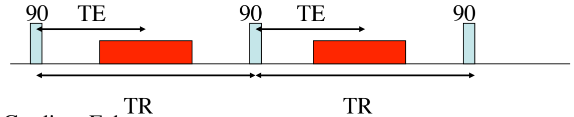
Bioengineering 280A
Principles of Biomedical Imaging

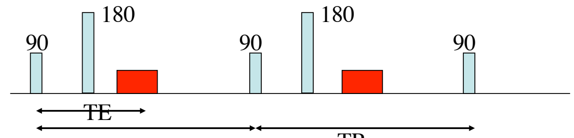
Fall Quarter 2015
MRI Lecture 5

Saturation Recovery Sequence



Gradient Echo

$$I(x,y) = \rho(x,y)\left[1-e^{-TR/T_1(x,y)}\right]e^{-TE/T_2^*(x,y)}$$



Spin Echo

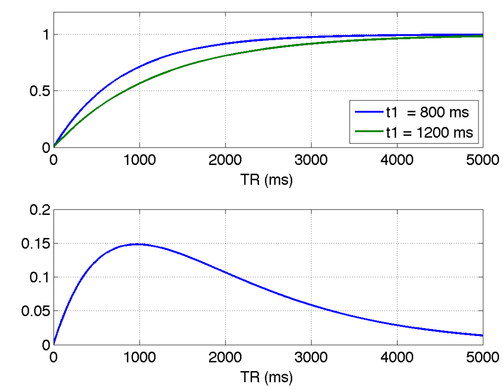
$$I(x,y) = \rho(x,y)\left[1-e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

T1-Weighted Scans

Make TE very short compared to either T_2 or T_2^*. The resultant image has both proton and T_1 weighting.

$$I(x,y) \approx \rho(x,y)\left[1-e^{-TR/T_1(x,y)}\right]$$

T1-Weighted Scans

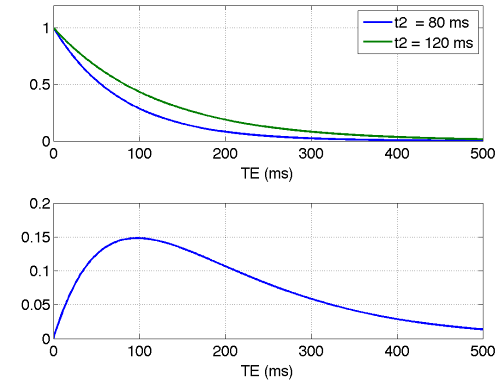


T2-Weighted Scans

Make TR very long compared to T_1 and use a spin-echo pulse sequence. The resultant image has both proton and T_2 weighting.

$$I(x,y) \approx \rho(x,y)e^{-TE/T_2}$$

T2-Weighted Scans

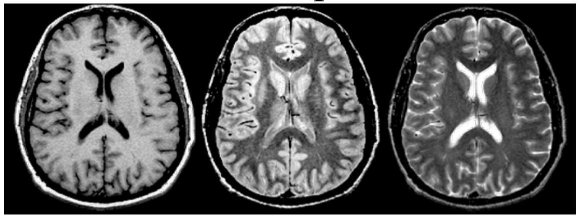


Proton Density Weighted Scans

Make TR very long compared to T_1 and use a very short TE. The resultant image is proton density weighted.

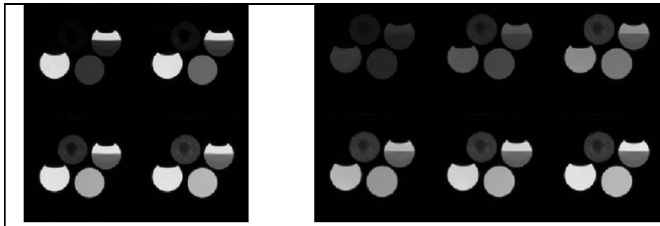
$$I(x,y) \approx \rho(x,y)$$

Example



T_1-weighted Density-weighted T_2-weighted

Tissue	Proton Density	T1 (ms)	T2 (ms)
Csf	1.0	4000	2000
Gray	0.85	1350	110
White	0.7	850	80

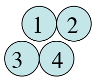


(a) Four images, all obtained with a common TR=5 seconds and TE=90, 50, 20, 15 ms (shown in reading order).

