Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2015
MRI Lecture 7

Slice Selection Problem

Consider a desired slice profile of the form $m(z) = M_0 rect(z/(10mm))$.

You are given an RF pulse of the form $p(t) = A \cdot \text{sinc}\left(\frac{t}{T}\right)$

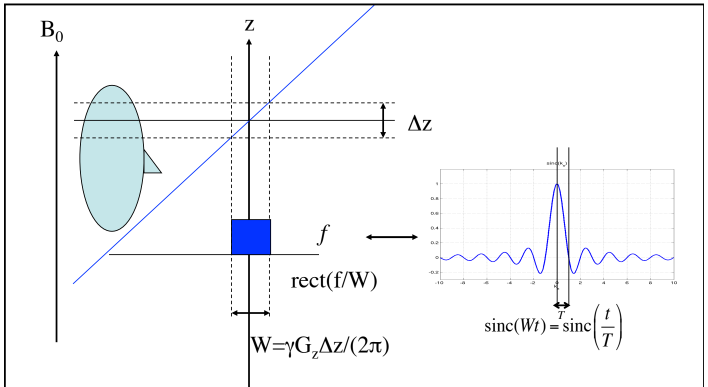
where $T = 400\ \mu\text{sec}$

PollEv.com/be280a

(1)What amplitude gradient should you use to achieve the desired slice profile?
HINT : Think about where the first zero in k_z space must appear. Then think about what gradient is needed so that the first zero in the RF occurs at the desired location in k-space. An alternate approach is to consider the frequency range of the spins that need to be excited within the slice and also consider the bandwidth of the RF pulse.

(2) What should the amplitude A of the RF pulse be?

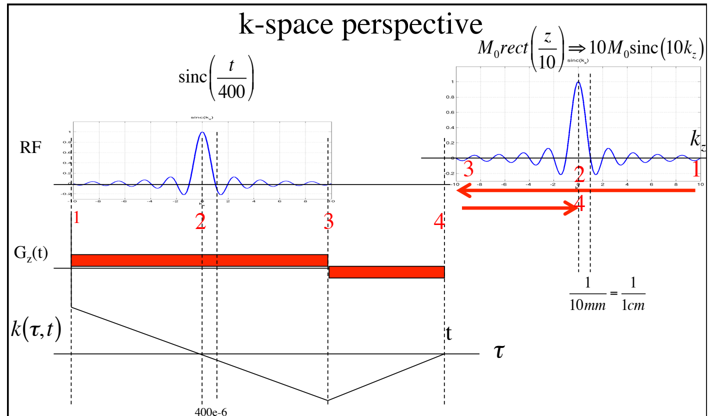
(3)What happens to the slice profile if you truncate the RF pulse to have a duration of 3.2 ms?



$$W = \frac{\gamma}{2\pi} G_z \Delta z = \frac{1}{T}$$

$$\Rightarrow G_z = \frac{1}{\frac{\gamma}{2\pi} T \Delta z} = \frac{1}{\frac{\gamma}{2\pi}(400e-6)(1cm)} = 0.587 \text{ G/cm}$$

k-space perspective



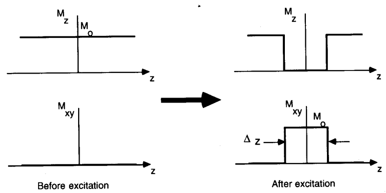
$$k(\tau,t) = \frac{\gamma}{2\pi} \int_\tau^t G_z(s)\, ds$$

$$\frac{\gamma}{2\pi} G_z(400\mu s) = \frac{1}{1} cm^{-1} \Rightarrow G_z = \frac{1}{\frac{\gamma}{2\pi}(400\mu s)(1cm)}$$

Slice Selection Problem

$m(z) = M_0 rect(z/(10mm)) \Rightarrow \theta = \frac{\pi}{2}$

$$\begin{aligned}\theta &= \gamma \int B_1(\tau)d\tau \\ &= \gamma A \int \mathrm{sinc}\left(\frac{\tau}{T}\right)d\tau \\ &= \gamma AF\left[\mathrm{sinc}\left(\frac{\tau}{T}\right)\right]\Big|_{f=0} \\ &= \gamma AT rect(fT)\Big|_{f=0} \\ &= \gamma AT \\ &\Rightarrow \gamma AT = \frac{\pi}{2} \Rightarrow A = \frac{\pi}{2}\frac{1}{\gamma T} = 0.1468G\end{aligned}$$



Slice Selection Problem

Truncated RF

$p(t) = A \cdot \mathrm{sinc}\left(\frac{t}{400\mu s}\right) rect\left(\frac{t}{8 \cdot 400\mu s}\right)$

$P(f) = A(400)(3200) rect(400f) * \mathrm{sinc}(8 \cdot 400 f)$

Note that

$f = \frac{\gamma}{2\pi} G_z z$

so

$P(z) \propto rect\left(400\frac{\gamma}{2\pi}G_z z\right) * \mathrm{sinc}\left(8 \cdot 400\frac{\gamma}{2\pi}G_z z\right)$

$= rect\left(\frac{z}{\Delta z}\right) * \mathrm{sinc}\left(\frac{z}{\Delta z/8}\right)$

$$\frac{Width}{Transition\ Width} = \frac{\Delta z}{\Delta z/8} = 8$$

Time-Bandwidth Product (TBW)

$\mathrm{sinc}(t/\tau) rect\left(\frac{t}{2N\tau}\right) \Rightarrow \tau rect(\tau f) * 2N\tau \mathrm{sinc}(2N\tau f)$

Duration $= 2N\tau$

Bandwidth $= \frac{1}{\tau} \quad \Rightarrow \Delta z = \frac{2\pi}{\gamma G_z \tau}$

N = number of zeros in Sinc

Transition Width $\approx \frac{1}{2N\tau} \quad \Rightarrow \Delta z' = \frac{2\pi}{\gamma G_z 2N\tau}$

Time − Bandwidth Product (TBW) $= 2N\tau\frac{1}{\tau} = 2N$

also, TBW $= \frac{\text{Bandwidth}}{\text{Transition Width}}$

For a fixed duration pulse, we can increase TBW by increasing the Bandwidth.
(Note: this will also lead to an increase in N).
This will require a higher B1 amplitude and a higher gradient to keep the slice width constant -- note that with higher TBW the physical transition width then decreases.

