BENG280A, Principles of Biomedical Imaging 11/9/15
Fall Quarter 2015

Exam

This exam is closed book and closed notes. The use of electronic devices is not permitted. By signing below, I am agreeing to these terms and confirming that I have not discussed the exam material with anyone else (aside from the instructor and the TA) and that the work on this exam is solely my own.

Printed Name ______________________________ Signature ______________________________

*Please sign above and make sure to put your name at the top of each page and to number the pages, and attach these pages to the rest of the exam [**this is worth 5 pts!**]*
When copying down the expressions, make sure that you've made an accurate transcription.
Please show all work to receive partial credit.
Clearly label all sketches to receive full credit!!!

Problem 1 (70 pts + BONUS)
A parallel beam CT imaging system is used to image an object defined as:

$$f(x,y) = rect\left(\frac{x}{2},\frac{y}{2}\right) ** \left[\delta(x-1,y-1)+\delta(x+1,y+1)+\delta(x-1,y+1)\right]$$

(a) (10 pts) Sketch the object. (**HINT**: for the remainder of the problem is there a simpler way to describe the object? – i.e. as the difference of two familiar objects)
(b) (5 pts) Sketch the projection at 0 degrees.
(c) (10 pts) Sketch the projection at 45 degrees.
(d) (10 pts) Derive the Fourier transform of the object
(e) (10 pts) Show that the projection-slice theorem holds for the projection at 0 degrees.
(f) (15 pts) Show that the projection-slice theorem holds for the projection at 45 degrees.
(g) (10 pts) Consider forming a crude back-projection image estimate using the projections at 0 and 90 degrees. Sketch what this estimate would look like.
(h) **BONUS** (20 pts): Derive and sketch the projection at 30 degrees. Show that the projection slice theorem holds. [It is **strongly** recommended that you complete Problem 2 before attempting this bonus]

x

Problem 2 (70 pts)
Consider the object shown in **Figure A**. For Parts e through h you may want to use the grids provided.

a) (10 pts) Write an expression for the object using rect and cosine functions.
b) (10 pts) Derive and sketch the Fourier transform of the object. (Make sure to indicate the peaks and zeroes).
c) (5 pts) Determine the values of Δk_x and Δk_y needed to avoid aliasing
d) (5 pts) Assume that the desired resolutions in the x and y directions are 1 cm and 2 cm, respectively, to generate the sampled object shown in **Figure B**. Determine $k_{x,\max}$ and $k_{y,\max}$
e) (10 pts) Assuming the sampling from part d, draw the phasor orientations as a function of space at $(k_x,k_y)=(0,0)$ and compute the vector sum of the product of the phasors and the sampled object.
f) (10 pts) Assuming the sampling from part d, draw the phasor orientations as a function of space at $(k_x,k_y)=(1/2,0)$ and compute the vector sum of the product of the phasors and the sampled object.
g) (10 pts) Assuming the sampling from part d, draw the phasor orientations as a function of space at $(k_x,k_y)=(1/2,1/4)$ and compute the vector sum of the product of the phasors and the sampled object.

h) (10 pts) Assuming the sampling from part d, draw the phasor orientations as a function of space at one of the values of (k_x, k_y) for which the Fourier transform is maximized. Indicate the value of (k_x, k_y) that you are using. Compute the vector sum of the product of the phasors and the sampled object.

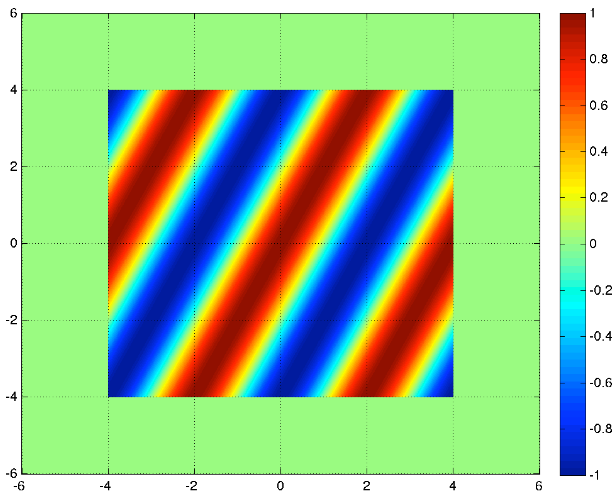


Figure A.

