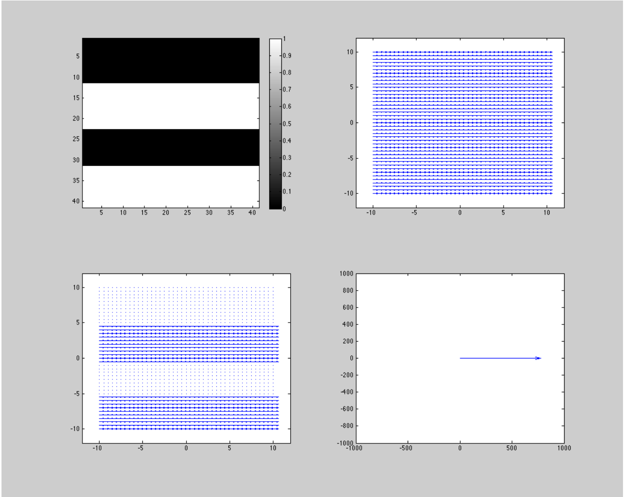
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2015
MRI Lecture 3



Signal Equation

Demodulate the signal to obtain

$$
\begin{aligned}
s(t) &= e^{j\omega_0 t} s_r(t) \\
&= \int_x \int_y m(x,y)\exp\left(-j\gamma \int_o^t \vec{G}(\tau)\cdot \vec{r}\, d\tau\right) dxdy \\
&= \int_x \int_y m(x,y)\exp\left(-j\gamma \int_o^t \left[G_x(\tau)x + G_y(\tau)y\right] d\tau\right) dxdy \\
&= \int_x \int_y m(x,y)\exp\left(-j2\pi\left(k_x(t)x + k_y(t)y\right)\right) dxdy
\end{aligned}
$$

Where

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

K-space

At each point in time, the received signal is the Fourier transform of the object

$$s(t) = M\left(k_x(t), k_y(t)\right) = F\left[m(x,y)\right]\Big|_{k_x(t),k_y(t)}$$

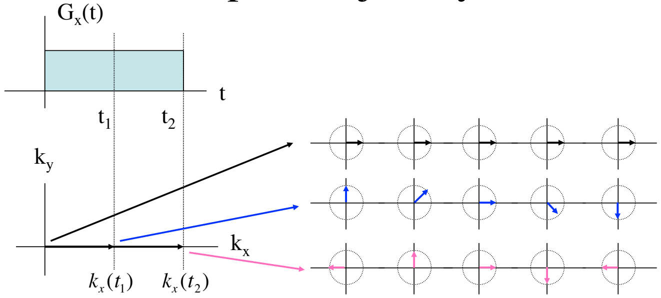
evaluated at the spatial frequencies:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Thus, the gradients control our position in k-space. The design of an MRI pulse sequence requires us to efficiently cover enough of k-space to form our image.

K-space trajectory



$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

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Units

Spatial frequencies (k_x, k_y) have units of 1/distance. Most commonly, 1/cm

Gradient strengths have units of (magnetic field)/ distance. Most commonly G/cm or mT/m

$\gamma/(2\pi)$ has units of Hz/G or Hz/Tesla.

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$= [Hz/Gauss][Gauss/cm][\sec]$$
$$= [1/cm]$$

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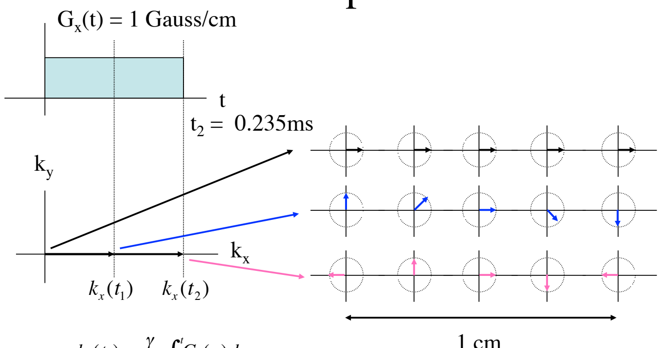
Example



$$k_x(t_2) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$
$$= 4257 Hz/G \cdot 1G/cm \cdot 0.235\times10^{-3} s$$
$$= 1\ cm^{-1}$$

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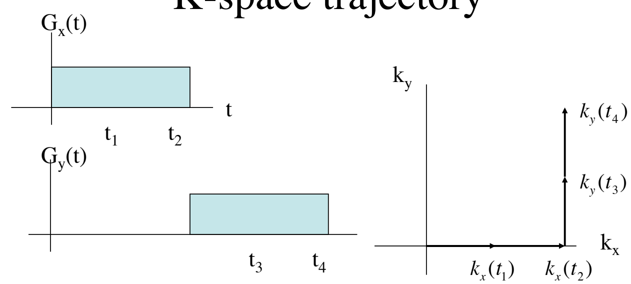
K-space trajectory