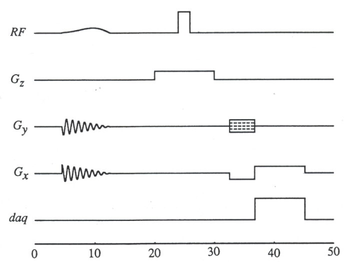
Multi-dimensional Excitation k-space

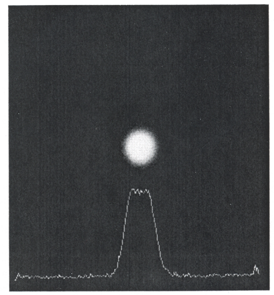
$$\begin{aligned}M_{xy}(t,\mathbf{r}) &= jM_0\int_{-\infty}^{t}\omega_1(\tau)\exp\left(-j\gamma\int_{\tau}^{t}\mathbf{G}(s)\cdot\mathbf{r}ds\right)d\tau \\ &= jM_0\int_{-\infty}^{t}\omega_1(\tau)\exp\left(j2\pi\mathbf{k}(\tau)\cdot\mathbf{r}\right)d\tau\end{aligned}$$

where $\quad \mathbf{k}(\tau) = -\frac{\gamma}{2\pi}\int_{\tau}^{t}\mathbf{G}(t')dt'$

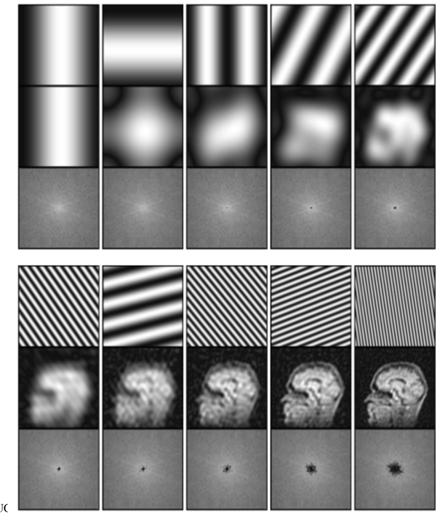
Pauly et al 1989

Excitation k-space



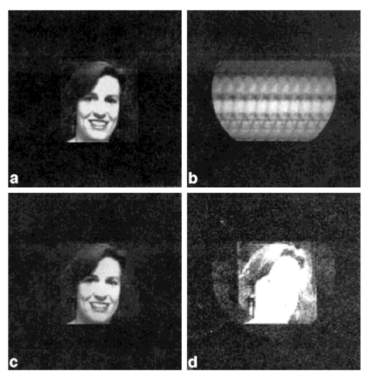


Pauly et al 1989



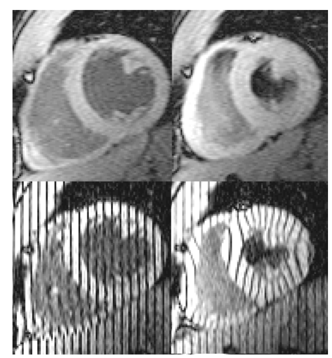
Hanson 2009

Excitation k-space

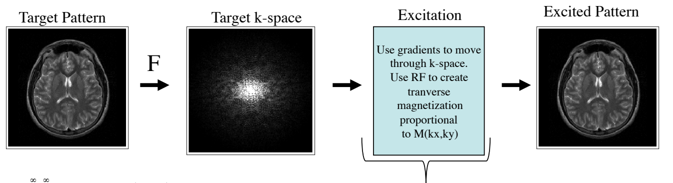


Panych MRM 1999

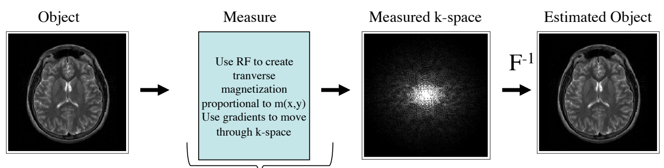
Cardiac Tagging



Imaging and Excitation



$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M(k_x,k_y)e^{j2\pi(k_x x+k_y y)}\,dk_x\,dk_y$; integration over k performed incrememtanlly by layering of spins in place



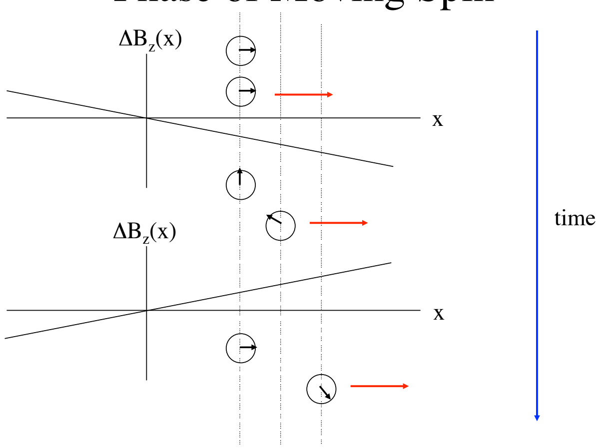
$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)e^{-j2\pi(k_x x+k_y y)}\,dx\,dy$; integration over space performed instantaneously by coil

Moving Spins (preview)

So far we have assumed that the spins are not moving (aside from thermal motion giving rise to relaxation), and contrast has been based upon T_1, T_2, and proton density. We were able to achieve different contrasts by adjusting the appropriate pulse sequence parameters.