(b) Six images obtained with a common TE=15 ms and TR=500, 1000, 2000, 3000, 4000, 5000 ms (shown in reading order).

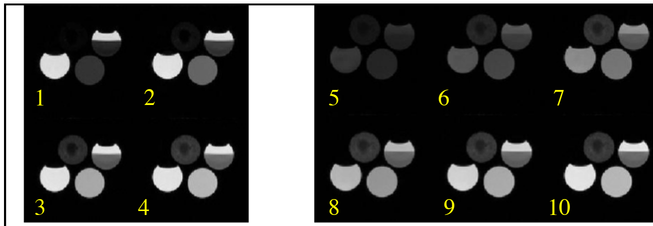
Figure 8: Phantom data which illustrates signal intensity and contrast for bottles filled with jello af varying consistency. Where is T_1 long/short? How long, how short? The same for T_2? Which bottles might be pure water? Which jello is most firm? What pictures are the most T_1-, T_2- and PD-weighted?

Hanson 2009



a) Which has the longest T1?
b) Which has the shortest T1?
c) Which has the longest T2?
d) Which has the shortest T2?
e) Which might be pure water?
f) Which has the most firm jello?

PollEv.com/be280a



(a) Four images, all obtained with a common TR=5 seconds and TE=90, 50, 20, 15 ms (shown in reading order).

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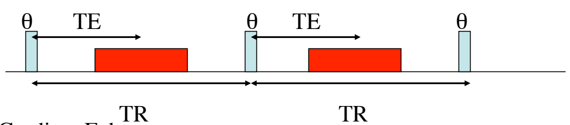
Figure 8: Phantom data which illustrates signal intensity and contrast for bottles filled with jello af varying consistency. Where is T_1 long/short? How long, how short? The same for T_2? Which bottles might be pure water? Which jello is most firm? What pictures are the most T_1-, T_2- and PD-weighted?

Hanson 2009

a) Which is the most T1 weighted?
b) Which is the most T2 weighted?
c) Which is the most PD weighted?

PollEv.com/be280a

FLASH sequence



Gradient Echo

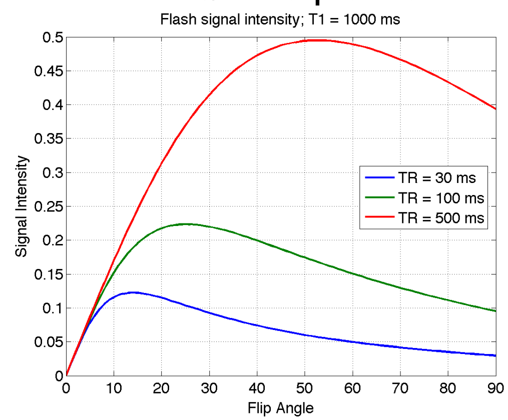
$$I(x,y) = \rho(x,y)\frac{\left[1-e^{-TR/T_1(x,y)}\right]\sin\theta}{\left[1-e^{-TR/T_1(x,y)}\cos\theta\right]}\exp(-TE/T_2^*)$$

Signal intensity is maximized at the Ernst Angle

$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

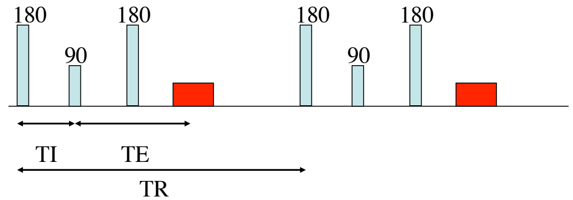
FLASH equation assumes no coherence from shot to shot. In practice this is achieved with RF spoiling.

FLASH sequence



$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

Inversion Recovery

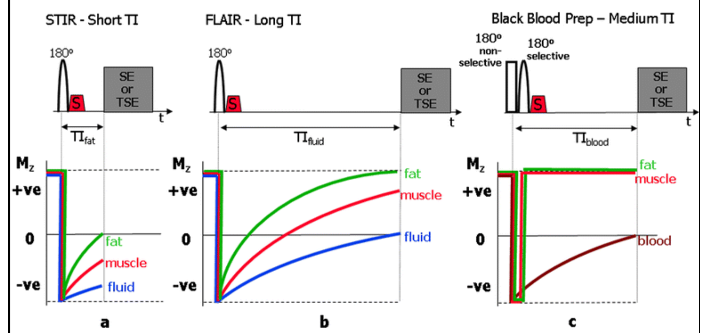


$$I(x,y) = \rho(x,y)\left[1 - 2e^{-TI/T_1(x,y)} + e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