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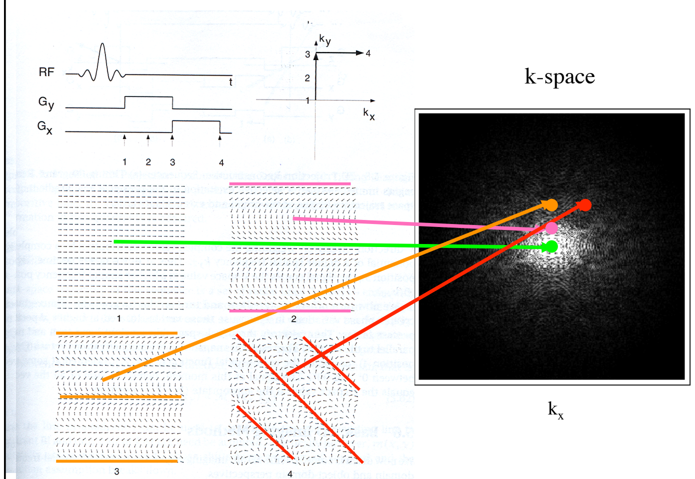
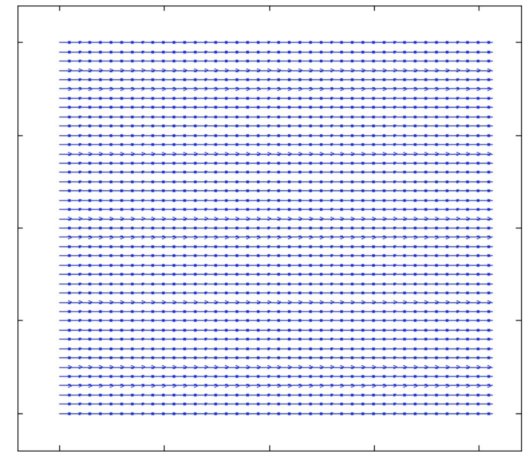
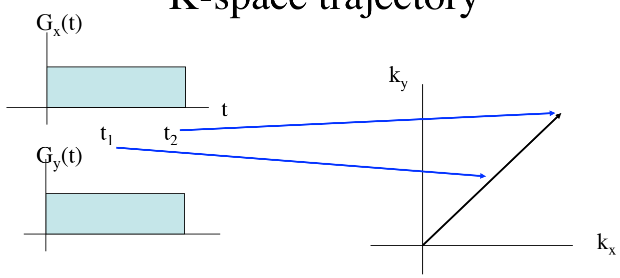
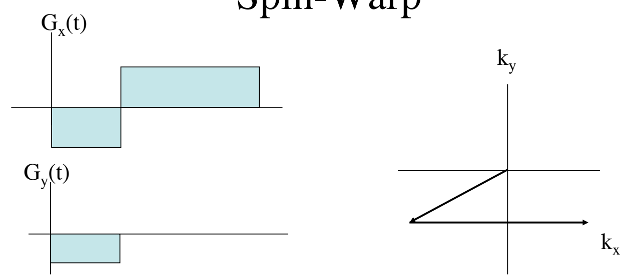