Biological samples are filled with moving spins, and we can also use MRI to image the movement. Examples: blood flow, diffusion of water in the white matter tracts. In addition, we can also sometimes induce motion into the object to image its mechanical properties, e.g. imaging of stress and strain with MR elastography.

Phase of Moving Spin



Phase of a Moving Spin

$$\begin{aligned}\varphi(t) &= -\int_0^t \Delta\omega(\tau)d\tau \\ &= -\int_0^t \gamma\Delta B(\tau)d\tau \\ &= -\int_0^t \gamma\vec{G}(\tau)\cdot\vec{r}(\tau)d\tau \\ &= -\gamma\int_0^t \left[G_x(\tau)x(\tau)+G_y(\tau)y(\tau)+G_z(\tau)z(\tau)\right]d\tau\end{aligned}$$

Phase of Moving Spin

Consider motion along the x-axis

$$x(t) = x_0 + vt + \frac{1}{2}at^2$$

$$\begin{aligned}
\varphi(t) &= -\gamma \int_0^t G_x(\tau)x(\tau)d\tau \\
&= -\gamma \int_0^t G_x(\tau)\left[x_0 + v\tau + \frac{1}{2}a\tau^2\right]d\tau \\
&= -\gamma\left[x_0\int_0^t G_x(\tau)d\tau + v\int_0^t G_x(\tau)\tau d\tau + \frac{a}{2}\int_0^t G_x(\tau)\tau^2 d\tau\right] \\
&= -\gamma\left[x_0 M_0 + vM_1 + \frac{a}{2}M_2\right]
\end{aligned}$$

Phase of Moving Spin

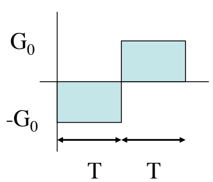
$$\varphi(t) = -\gamma\left[x_0 M_0 + vM_1 + \frac{a}{2}M_2\right]$$

$M_0 = \int_0^t G_x(\tau)d\tau$ Zeroth order moment

$M_1 = \int_0^t G_x(\tau)\tau d\tau$ First order moment

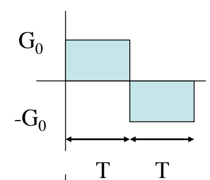
$M_2 = \int_0^t G_x(\tau)\tau^2 d\tau$ Second order moment

Flow Moment Example

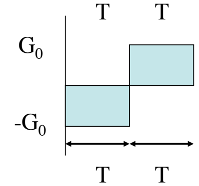


$$\begin{aligned}
M_0 &= \int_0^t G_x(\tau)d\tau = 0 \\
M_1 &= \int_0^t G_x(\tau)\tau d\tau \\
&= -\int_0^T G_0\tau d\tau + \int_T^{2T} G_0\tau d\tau \\
&= G_0\left[-\frac{\tau^2}{2}\bigg|_0^T + \frac{\tau^2}{2}\bigg|_T^{2T}\right] \\
&= G_0\left[-\frac{T^2}{2} + \frac{4T^2}{2} - \frac{T^2}{2}\right] = G_0T^2
\end{aligned}$$

Phase Contrast Angiography (PCA)



$$\varphi_1 = -\gamma v_x M_1 = \gamma v_x G_0 T^2$$

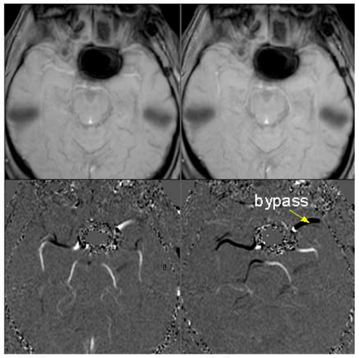


$$\varphi_2 = -\gamma v_x M_1 = -\gamma v_x G_0 T^2$$

$$\Delta\varphi = \varphi_1 - \varphi_2 = 2\gamma v_x G_0 T^2$$

$$v_x = \frac{\Delta\varphi}{2G_0T^2}$$

PCA example



White =
Flow direction AP (↓)

White =
Flow direction RL (→)

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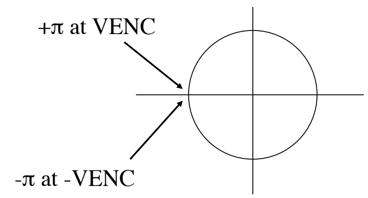
http://www.medical.philips.com/main/products/mri/assets/images/case_of_week/cotw_51_s5.jpg

Aliasing in PCA

Define VENC as the velocity at at which the phase is 180 degrees.

$$VENC \equiv \frac{\pi}{\gamma G_0 T^2}$$

Because of phase wrapping the velocity of spins flowing faster than VENC is ambiguous.



