BENG280A, Principles of Biomedical Imaging 10/31/15
Fall Quarter 2015

HOMEWORK #5
Due at 5 pm on Friday 11/6/15

Homework policy: Homeworks can be turned in during class prior to the due date or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material. It is recommended that you make a copy of the homework for yourself (e.g. scan it in) before you turn it in.

Required Readings: MRI notes by Lars Hanson (PDF on the website) (Read sections 2-5 and 9.1-9.7).

Textbook reading: Skim sections 2.1, 2.2, and 3.1-3.5 (read this for general understanding). Read sections: 4.1-4.5, 5.1-5.5, 5.6.2 for general understanding. Focus on Sections 5.4, 5.5, and 5.6.2.

Videos: View the MRI safety video at http://www.youtube.com/watch?v=xefyXb5u658. Also, watch the following video http://www.youtube.com/watch?v=9SOUJP5dFEg to see what happens when a magnet loses its field.

Problem 1

Consider the object $f(x,y) = \cos\left(\frac{1}{\sqrt{3}}\pi x + 2\pi y\right)$

a. Sketch the object by hand, labeling critical points, such as the zero-crossings and the distances between peaks. Also use MATLAB to make an image of the object and compare the MATLAB result to your sketch.
b. Consider sampling the object in both the x and y directions with sample intervals of Δ_x and Δ_y, respectively. Indicate what sample intervals should be used to avoid aliasing.
c. Now consider imaging the object with a parallel beam CT imaging system. At what angle will the projection be non-zero?
d. We now wish to sample the non-zero projection. What sampling interval should we use to avoid aliasing?
e. Now consider the object $g(x,y) = \left(f(x,y)\right)^2$. Answer items (c) and (d) for this object

Problem 2

In this problem, we will continue our examination of the square object from the prior homeworks. Assume an FOV of 40 mm and a beamwidth Δs = 1 mm

a) Determine the appropriate detector size Δr and the number of radial samples needed to span the FOV. Assume that the middle two samples are acquired at coordinates of -Δr/2 and Δr/2 and use this to define the sampling grid for your projections.
b) Determine the number of angular samples required. For the simulations, round this up to the nearest multiple of 4. Use this number of angular samples when reconstructing your image.
c) Now consider the effect of the finite beamwidth. Take the code from prior homeworks and use it to compute the projections on a 10x oversampled grid, where the spacing is Δr/10. Use the MATLAB conv function to filter these projections with a rect function of the appropriate width. Compare the original projections (on the oversampled grid) with the smoothed projections and provide plots of these comparisons for theta = 0, 45, and 90 degrees (Hint: compare with the class notes).
d) Resample your smoothed projection to get back to the desired spacing of Δr (i.e. take every 10^{th} sample of the oversampled projection). Now take the resampled smoothed projections and pass them through the Ram-Lak filter from last week's homework. Backproject to reconstruct the

image with the filtered and smoothed projections. Compare the image with what you obtained last week with the filtered ideal projections. Note that for your Ram-Lak filter, you will want to use $k_{max} = 1/\Delta s$.

e) Examine the effects of reducing the sampling of the projection. Instead of resampling the smoothed projections with a spacing of Δr, consider what happens when you use spacings of $2\Delta r$, $4\Delta r$, and $8\Delta r$. Show the reconstructed images and comment on what you see.

f) Examine the effects of undersampling in the angular dimension. See what happens when you only use every 2nd, 4th, or 8th angle. Show the reconstructed images and comment on what you see.

Problem 3

From the safety video, answer the following questions: (a) What are helium and nitrogen used for in the MRI system? (b) What does the term quench mean? (c) Why is it dangerous to smoke near an MRI system? Find a example (on the web) of a large object that's been pulled into the magnet and include a copy of the image.

Problem 4 on next page.

Problem 4

(A) k-space coordinates	(B) Phasor diagram	(C)Vector sum for object 1	(D) Vector sum for Object 2	(E) Vector sum for Object 3
-0.5, 0				
0.25, -0.25				
-0.25, 0.25				
0, 0.25				
-0.5, 0.5				
0, -0.5				
0.25 0.25				
0, -0.25				
0,0				

a) Each of the phasor diagrams in Figure 1 corresponds to one of the k-space locations in column A of the table. In column (B), indicate the correct phasor diagram (labeled (a) through (i)). Note that for each (kx,ky) location, the phasor diagram is a representation of exp(-j*2*pi(kx*x + ky*y)). Also note the negative sign in the definition of the phasor.

b) For each object shown in Figure 2, write down the vector sum in the appropriate column. Determine where the peaks in the vector sum and provide an explanation below why the peaks occur where they do.