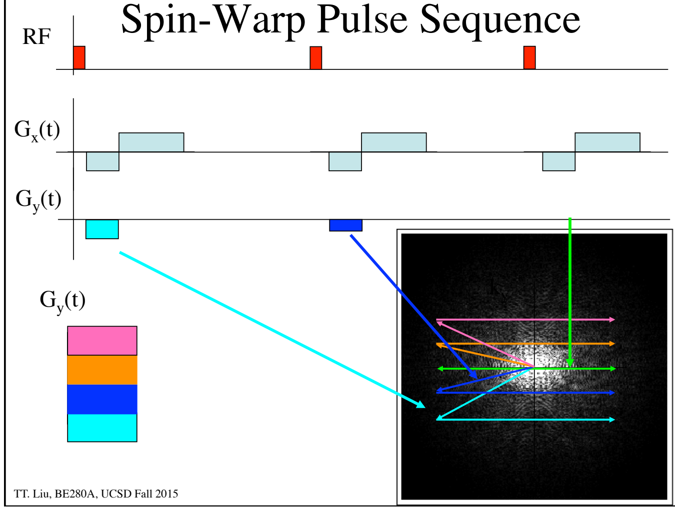
K-space trajectory

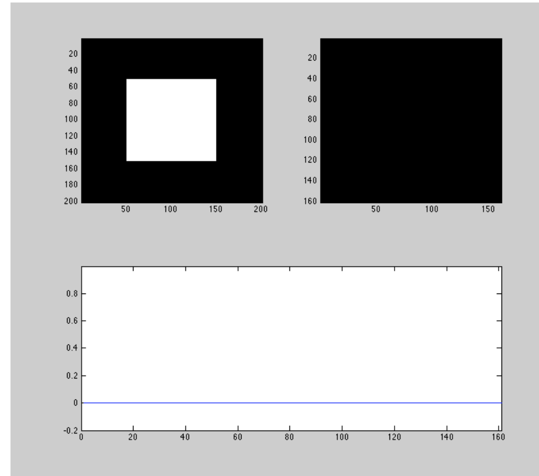


Spin-Warp



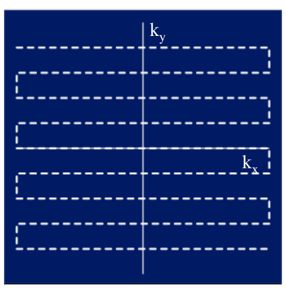
Spin-Warp Pulse Sequence



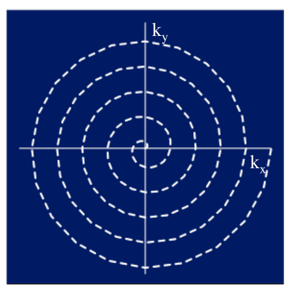


K-space trajectories

EPI



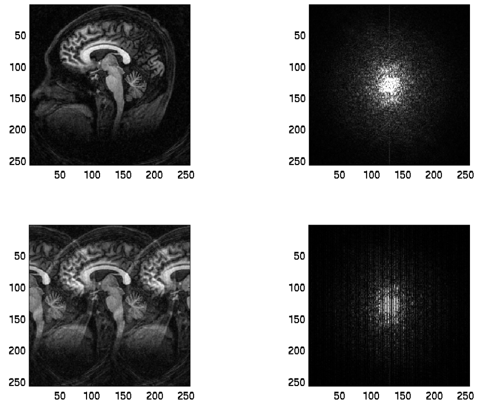
Spiral



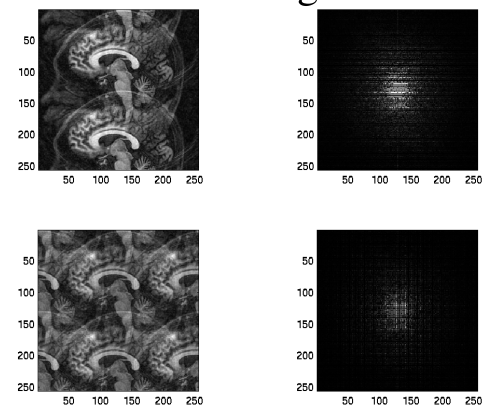
Credit: Larry Frank

Very Expensive Bagpipes
i.e.
Prompting fMRI Subjects
with Gradient Waveforms

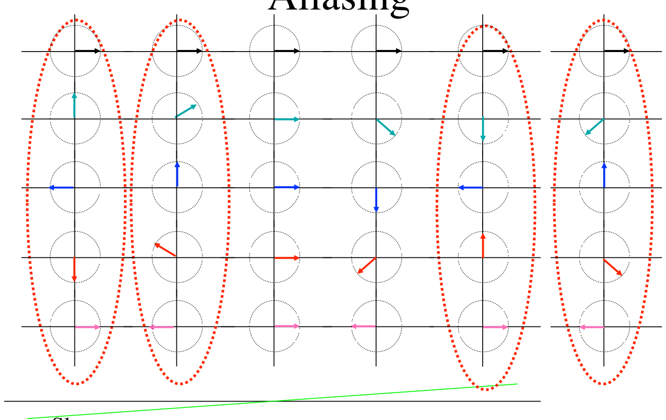
Sampling in k-space



Aliasing

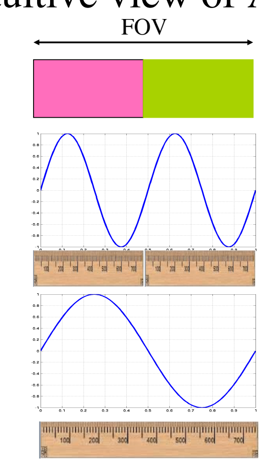


Aliasing



$\Delta B_z(x)=G_x x$

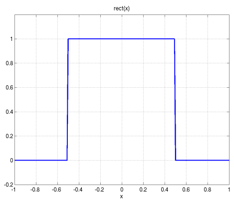
Intuitive view of Aliasing



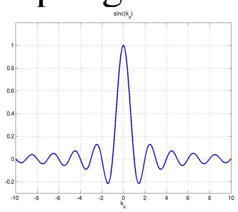
$k_x = 2/FOV$

$k_x = 1/FOV$

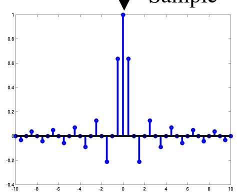
Fourier Sampling



F



Sample

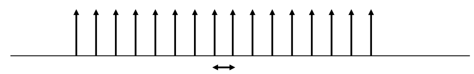


Instead of sampling the signal, we sample its Fourier Transform

??? F^{-1}

Fourier Sampling

$(1/\Delta k_x)\ comb(k_x/\Delta k_x)$



k_x

Δk_x

$$G_S(k_x) = G(k_x)\frac{1}{\Delta k_x}comb\left(\frac{k_x}{\Delta k_x}\right)$$
$$= G(k_x)\sum_{n=-\infty}^{\infty}\delta(k_x - n\Delta k_x)$$
$$= \sum_{n=-\infty}^{\infty}G(n\Delta k_x)\delta(k_x - n\Delta k_x)$$