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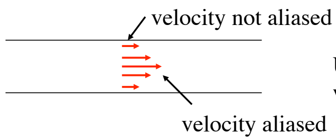
Aliasing Solutions



Use data from regions with slower flow

Use multiple VENC values so that the phase differences are smaller than π radians.

$$\varphi_1 = \pi \frac{v_x}{VENC_1}$$

$$\varphi_2 = \pi \frac{v_x}{VENC_2}$$

$$\varphi_1 - \varphi_2 = \pi v_x \left(\frac{1}{VENC_1} - \frac{1}{VENC_2} \right)$$

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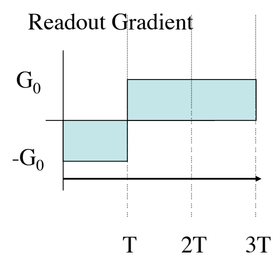
Flow Artifacts



During readout moving spins within the object will accumulate phase that is in addition to the phase used for imaging. This leads to

1) Net phase at echo time TE = 2T.
2) An apparent shift in position of the object.
3) Blurring of the object due to a quadratic phase term.

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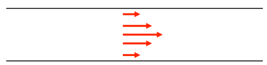
Flow Artifacts

Plug Flow

All moving spins in the voxel experience the same phase shift at echo time.

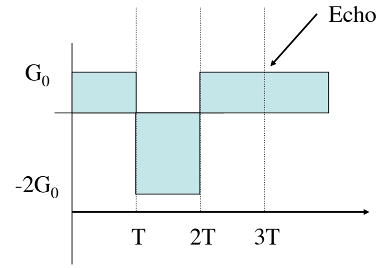
Laminar Flow



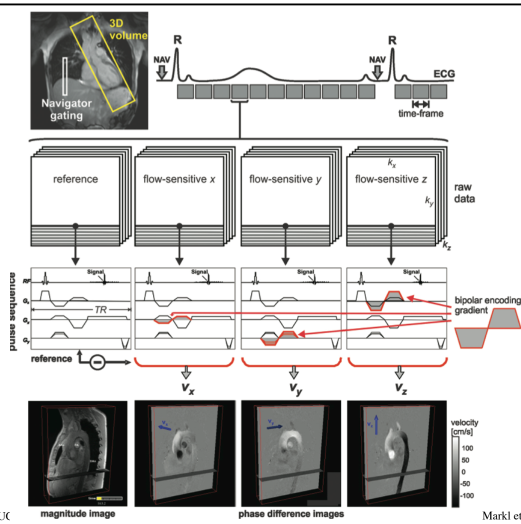
Spins have different phase shifts at echo time. The dephasing causes the cancelation and signal dropout.

Flow Compensation

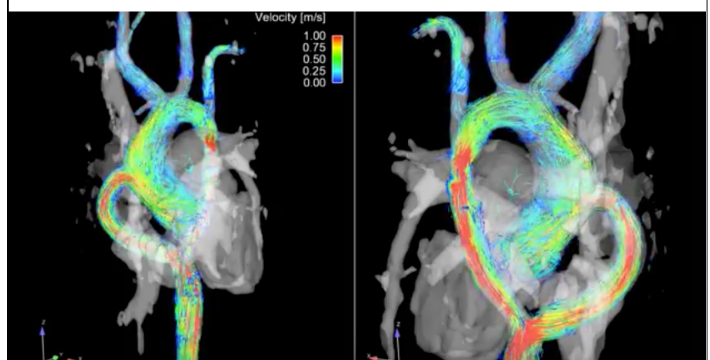
Readout Gradient



At TE both the first and second order moments are zero, so both stationary and moving spins have zero net phase.



Markl et al JMRI 2012



https://www.youtube.com/watch?v=1ORpeB6j0gc

Velocity k-space

A bipolar gradient introduces a phase modulation across velocities of the form $\varphi(v_x) = -\gamma v_x G_0 T^2$

The MRI signal (with no spatial encoding) acquired across a volume of spins with varying velocities is:

$$
\begin{aligned}
M(k_{v_x}) &= \int_{-\infty}^{\infty} m(v_x) e^{j\varphi(v_x)} dv_x \\
&= \int_{-\infty}^{\infty} m(v_x) e^{-j\gamma v_x G_0 T^2} dv_x \\
&= \int_{-\infty}^{\infty} m(v_x) e^{-j2\pi k_{v_x} v_x} dv_x \\
&= F\left[m(v_x)\right] \text{ with } k_{v_x} = \frac{\gamma}{2\pi} G_0 T^2
\end{aligned}
$$

By making measurements with bipolar gradients of varying amplitudes/durations and taking the inverse transform of the measurements, we can obtain the velocity distribution.

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Velocity k-space

$$M(k_x, k_{v_x}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x, v_x) e^{-j2\pi k_x x} e^{-j2\pi k_{v_x} v_x} dx dv_x$$

In addition, we can apply imaging gradients so that we can eventually obtain the velocity distribution at each point in space. A full k-space acqusition would then yield 6 dimensions -- 3 spatial dimensions and 3 velocity dimensions.