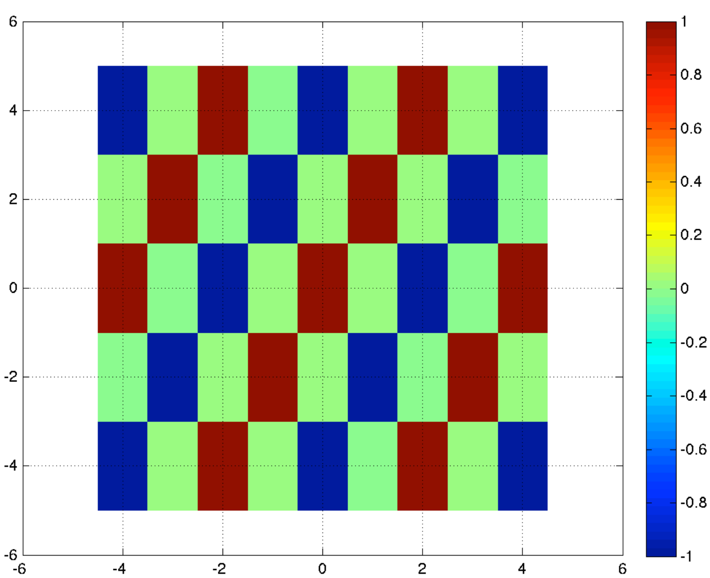
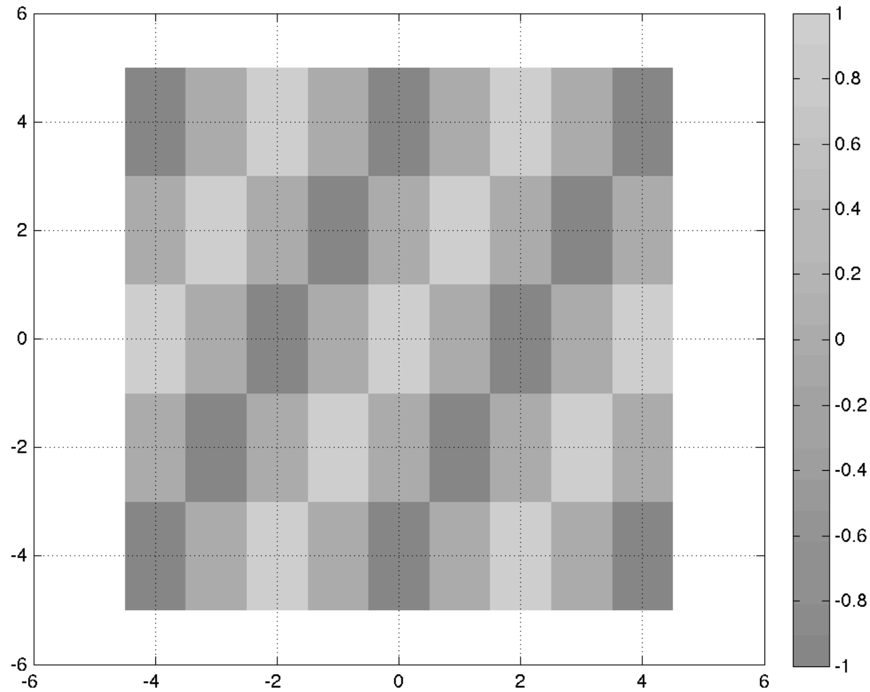