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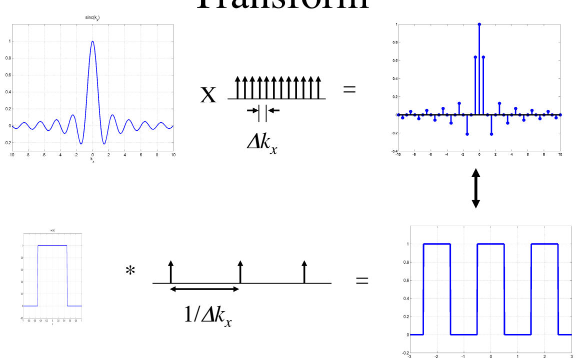
Fourier Sampling -- Inverse Transform



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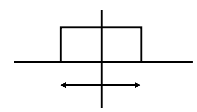
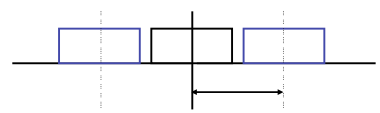
Nyquist Condition



FOV (Field of View)



$1/\Delta k_x$

To avoid overlap, $1/\Delta k_x >$ FOV, or equivalently, $\Delta k_x < 1/$FOV

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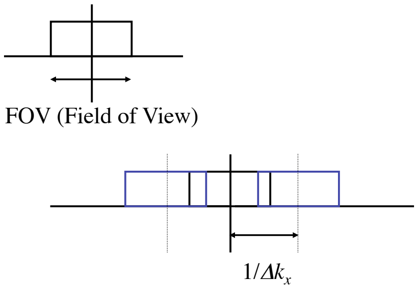
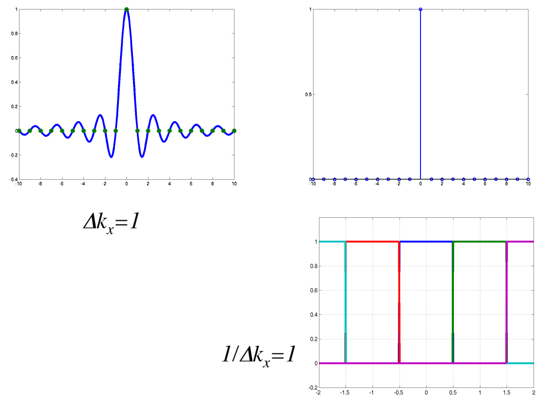
Aliasing



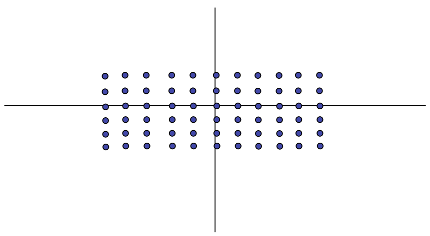
Aliasing occurs when $1/\Delta k_x <$ FOV

Aliasing Example



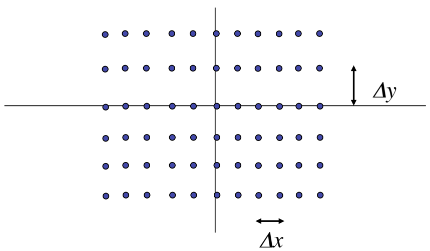
2D Comb Function

$$comb(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m, y-n)$$
$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m)\delta(y-n)$$
$$= comb(x)comb(y)$$



Scaled 2D Comb Function

$$comb(x/\Delta x, y/\Delta y) = comb(x/\Delta x)comb(y/\Delta y)$$
$$= \Delta x \Delta y \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m\Delta x)\delta(y-n\Delta y)$$