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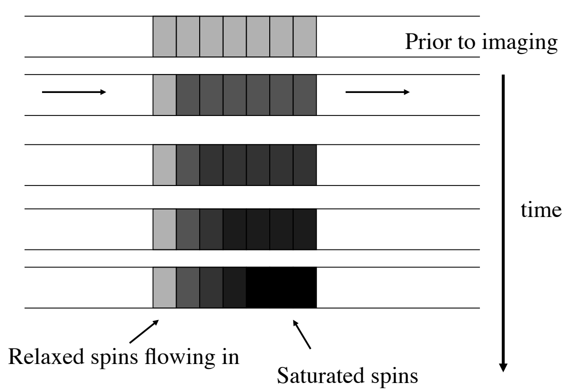
Inflow Effect



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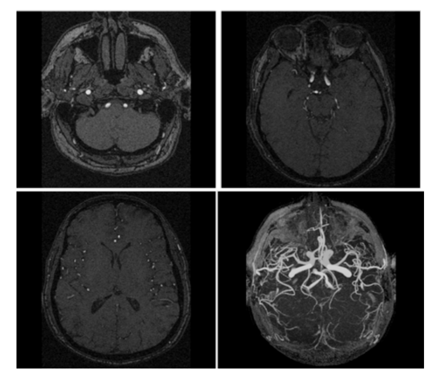
Time of Flight Angiography



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Cerebral Blood Flow (CBF)

CBF = Perfusion
= Rate of delivery of arterial blood to a capillary bed in tissue.

Units: $\frac{\text{(ml of Blood)}}{\text{(100 grams of tissue)(minute)}}$

Typical value is 60 ml/(100g-min) or 60 ml/(100 ml-min) = 0.01 s^{-1}, assuming average density of brain equals 1 gm/ml

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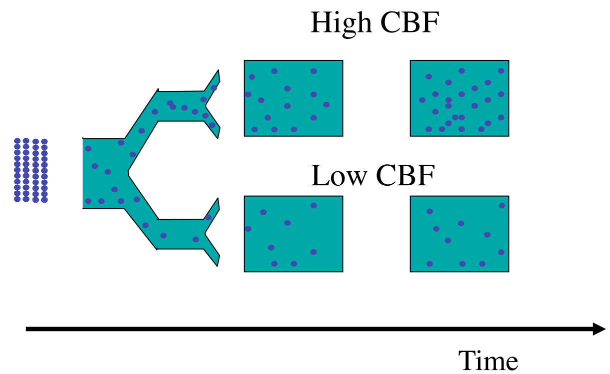

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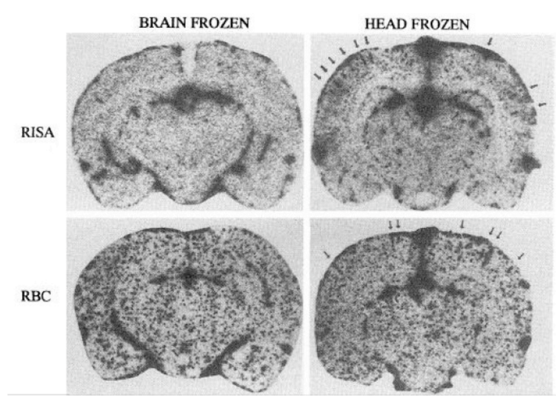

Bereczki et al 1992

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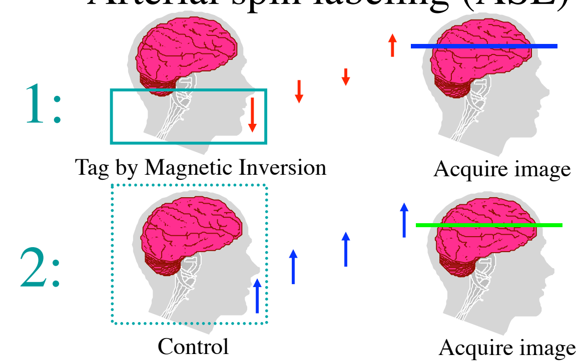
Arterial spin labeling (ASL)



Control - Tag = $\Delta M \propto$ CBF

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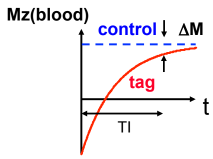
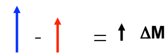
ASL Signal Equation

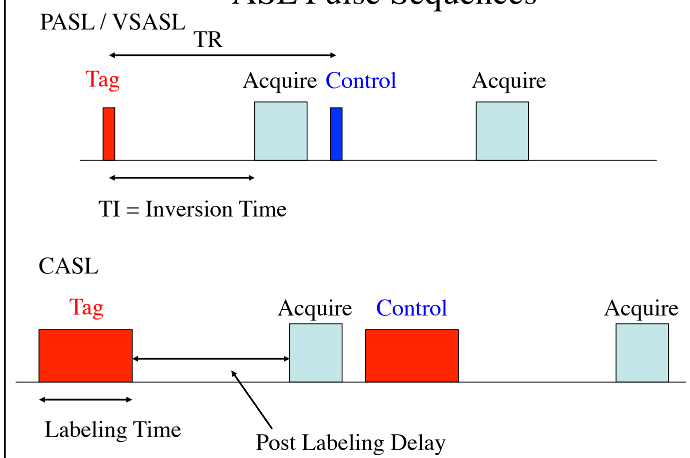




$$\Delta M = CBF \cdot A_{eff}$$

A_{eff} is the effective area of the arterial bolus. It depends on both physiology and pulse sequence parameters.