Intensity is zero when inversion time is

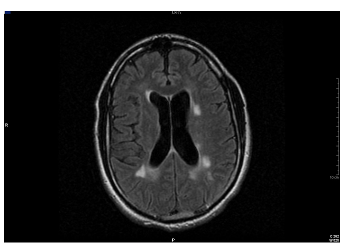
$$TI = -T_1 \ln\left[\frac{1+\exp(-TR/T_1)}{2}\right]$$

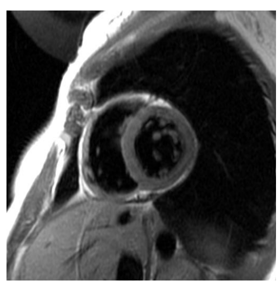
Inversion Recovery

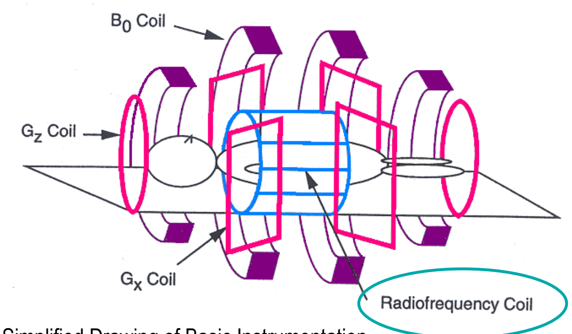


Biglands *et al*. *Journal of Cardiovascular Magnetic Resonance* 2012 **14**:66 doi:10.1186/1532-429X-14-66

Inversion Recovery



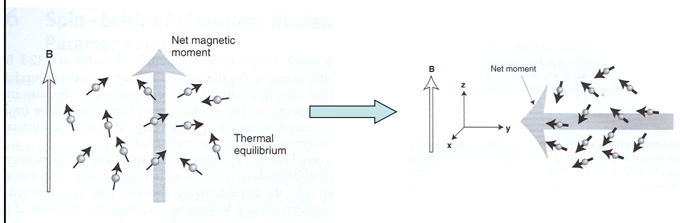


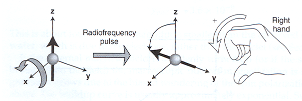


Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

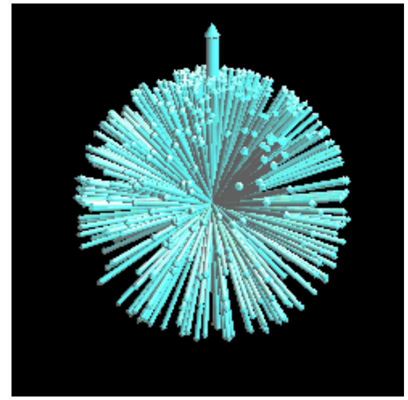
RF Excitation





From Levitt, Spin Dynamics, 2001

RF Excitation



http://www.drcmr.dk/main/content/view/213/74/

RF Excitation

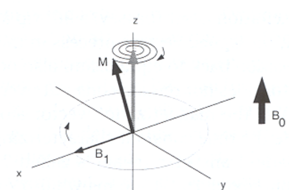
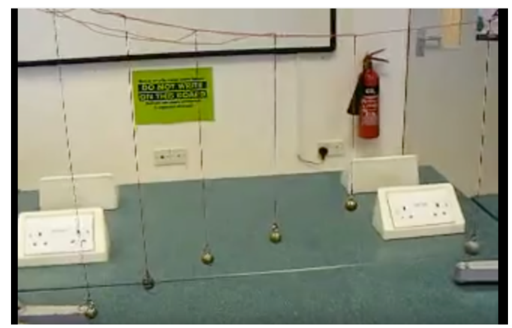


Image & caption: Nishimura, Fig. 3.2

At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

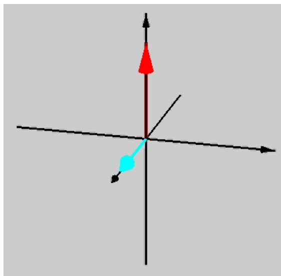
B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