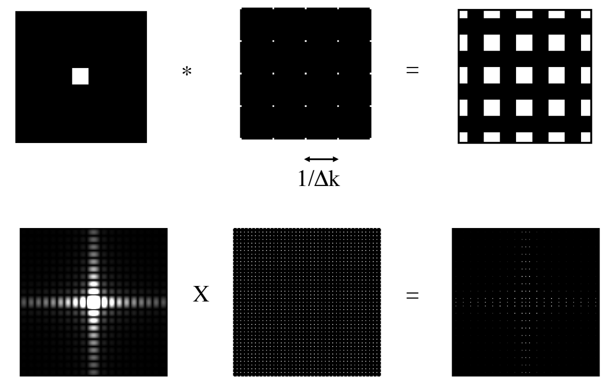


2D k-space sampling

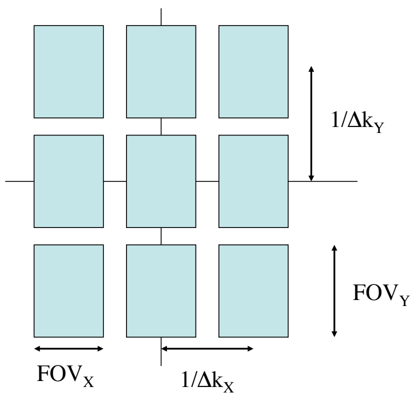
$$G_S(k_x,k_y) = G(k_x,k_y)\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right)$$

$$= G(k_x,k_y)\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(k_x - m\Delta k_x, k_y - n\Delta k_y)$$

$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}G(m\Delta k_x, n\Delta k_y)\delta(k_x - m\Delta k_x, k_y - n\Delta k_y)$$



Nyquist Conditions



$1/\Delta k_Y > FOV_Y$

$1/\Delta k_X > FOV_X$

Windowing

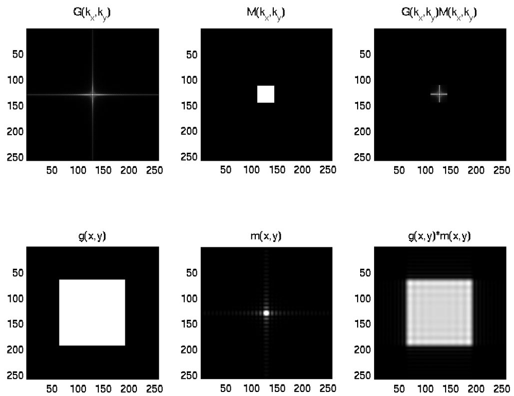
Windowing the data in Fourier space

$$G_W(k_x,k_y) = G(k_x,k_y)W(k_x,k_y)$$

Results in convolution of the object with the inverse transform of the window

$$g_W(x,y) = g(x,y) * w(x,y)$$

Resolution



Windowing Example

$$W(k_x,k_y) = rect\left(\frac{k_x}{W_{k_x}}\right)rect\left(\frac{k_y}{W_{k_y}}\right)$$

$$w(x,y) = F^{-1}\left[rect\left(\frac{k_x}{W_{k_x}}\right)rect\left(\frac{k_y}{W_{k_y}}\right)\right]$$
$$= W_{k_x}W_{k_y}\,\mathrm{sinc}\left(W_{k_x}x\right)\mathrm{sinc}\left(W_{k_y}y\right)$$

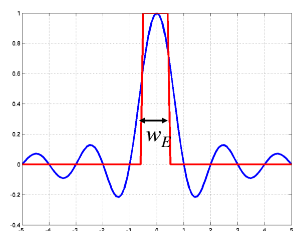
$$g_W(x,y) = g(x,y) ** W_{k_x}W_{k_y}\,\mathrm{sinc}\left(W_{k_x}x\right)\mathrm{sinc}\left(W_{k_y}y\right)$$

Effective Width

$$w_E = \frac{1}{w(0)}\int_{-\infty}^{\infty} w(x)dx$$

Example

$$w_E = \frac{1}{1}\int_{-\infty}^{\infty}\mathrm{sinc}(W_{k_x}x)dx$$
$$= F\left[\mathrm{sinc}(W_{k_x}x)\right]\Big|_{k_x=0}$$
$$= \frac{1}{W_{k_x}}rect\left(\frac{k_x}{W_{k_x}}\right)\Bigg|_{k_x=0}$$
$$= \frac{1}{W_{k_x}}$$



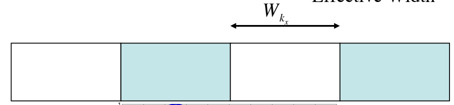
$-\frac{1}{W_{k_x}}$ $\quad \frac{1}{W_{k_x}}$

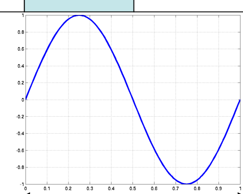
Resolution and spatial frequency

With a window of width W_{k_x} the highest spatial frequency is $W_{k_x}/2$.

This corresponds to a spatial period of $2/W_{k_x}$.