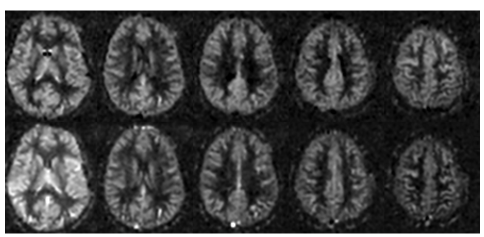
ASL Pulse Sequences



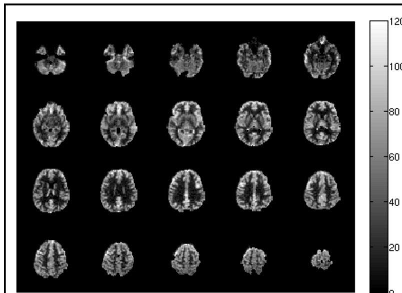
Multislice CASL and PICORE

CASL

PICORE QUIPSS II

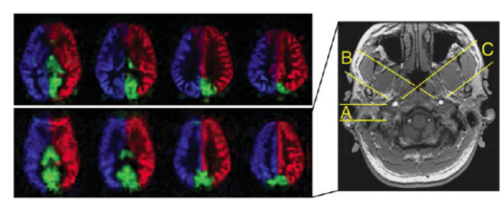


Credit: E. Wong



Whole brain non-invasive measures of Cerebral Blood Flow obtained with arterial spin labeling (ASL) MRI (Courtesy of D. Shin)

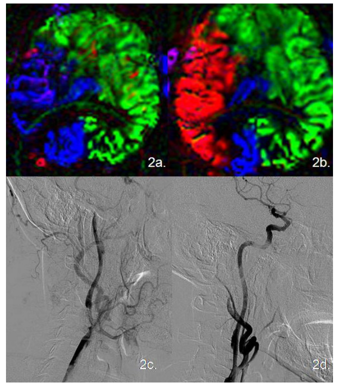
Vascular territory imaging using vessel-encoded ASL (Wong 2007)



Clinical Example: Pre and Post Right ICA stent placement

Patient had severe right ICA stenosis. The right ICA territory (red) is almost non-existent before stent placement. Blood flow was partially compensated by posterior cerebral artery (blue). After Stent placement, right ICA area is almost normal.

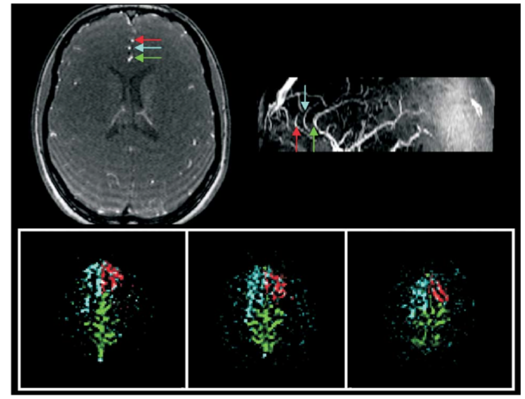
Vessel Encoded ASL



Before After

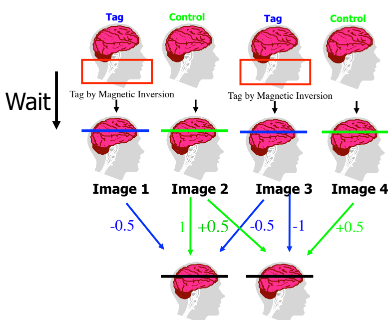
From Eric Wong, UCSD and Wu Bing, Peking University

Super-selective ASL

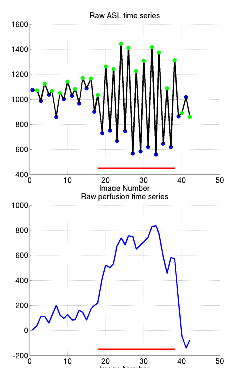


Helle et al, MRM 2010

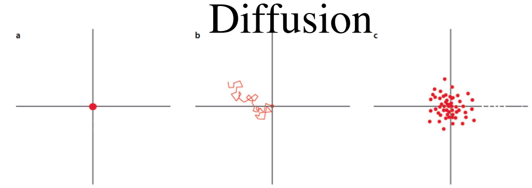
ASL Time Series



Perfusion Images



Diffusion



1D Random Walk:

Assume N independent steps of ±d at time interval τ

$x=\sum_{i=1}^{N} s_i d$ where $s_i \in (-1,1)$ and $\langle s_i s_j \rangle = \delta[i-j]$

$\langle x \rangle = 0$ --- mean displacement is zero

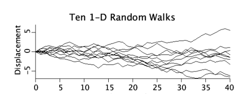
$\langle x^2 \rangle = d^2 \left\langle \sum_{j=1}^{N} \sum_{i=1}^{N} s_i s_j \right\rangle = Nd^2$

After some time $t = N\tau$

$\langle x^2 \rangle = \frac{t}{\tau} d^2 = 2Dt$

where $D \equiv \frac{d^2}{\tau}$ is the Diffusivity

RMS displacement is: $\sigma = \sqrt{x^2} = \sqrt{2Dt}$



Example

$D = 2\mu m^2 / ms$

$T = 100ms$

$\sigma = \sqrt{2Dt}$

$= \sqrt{2 \cdot 2\mu m^2 / ms \cdot 100ms}$

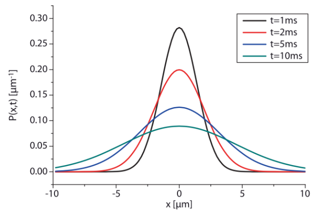
$= 20\mu m$

Stilejies et al 2013

Diffusion

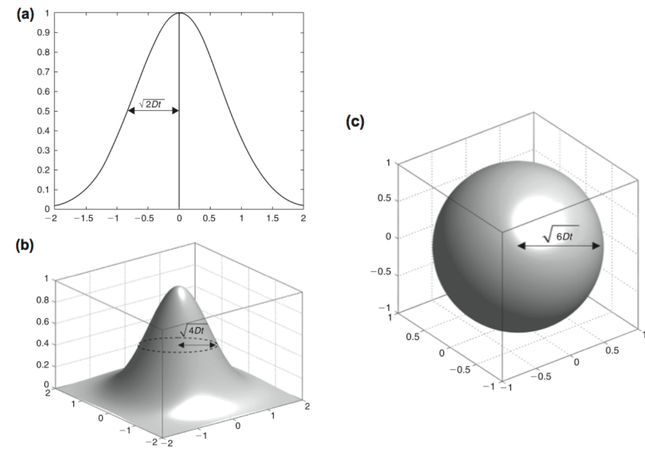
From the Central Limit Theorem

$$p(x,t) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(\frac{-x^2}{2\sigma^2}\right)$$
$$= \frac{1}{\sqrt{4\pi Dt}}\exp\left(\frac{-x^2}{4Dt}\right)$$