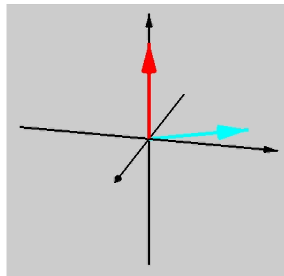
http://www.eecs.umich.edu/%7EdnollBME516/



https://www.youtube.com/watch?v=kODOL-QBzSM

RF Excitation

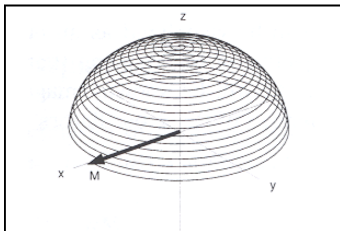


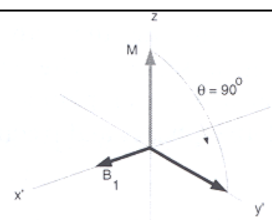


http://www.eecs.umich.edu/%7EdnollBME516/

Rotating Frame of Reference

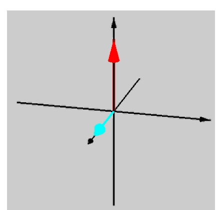
Reference everything to the magnetic field at isocenter.



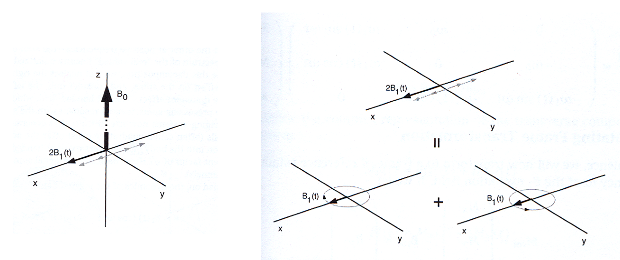


a) Laboratory frame behavior of **M** b) Rotating frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3

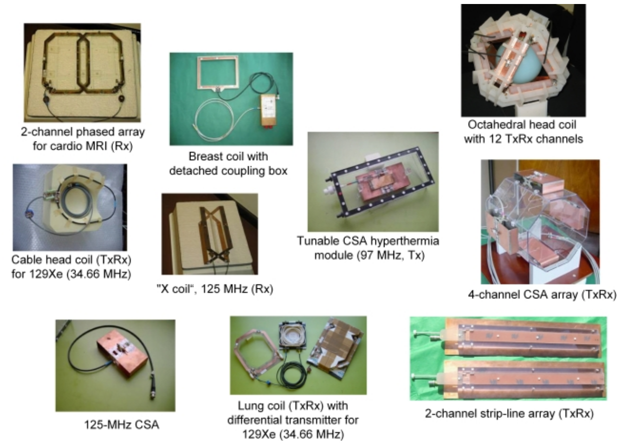


http://www.eecs.umich.edu/%7EdnollBME516/

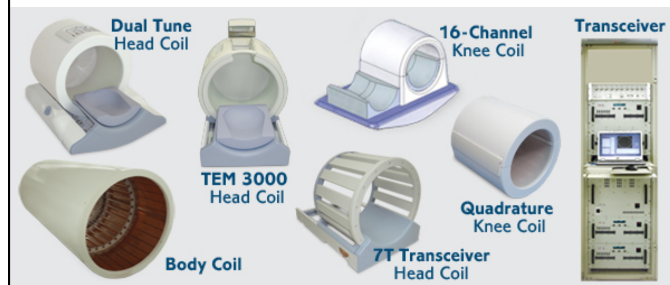


$$\mathbf{B_1}(t) = 2B_1(t)\cos(\omega t)\mathbf{i}$$
$$= B_1(t)\left(\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}\right) + B_1(t)\left(\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\right)$$

Nishimura 1996



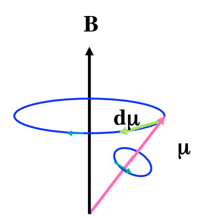
http://www.berlin.ptb.de/en/org/8/81/811/ResearchTopics/CoilDevelopment.html



http://www.mrinstruments.com/

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma \mathbf{B}$$

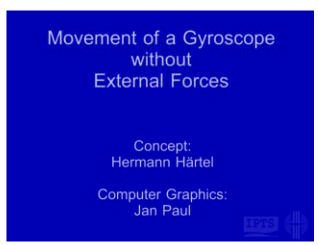


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma B$$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertelmpg_e/gyros_free.mpg