$\frac{1}{W_{k_x}}$ = Effective Width



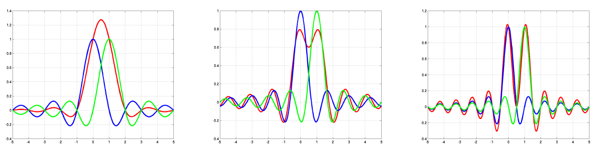


$\frac{2}{W_{k_x}}$

Windowing Example

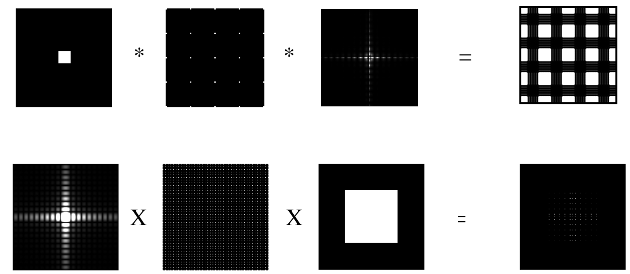
$$g(x,y) = [\delta(x) + \delta(x-1)]\delta(y)$$

$$\begin{aligned} g_W(x,y) &= [\delta(x) + \delta(x-1)]\delta(y) ** W_{k_x} W_{k_y} \operatorname{sinc}(W_{k_x} x)\operatorname{sinc}(W_{k_y} y) \\ &= W_{k_x} W_{k_y}\left([\delta(x) + \delta(x-1)] * \operatorname{sinc}(W_{k_x} x)\right)\operatorname{sinc}(W_{k_y} y) \\ &= W_{k_x} W_{k_y}\left(\operatorname{sinc}(W_{k_x} x) + \operatorname{sinc}(W_{k_x}(x-1))\right)\operatorname{sinc}(W_{k_y} y) \end{aligned}$$



$W_{k_x} = 1$ $\qquad$ $W_{k_x} = 1.5$ $\qquad$ $W_{k_x} = 2$

Sampling and Windowing



Sampling and Windowing

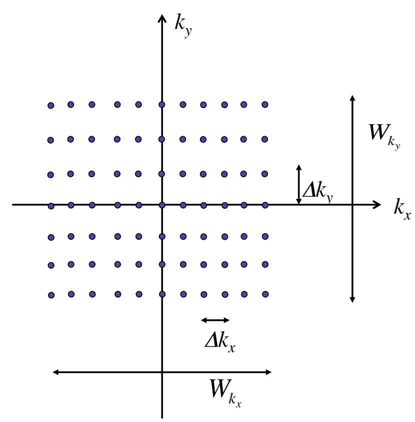
Sampling and windowing the data in Fourier space

$$G_{SW}(k_x,k_y) = G(k_x,k_y)\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right) rect\left(\frac{k_x}{W_{k_x}},\frac{k_y}{W_{k_y}}\right)$$

Results in replication and convolution in object space.

$$g_{SW}(x,y) = W_{k_x} W_{ky} g(x,y) ** comb(\Delta k_x x, \Delta k_y y) ** \operatorname{sinc}(W_{k_x} x)\operatorname{sinc}(W_{k_y} y)$$

Sampling and Windowing



$\Delta k_Y < 1/(FOV_Y)$

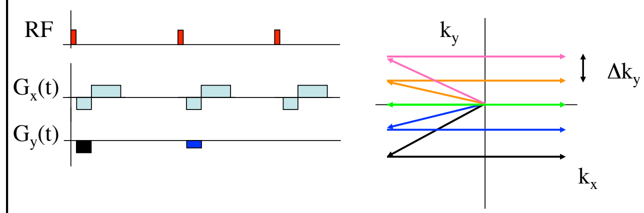
$\Delta k_X < 1/(FOV_X)$

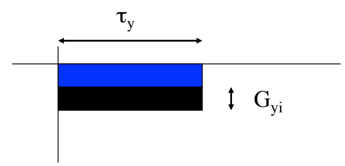
$W_{k_x} > 1/\delta_x$

$W_{k_y} > 1/\delta_y$

GE Medical Systems 2003

Sampling in k_y

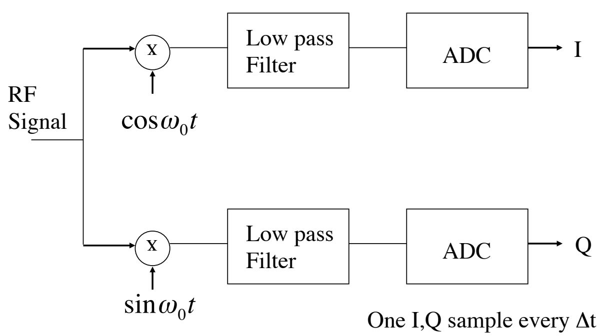




$$\Delta k_y = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

$$FOV_y = \frac{1}{\Delta k_y}$$

Sampling in k_x

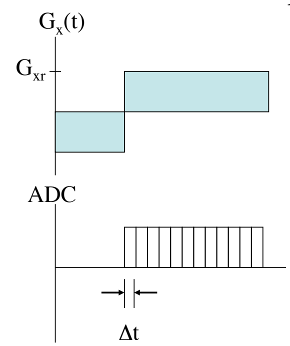


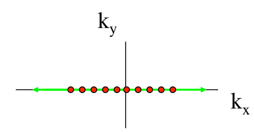
One I,Q sample every Δt

M= I+jQ

Note: In practice, there are number of ways of implementing this processing.

