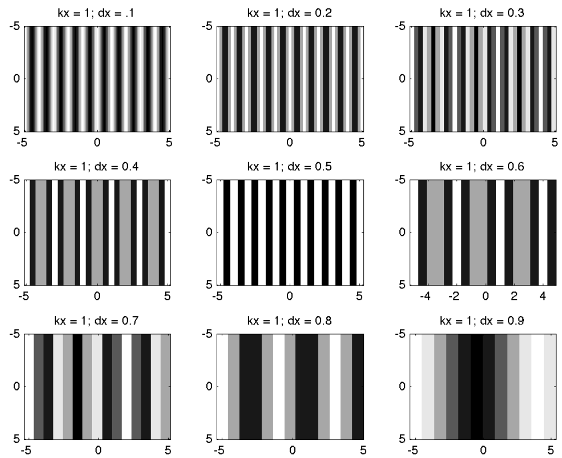


Aliasing Example



Aliasing Example





Example

1. Consider the function $g(x) = \cos^2(2\pi k_0 x)$. Sketch this function. You sample this signal in the spatial domain with a sampling rate $K_S = 1/\Delta x$ (e.g. samples spaced at intervals of Δx). What is the minimum sampling rate that you can use without aliasing? Give an intuitive explanation for your answer.

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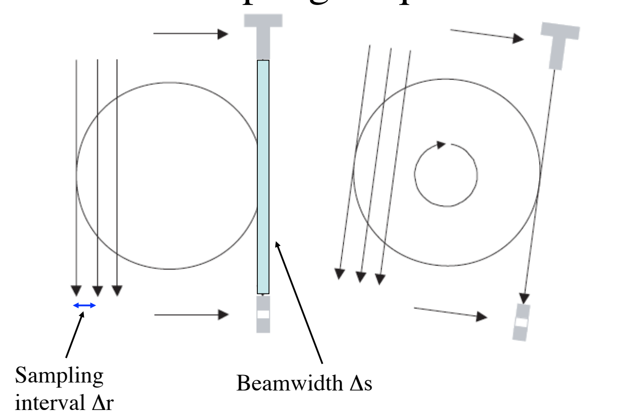
Detector Sampling Requirements



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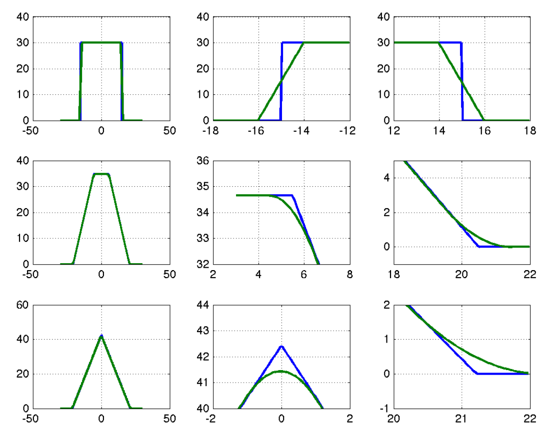
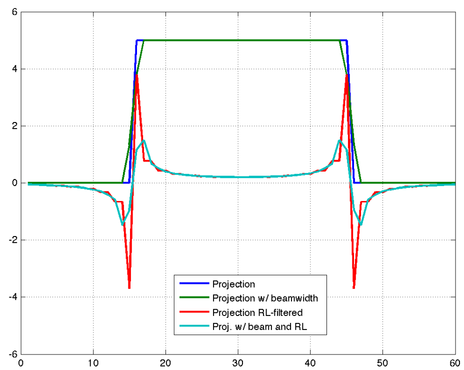
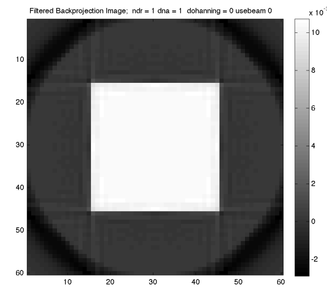
Smoothing of Projections

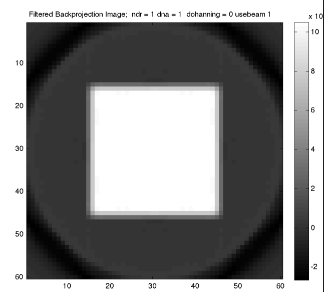


Smoothing of Projections



Smoothing of Projections



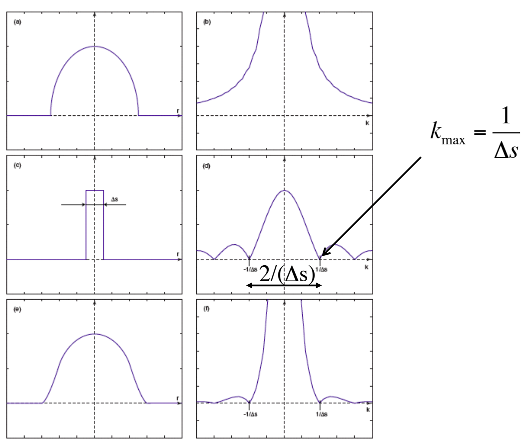


Smoothing of Projection

Projection

Beam Width

Smoothed Projection



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Smoothing and Sampling of Projection

$$g_s(l,\theta) = rect(l/\Delta s) * g(l,\theta)$$

$$G_s(k_x,\theta) = \Delta s \, \mathrm{sinc}(k_x \Delta s) G(k_x,\theta)$$

Approximate highest frequency component as occuring at first zero of the sinc function $k_{max} = \frac{1}{\Delta s}$