Rotating Frame Bloch Equation

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}; \quad \omega_{rot} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$$

Note: we use the RF frequency to define the rotating frame. If this RF frequency is on-resonance, then the main B0 field doesn't cause any precession in the rotating frame. However, if the RF frequency is off-resonance, then there will be a net precession in the rotating frame that is give by the difference between the RF frequency and the local Larmor frequency.

$$Let\ \mathbf{B}_{rot} = B_1(t)\mathbf{i} + B_0\mathbf{k}$$

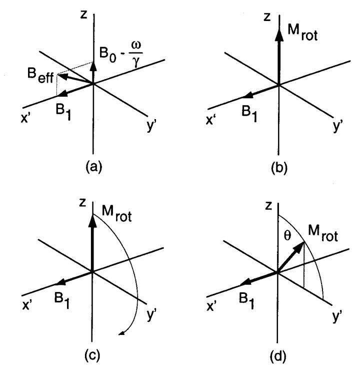
$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$

$$= B_1(t)\mathbf{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k}$$

If $\omega = \omega_0$

$$= \gamma B_0$$

Then $\mathbf{B}_{eff} = B_1(t)\mathbf{i}$



Flip angle

$$\theta = \int_0^{\tau} \omega_1(s)ds$$

where

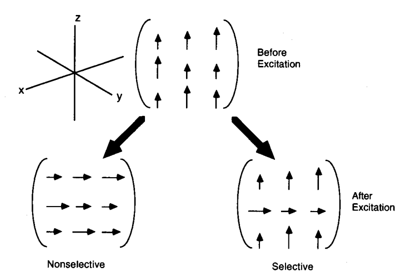
$$\omega_1(t) = \gamma B_1(t)$$

Nishimura 1996

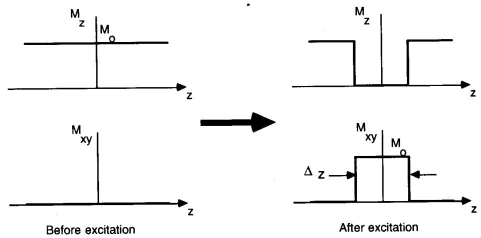
Example

$\tau = 400\ \mu\text{sec};\ \theta = \pi/2$

$$B_1 = \frac{\theta}{\gamma\tau} = \frac{\pi/2}{2\pi(4257 Hz/G)(400e-6)} = 0.1468\ G$$



Nishimura 1996



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Nishimura 1996

Let $\mathbf{B}_{rot} = B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z\right)\mathbf{k}$

$$\mathbf{B}_{eff} = \mathbf{B}_{rot} + \frac{\omega_{rot}}{\gamma}$$
$$= B_1(t)\mathbf{i} + \left(B_0 + \gamma G_z z - \frac{\omega}{\gamma}\right)\mathbf{k}$$

If $\omega = \omega_0$

$$\mathbf{B}_{eff} = B_1(t)\mathbf{i} + \left(\gamma G_z z\right)\mathbf{k}$$

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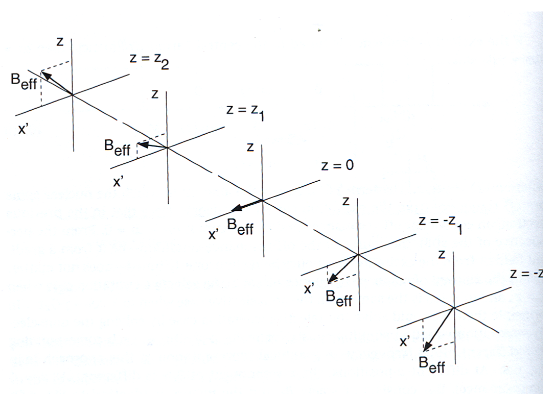