Sampling in k_x



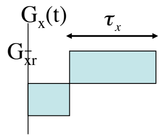


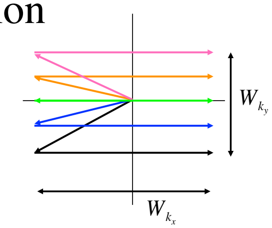
$$\Delta k_x = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

$$FOV_x = \frac{1}{\Delta k_x}$$

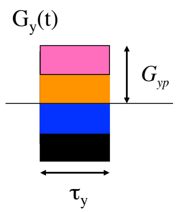
Resolution

$$\delta_x = \frac{1}{W_{k_x}} = \frac{1}{2k_{x,\max}} = \frac{1}{\frac{\gamma}{2\pi}G_{xr}\tau_x}$$





$$\delta_y = \frac{1}{W_{k_y}} = \frac{1}{2k_{y,\max}} = \frac{1}{\frac{\gamma}{2\pi}2G_{yp}\tau_y}$$



Example

Goal:

$FOV_x = FOV_y = 25.6$ cm

$\delta_x = \delta_y = 0.1$ cm

Readout Gradient:

$$FOV_x = \frac{1}{\frac{\gamma}{2\pi}G_{xr}\Delta t}$$

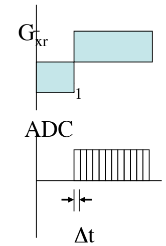
Pick $\Delta t = 32\ \mu$sec

$$G_{xr} = \frac{1}{FOV_x\frac{\gamma}{2\pi}\Delta t} = \frac{1}{(25.6cm)(42.57\times10^6 T^{-1}s^{-1})(32\times10^{-6}s)}$$

$= 2.8675 \times 10^{-5}$ T/cm

$= .28675$ G/cm

1 Gauss $= 1\times10^{-4}$ Tesla



Example

Readout Gradient:

$$\delta_x = \frac{1}{\frac{\gamma}{2\pi}G_{xr}\tau_x}$$

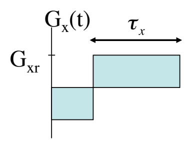
$$\tau_x = \frac{1}{\delta_x\frac{\gamma}{2\pi}G_{xr}} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(0.28675\ \text{G/cm})}$$

$= 8.192$ ms

$= N_{read}\Delta t$

where

$$N_{read} = \frac{FOV_x}{\delta_x} = 256$$



Example

Phase - Encode Gradient:

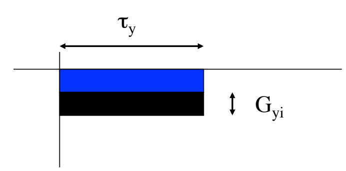
$$FOV_y = \frac{1}{\frac{\gamma}{2\pi}G_{yi}\tau_y}$$

Pick $\tau_y = 4.096$ *m*sec

$$G_{yi} = \frac{1}{FOV_y\frac{\gamma}{2\pi}\tau_y} = \frac{1}{(25.6cm)(42.57\times10^6 T^{-1}s^{-1})(4.096\times10^{-3}s)}$$

$= 2.2402\times10^{-7}$ T/cm

$= .00224$ G/cm



Example

Phase - Encode Gradient :

$$\delta_y = \frac{1}{\frac{\gamma}{2\pi} 2 G_{yp} \tau_y}$$

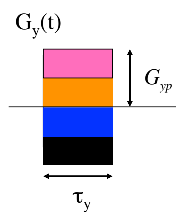
$$G_{yp} = \frac{1}{\delta_y 2 \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(4.096 \times 10^{-3}\ s)}$$

$$= 0.2868 \text{ G/cm}$$

$$= \frac{N_p}{2} G_{yi}$$

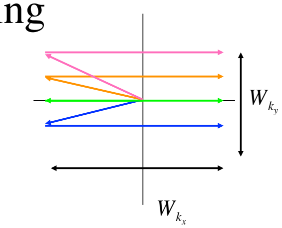
where

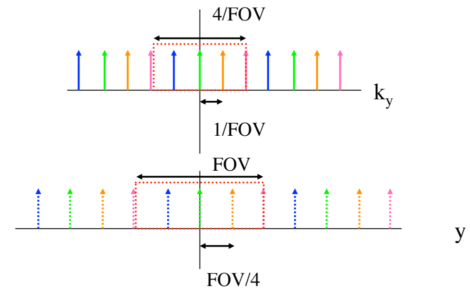
$$N_p = \frac{FOV_y}{\delta_y} = 256$$



Sampling

In practice, an even number (typically power of 2) sample is usually taken in each direction to take advantage of the Fast Fourier Transform (FFT) for reconstruction.