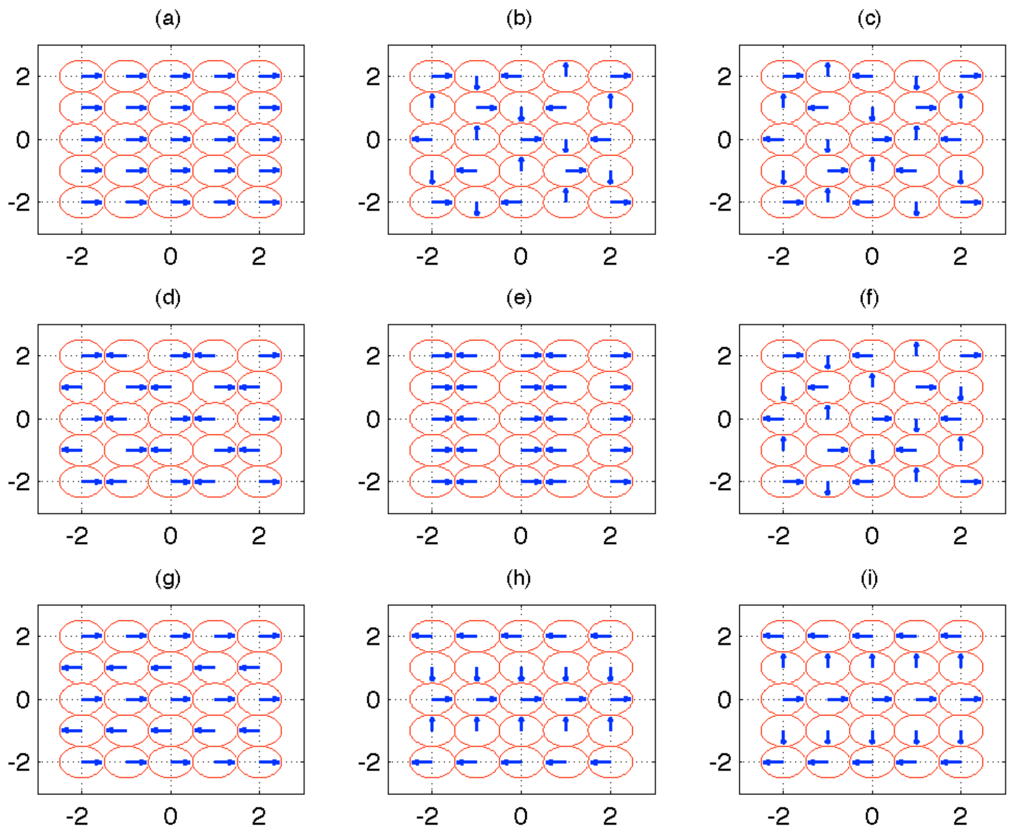

Figure 1. Phasor Diagrams

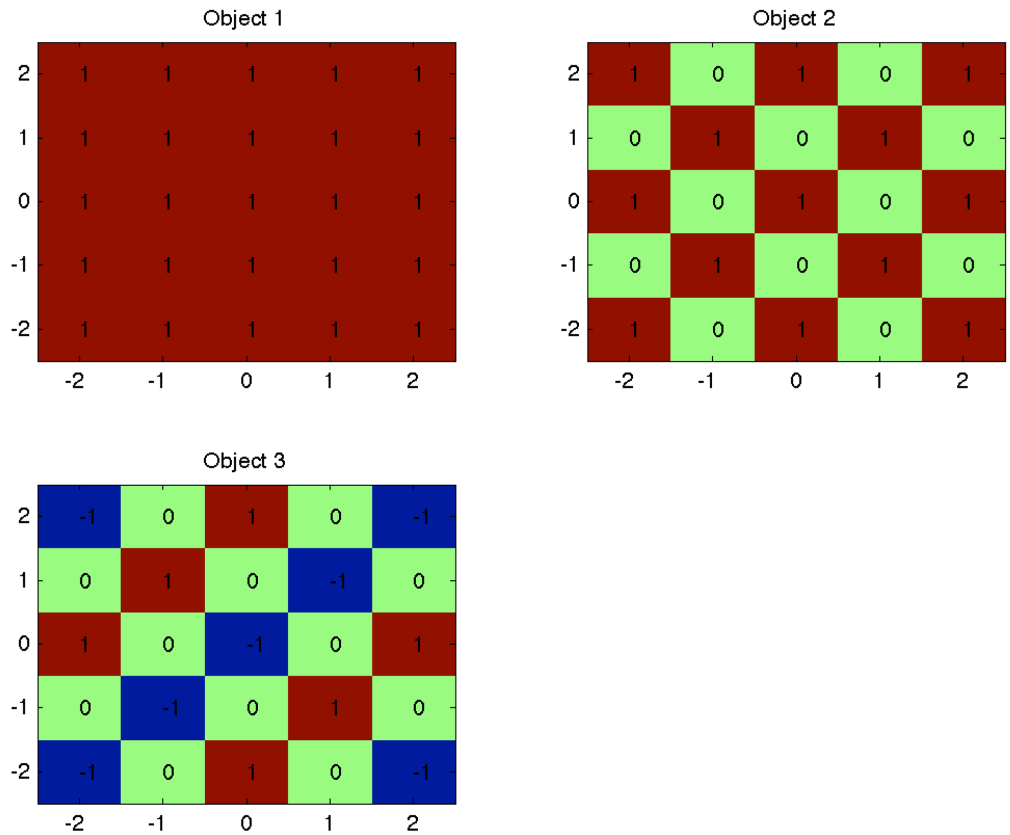

Figure 2. Objects