Figure B.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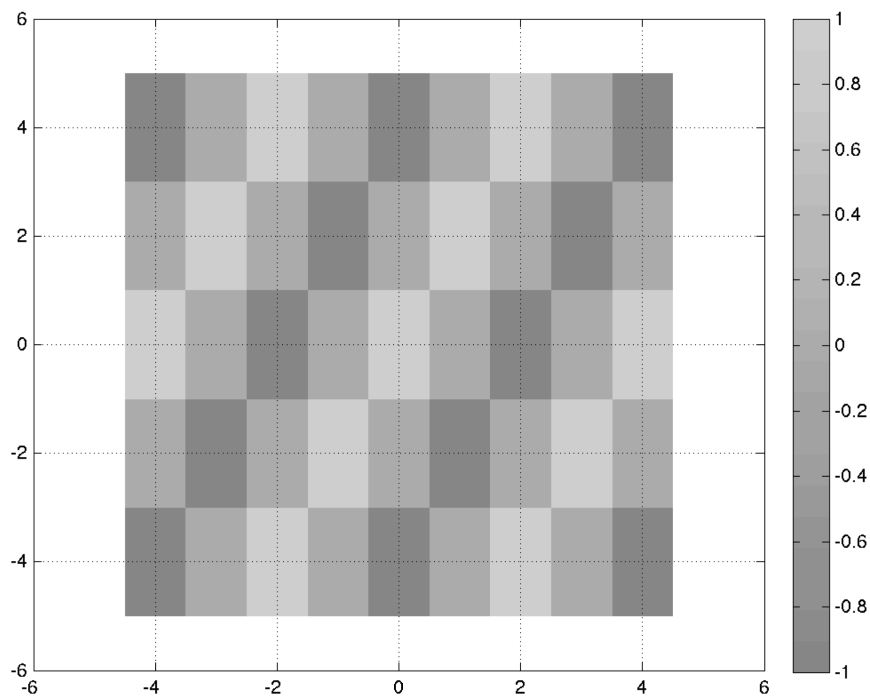
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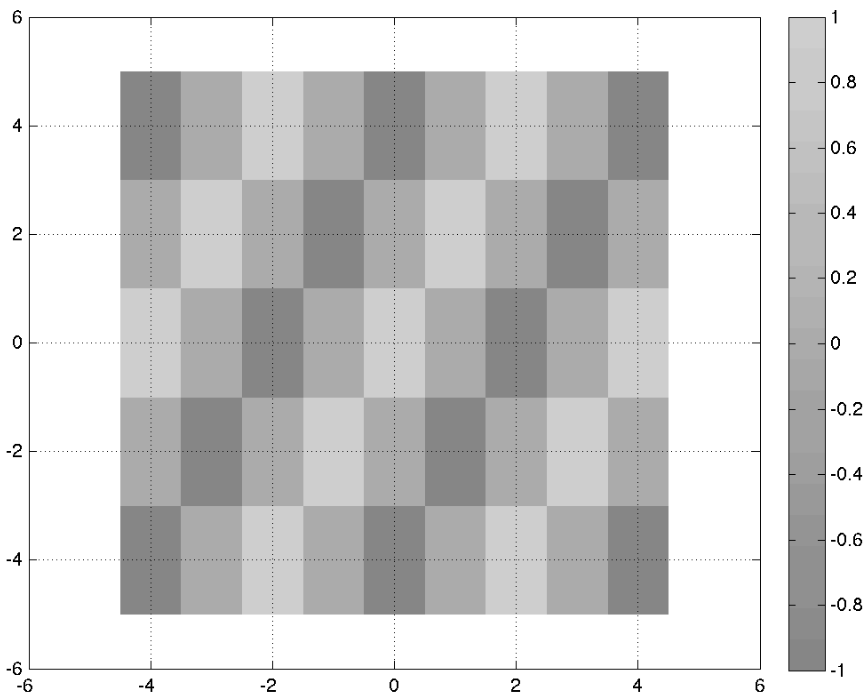
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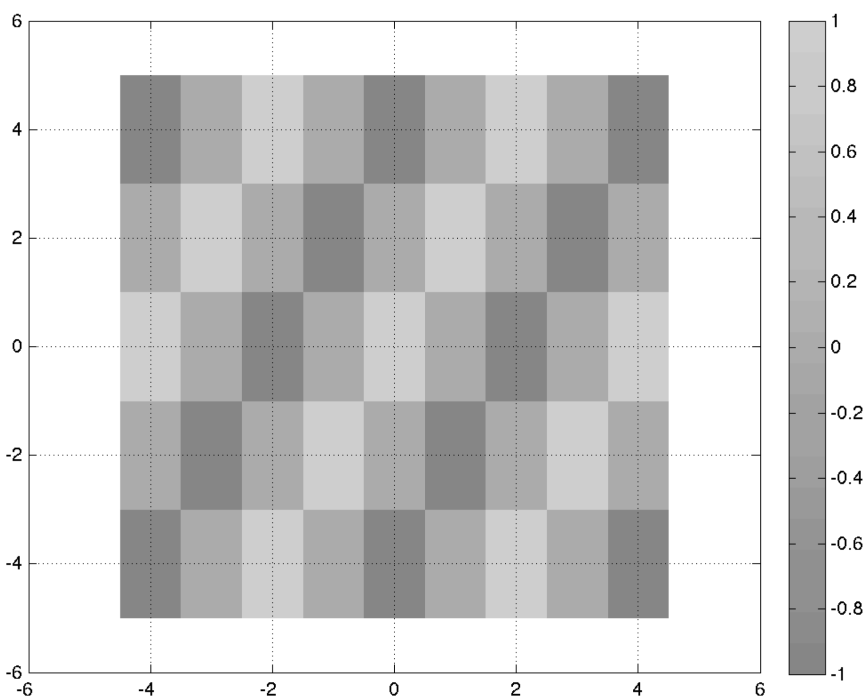
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Basic Properties