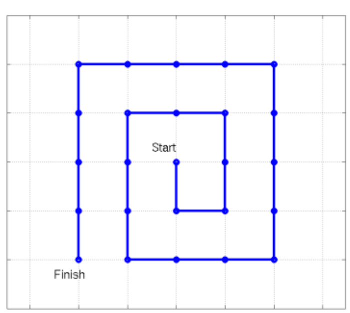




Example

Consider the k-space trajectory shown below. ADC samples are acquired at the points shown with $\Delta t = 10\ \mu\text{sec}$. The desired FOV (both x and y) is 10 cm and the desired resolution (both x and y) is 2.5 cm. Draw the gradient waveforms required to achieve the k-space trajectory. Label the waveform with the gradient amplitudes required to achieve the desired FOV and resolution. Also, make sure to label the time axis correctly.

$$Assume\ \gamma/(2\pi) = 4000 Hz/G$$



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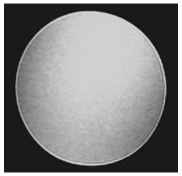
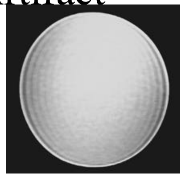
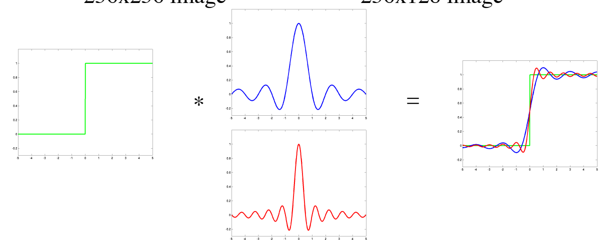
Gibbs Artifact



256x256 image

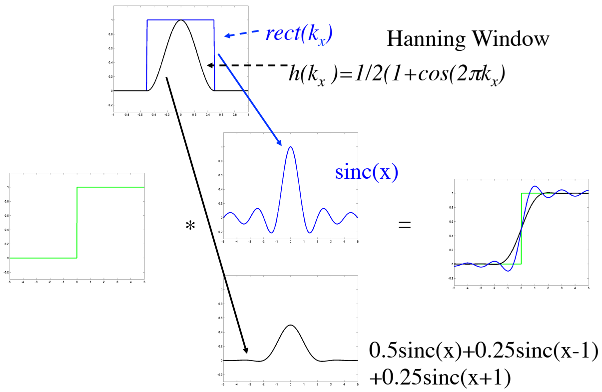


256x128 image



Images from http://www.mritutor.org/mritutor/gibbs.htm

Apodization



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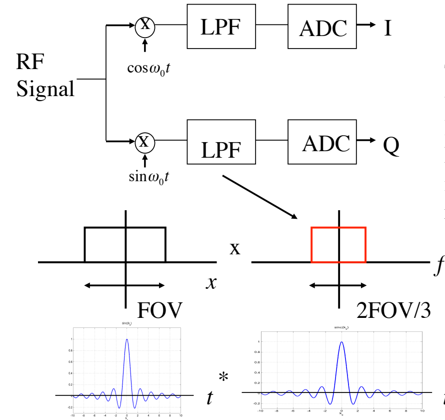
Images from http://www.mritutor.org/mritutor/gibbs.htm

Aliasing and Bandwidth



Temporal filtering in the readout direction limits the readout FOV. So there should never be aliasing in the readout direction.

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Aliasing and Bandwidth

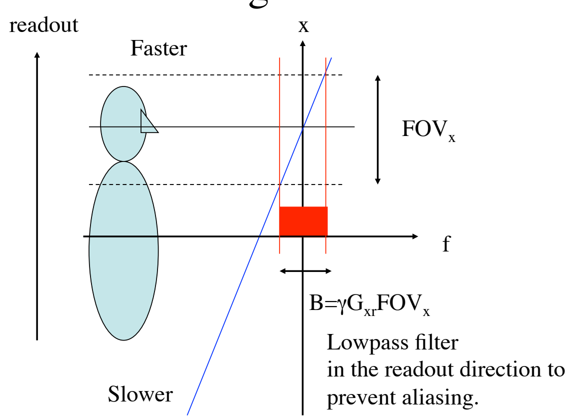

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Figure 7-31 Default Axial Directions

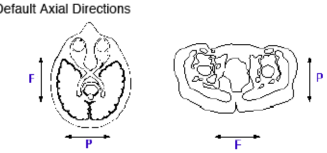
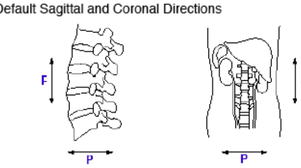


Figure 7-32 Default Sagittal and Coronal Directions



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