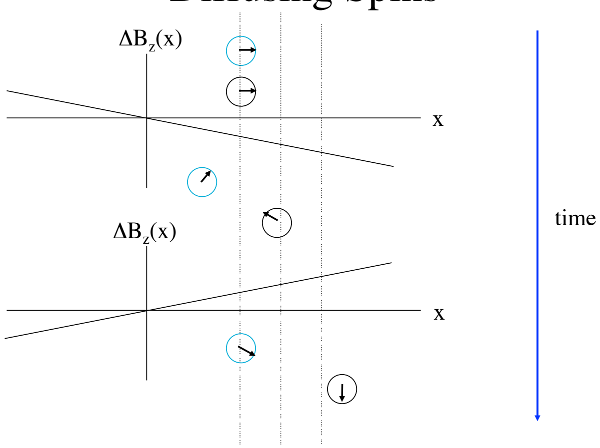
$p(x,t)$ with $D = 1\mu m^2 / ms$

Stilejies et al 2013

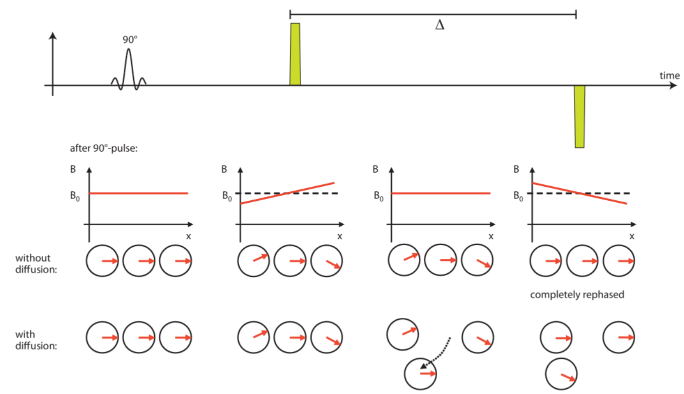


Jones 2014

Diffusing Spins

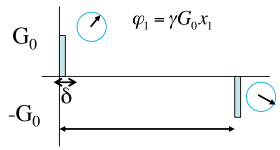


Diffusion Pulse Sequence



Stilejies et al 2013

Narrow Pulse Approximation



$\varphi_1 = \gamma G_0 x_1$

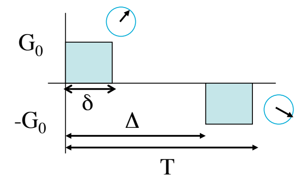
$\varphi_2 = -\gamma G_0 x_2$

$$\varphi = \varphi_1 + \varphi_2 = \gamma G_0 \delta (x_1 - x_2) = \gamma G_0 \delta x$$

$$\begin{aligned} \langle \varphi^2 \rangle &= \gamma^2 G_0^2 \delta^2 \langle x^2 \rangle \\ &= 2\gamma^2 G_0^2 \delta^2 DT \\ &= 2bT \end{aligned}$$

$$\begin{aligned} S \propto \langle e^{j\varphi} \rangle &= 1 + j\langle \varphi \rangle - \frac{1}{2}\langle \varphi^2 \rangle + \ldots \\ &\approx 1 - \frac{1}{2}\langle \varphi^2 \rangle \\ &\approx \exp\left(-\frac{1}{2}\langle \varphi^2 \rangle\right) \\ &= \exp(-\gamma^2 G^2 \delta^2 DT) \\ &= \exp(-bD) \quad \text{where } b = \gamma^2 G^2 \delta^2 T \end{aligned}$$

Long diffusion gradients



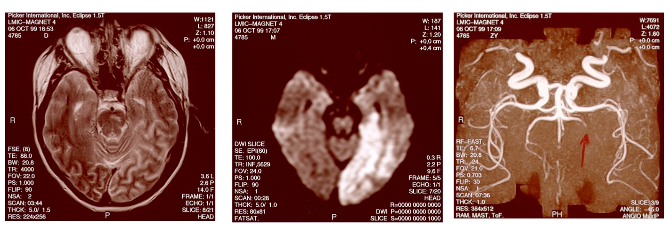
$$\varphi = \gamma \int_0^T G(\tau) x(\tau) d\tau$$

$$\begin{aligned} \langle \varphi^2 \rangle &= \gamma^2 \left\langle \int_0^T G(\tau) x(\tau) d\tau \int_0^T G(s) x(s) ds \right\rangle \\ &= \gamma^2 \int_0^T \int_0^T G(\tau) G(s) \langle x(\tau) x(s) \rangle ds\, d\tau \\ &= 2\gamma^2 D \int_0^T \int_\tau^T G(\tau) G(s) (s - \tau) ds\, d\tau \\ &= 2bD \end{aligned}$$

$$\begin{aligned} S &\propto \exp\left(-\frac{1}{2}\langle \varphi^2 \rangle\right) \\ &= \exp(-bD) \quad \text{where } b = \gamma^2 G^2 \delta^2 \left(\Delta - \frac{\delta}{3}\right) \end{aligned}$$