Linearity

$$F\left[ag(x,y)+bh(x,y)\right]=aG(k_x,k_y)+bH(k_x,k_y)$$

Scaling

$$F\left\{g(ax)\right\}=\frac{1}{|a|}G\left(\frac{k_x}{a}\right) \qquad F\left[g(ax,by)\right]=\frac{1}{|ab|}G\left(\frac{k_x}{a},\frac{k_x}{b}\right)$$

Separability

$$F\left[g(x)h(y)\right]=G\left(k_x\right)H\left(k_y\right)$$

Duality

$$F\left\{G(x)\right\}=g(-k_x)$$

Shift

$$F\left[g(x-a,y-b)\right]=G(k_x,k_y)e^{-j2\pi(k_xa+k_yb)}$$

Convolution

$$F\left[g(x,y)**h(x,y)\right]=G(k_x,k_y)H(k_x,k_y)$$

Multiplication

$$F\left[g(x,y)h(x,y)\right]=G(k_x,k_y)**H(k_x,k_y)$$

Modulation

$$F\left[g(x,y)e^{j2\pi(xa+yb)}\right]=G(k_x-a,k_y-b)$$

Transform Pairs

$$\delta(x) \Leftrightarrow 1$$

$$\delta(x - x_0) \Leftrightarrow e^{-j2\pi k_x x_0}$$

$$1 \Leftrightarrow \delta(k_x)$$

$$rect(x) \Leftrightarrow \mathrm{sinc}(k_x)$$

$$\mathrm{sinc}(x) \Leftrightarrow rect(k_x)$$

$$e^{j2\pi k_0 x} \Leftrightarrow \delta(k_x - k_0)$$

$$\cos 2\pi k_0 x \Leftrightarrow \frac{1}{2}\left(\delta(k_x - k_0) + \delta(k_x + k_0)\right)$$

$$\sin 2\pi k_0 x \Leftrightarrow \frac{1}{2j}\left(\delta(k_x - k_0) - \delta(k_x + k_0)\right)$$

$$\Pi(x)\Pi(y) \Leftrightarrow \mathrm{sinc}(k_x)\mathrm{sinc}(k_y)$$

$$\Lambda(x) \Leftrightarrow \mathrm{sinc}^2(k_x)$$

$$comb(x) \Leftrightarrow comb(k_x)$$

Useful facts and definitions

$$\mathrm{sinc}(k_x) = \frac{\sin(\pi k_x)}{\pi k_x}$$

$$\delta(x, y) = \delta(x)\delta(y)$$

$$\mathrm{sinc}(k_x, k_y) = \mathrm{sinc}(k_x)\mathrm{sinc}(k_y)$$

$$rect(x, y) = rect(x)rect(y)$$

$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & otherwise \end{cases}$$

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$