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Nishimura 1996

https://www.youtube.com/watch?v=kODOL-QBzSM

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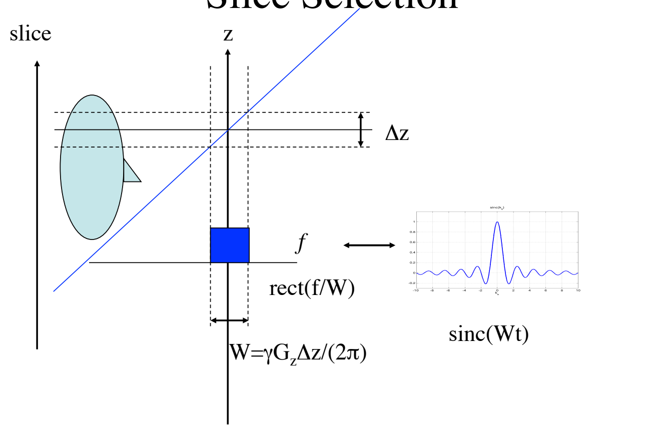
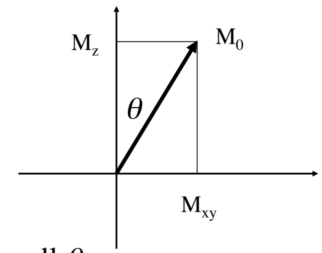
Slice Selection



Small Tip Angle Approximation

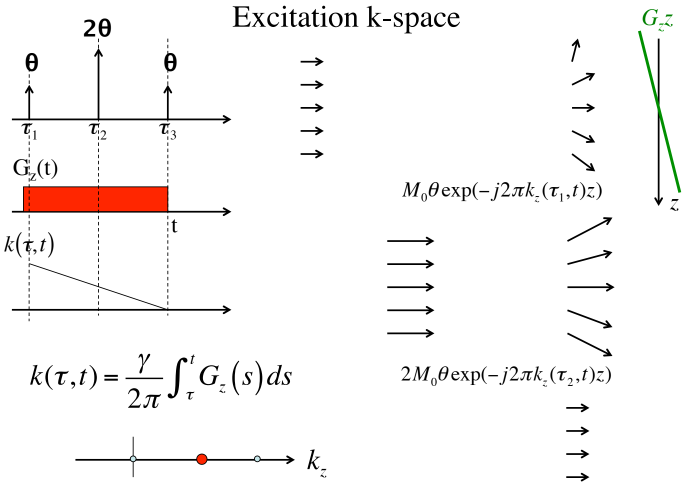


For small θ

$$M_z = M_0 \cos\theta \approx M_0$$

$$M_{xy} = M_0 \sin\theta \approx M_0\theta$$

Excitation k-space

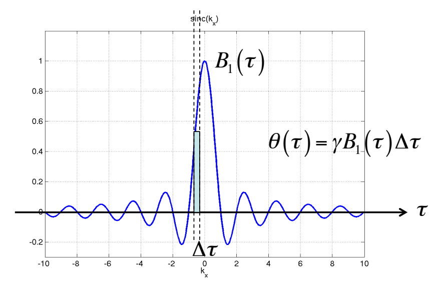


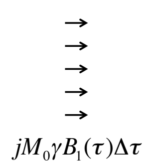
$$k(\tau,t) = \frac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$$

Consider analogy with 3D Printing

Excitation k-space

At each time increment of width $\Delta\tau$, the excitation $B_1(\tau)$ produces an increment in magnetization of the form $\Delta M_{xy} \approx jM_0\theta(\tau) = jM_0\gamma B_1(\tau)\Delta\tau$ (small tip angle approximation)



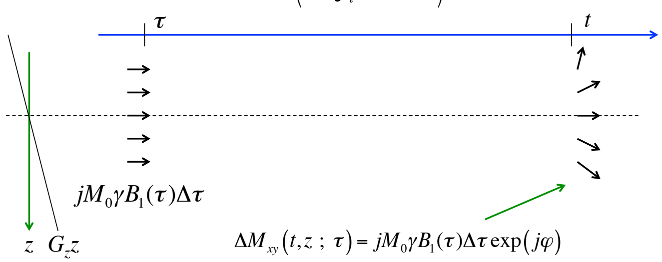


Excitation k-space

In the presence of a gradient, this will accumulate phase of the form

$\varphi = -\gamma\int_{\tau}^{t} zG_z(s)ds$, such that the incremental magnetization at time t is

$$\Delta M_{xy}(t,z\ ;\ \tau) = jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t} zG_z(s)ds\right)\Delta\tau$$



$$\Delta M_{xy}(t,z\ ;\ \tau) = jM_0\gamma B_1(\tau)\Delta\tau\exp(j\varphi)$$
$$= jM_0\gamma B_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t} zG_z(s)ds\right)\Delta\tau$$

Excitation k-space

Integrating over all time increments $d\tau$, we obtain

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t} zG_z(s)ds\right)d\tau$$
$$= jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp(-j2\pi k(\tau,t)z)d\tau$$

where $k(\tau,t) = \frac{\gamma}{2\pi}\int_{\tau}^{t} G_z(s)ds$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.