Diffusion Weighted Images

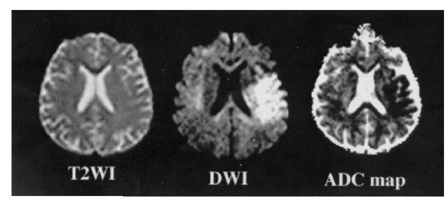
T2 weighted | Diffusion Weighted | Angiogram

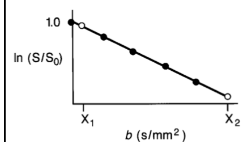


After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

http://lehighmri.com/cases/dwi/patient-b.html

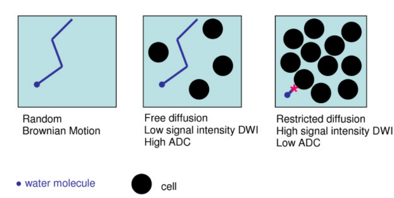
Apparent Diffusion Coefficient (ADC)





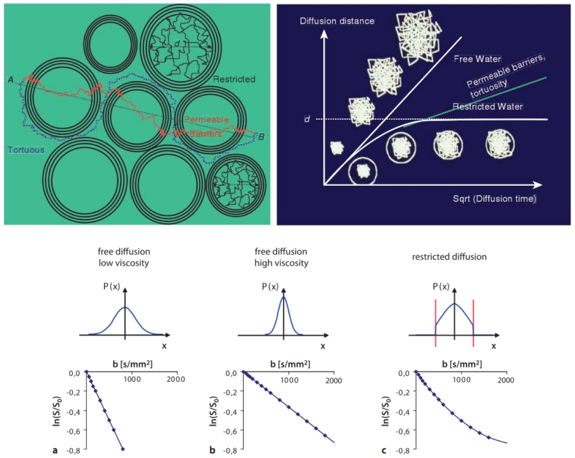
$$S = S_0 \exp(-b \cdot ADC)$$

$$\ln\left(\frac{S}{S_0}\right) = -b \cdot ADC$$



http://www.nature.com/jcbfm/journal/v18/n6/images/9590397f2.jpg; Neil 1997

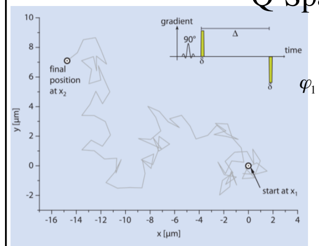
 http://www.researchgate.net/profile/Elizabeth_OFlynn/publication/50362785/figure/fig4/Diagram-illustrating-free-and-restricted-diffusion-of-water-in-different-tissues.-ADC.bcr2815-4



Le Bihan 2015; Stilejies et al 2013

Q-Space Imaging

$\varphi_2 = -\gamma G_0 x_2 = -q x_2$



$\varphi_1 = \gamma G_0 x_1 = q x_1$

$\varphi = \varphi_1 + \varphi_2 = \gamma G_0 \delta (x_1 - x_2) = -q(x_2 - x_1) = -qx$

$$
\begin{aligned}
E(q) &= \int\int \rho(x_1) P(x_1, x_2, \Delta) e^{-jq(x_2 - x_1)}\, dx_1 dx_2 \\
&= \int\int \rho(x_1) P(x_1, x_1 + x, \Delta) e^{-jqx}\, dx_1 dx \\
&= \int \bar{P}(x, \Delta) e^{-jqx} dx
\end{aligned}
$$

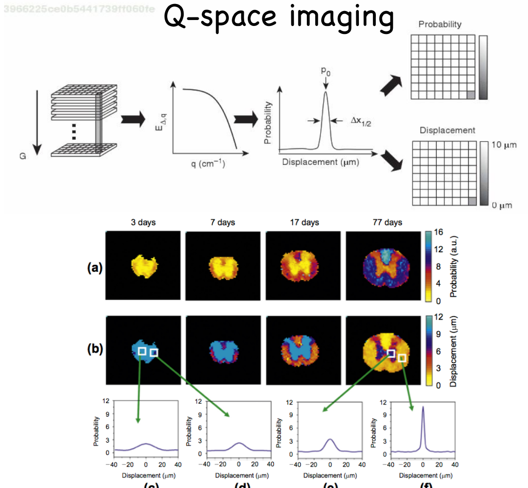
Average over all starting positions to obtain Mean Propagator

$\bar{P}(x,\Delta) = \int \rho(x_1) P(x_1, x_1 + x, \Delta) dx_1$

this is a density-weighted average probability of moving a distance x in an time interval Δ

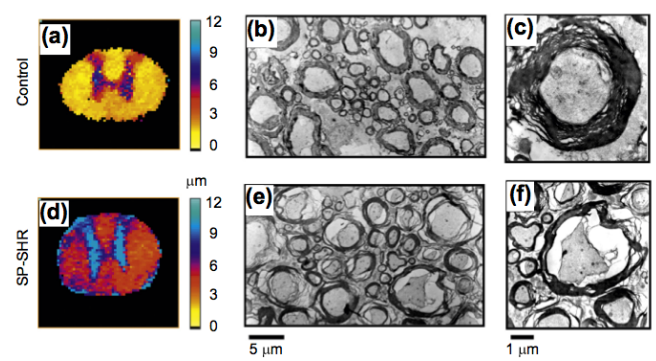
Stilejies et al 2013

Q-space imaging



Assaf and Cohen 2014

Q-space imaging



Assaf and Cohen 2014