Nyquist criterion: $k_S = 2k_{max} = \frac{2}{\Delta s}$

Required sampling period $= \frac{1}{k_S} = \frac{\Delta s}{2}$

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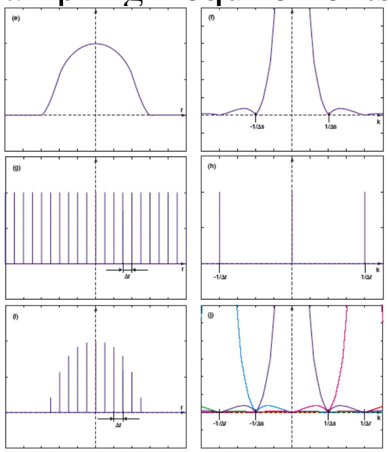
Sampling Requirements

Smoothed Projection

Detectors
$\Delta r \leq \Delta s/2$

Sampled Smooth Projection



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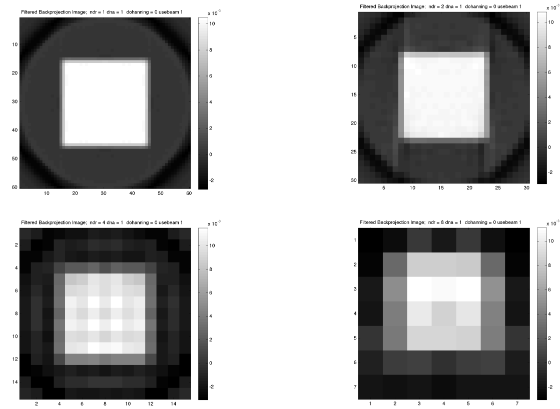
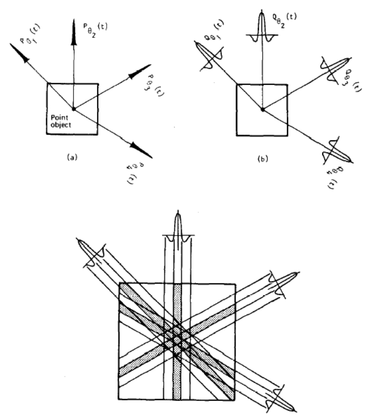
Sampling of Projections



View Aliasing



Kak and Slaney

View Sampling Requirements

View Sampling -- how many views?

Basic idea is that to make the maximum angular sampling the same as the projection sampling.

$$\frac{\pi FOV}{N_{views}} = \Delta r$$

$$N_{views,360} = \frac{\pi FOV}{\Delta r} = \pi N_{proj} \quad \text{(for 360 degrees)}$$

$$N_{views,180} = \frac{\pi N_{proj}}{2} \quad \text{(for 180 degrees)}$$

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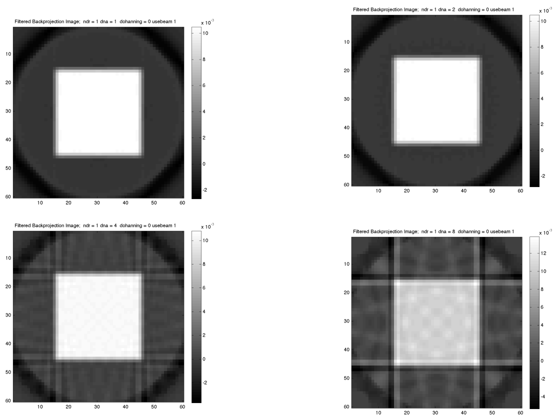
View Aliasing



 Kak and Slaney

Example

beamwidth Δs = 1 mm

Field of View (FOV) = 50 cm

$\Delta r = \Delta s/2$ = 0.5 mm

500 mm/ 0.5 mm = N = 1000 detector samples

$\pi * N$ = 3146 views per 360 degrees

$\approx$ 1500 views per 180 degrees

CT "Rule of Thumb"

$$N_{view} = N_{\det ectors} = N_{pixels}$$

 Suetens 2002

Example

Consider a rectangular object of width 20mm and height 40mm centered at (-10mm, -10mm). The attenuation coefficient of the object is 1 mm^{-1}. The object is imaged with a 1st generation CT scanner with a beamwidth of 1mm. The desired FOV is 100 mm.

Determine the appropriate detector size Δr and the number of radial samples needed to span the FOV. Assume that the middle two samples are acquired at coordinates of -$\Delta r/2$ and $\Delta r/2$.

Determine the number of angular samples required.

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