For a historical perspective see
http://www.sciencedirect.com/science/article/pii/S1090780711002655

Excitation k-space

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp(-j2\pi k(\tau,t)z)d\tau$$

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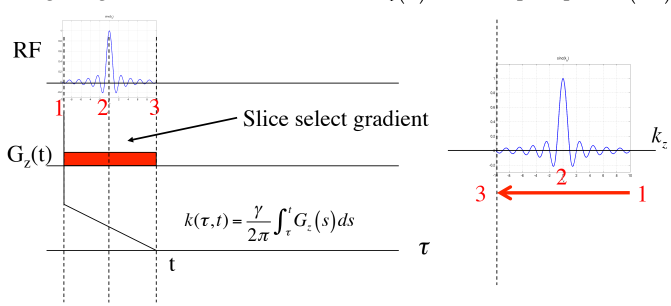
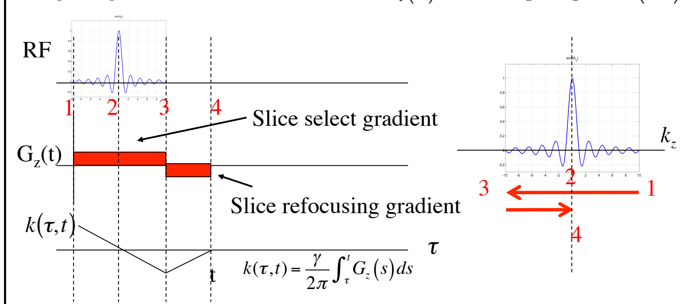


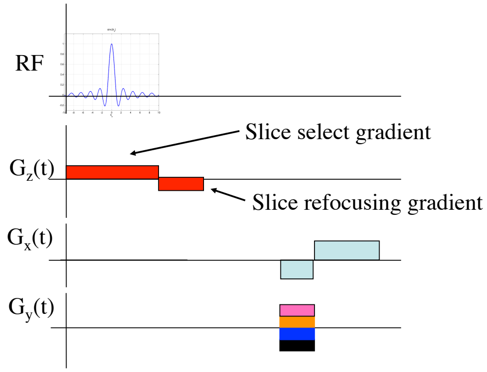
Refocusing

$$M_{xy}(t,z) = jM_0\int_{-\infty}^{t}\gamma B_1(\tau)\exp(-j2\pi k(\tau,t)z)d\tau$$

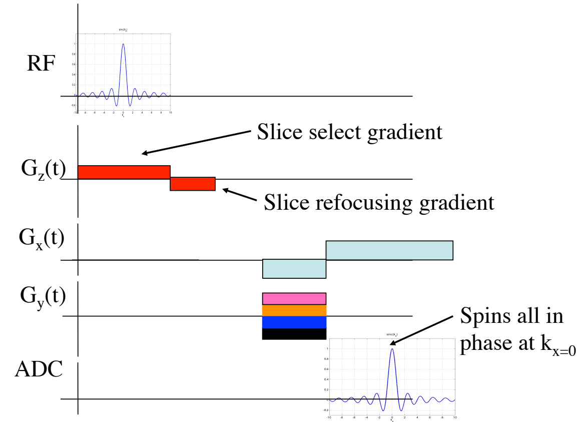
This has the form of a Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k-space point $k(\tau,t)$.



Slice Selection



Gradient Echo



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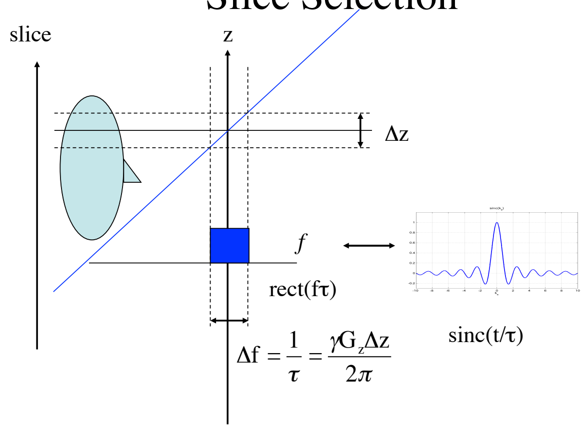
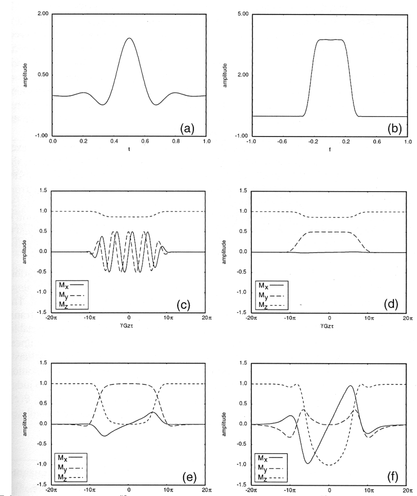




Nishimura 1996

Example

$M_{xy}(x) = M_0 \cos(4\pi x)$

$F\left(M_{xy}(x)\right) = \frac{M_0}{2}\left(\delta(k_x - 2) + \delta(k_x + 2)\right)$

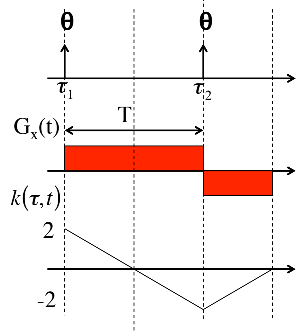
$g_{\max} = 4\ G/cm$

$\frac{\gamma}{2\pi} g_{\max} T = 4\ \text{cm}^{-1};\ \ \text{T} = 235\ \mu\text{sec}$

with small tip angle approximation --> $\theta = \frac{1}{2}$

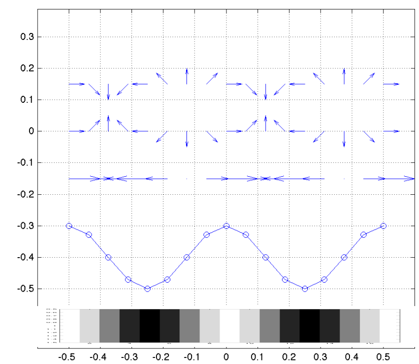
Compare with $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6} = 0.5236$

Question: Should we use $\theta = \frac{\pi}{4}$ instead?



Exercise: Sketch the quiver diagrams corresponding to the contributions of the two RF pulses and show that they produce the desired pattern.

Exercise: Sketch the quiver diagrams corresponding to the contributions of the two RF pulses and show that they produce the desired pattern. (Patterns shown below scaled for display purposes)



Multi-dimensional Excitation k-space

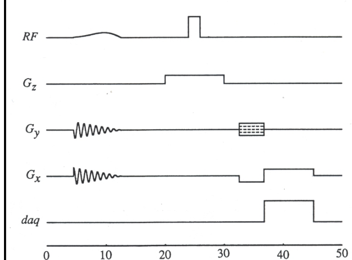
$$M_{xy}(t,\mathbf{r}) = jM_0 \int_{-\infty}^{t} \omega_1(\tau) \exp\left(-j\gamma \int_{\tau}^{t} \mathbf{G}(s)\cdot \mathbf{r}\, ds\right) d\tau$$

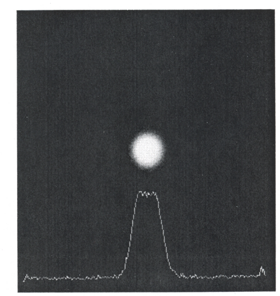
$$= jM_0 \int_{-\infty}^{t} \omega_1(\tau) \exp\left(j2\pi \mathbf{k}(\tau)\cdot \mathbf{r}\right) d\tau$$

where $\mathbf{k}(\tau) = -\frac{\gamma}{2\pi}\int_{\tau}^{t} \mathbf{G}(t')\,dt'$

Pauly et al 1989

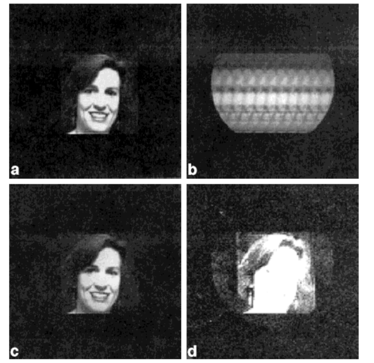
Excitation k-space





Pauly et al 1989

Excitation k-space



Panych MRM 1999

Cardiac Tagging