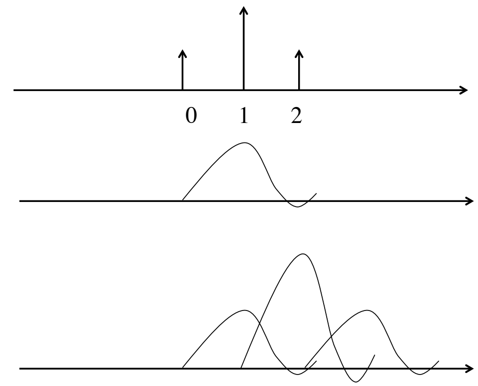
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2015
X-Rays Lecture 3

Linear Shift Invariant System



Convolution

$$g[m] = g[0]\delta[m] + g[1]\delta[m-1] + g[2]\delta[m-2]$$
$$h[m',k] = L[\delta[m-k]] = h[m'-k]$$

$$\begin{aligned} y[m'] &= L\left[g[m]\right] \\ &= L\left[g[0]\delta[m] + g[1]\delta[m-1] + g[2]\delta[m-2]\right] \\ &= L\left[g[0]\delta[m]\right] + L\left[g[1]\delta[m-1]\right] + L\left[g[2]\delta[m-2]\right] \\ &= g[0]L\left[\delta[m]\right] + g[1]L\left[\delta[m-1]\right] + g[2]L\left[\delta[m-2]\right] \\ &= g[0]h[m'-0] + g[1]h[m'-1] + g[2]h[m'-2] \\ &= \sum_{k=0}^{2} g[k]h[m'-k] \end{aligned}$$

1D Convolution

$$\begin{aligned} I(x) &= \int_{-\infty}^{\infty} g(\xi)h(x;\xi)d\xi \\ &= \int_{-\infty}^{\infty} g(\xi)h(x-\xi)d\xi \\ &= g(x) * h(x) \end{aligned}$$

Flip and shift

$$\int_{-\infty}^{\infty} \underbrace{g(\xi)h(x-\xi)}_{\text{Multiply}}d\xi$$

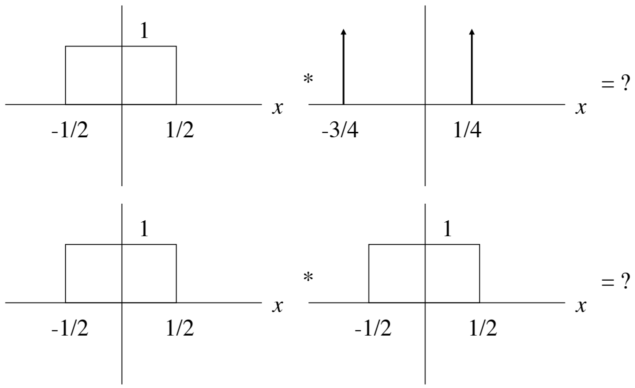
Multiply

Integrate the product

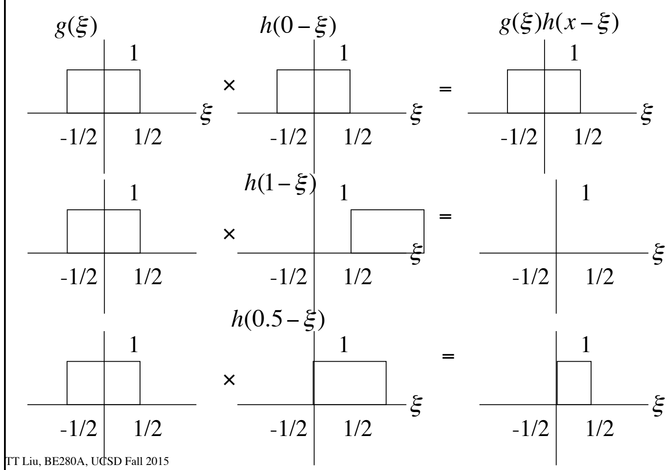
Useful fact:

$$\begin{aligned} g(x) * \delta(x-\Delta) &= \int_{-\infty}^{\infty} g(\xi)\delta(x-\Delta-\xi)d\xi \\ &= g(x-\Delta) \end{aligned}$$

1D Convolution Examples



$h(x-\xi)$ for different values of x



2D Convolution

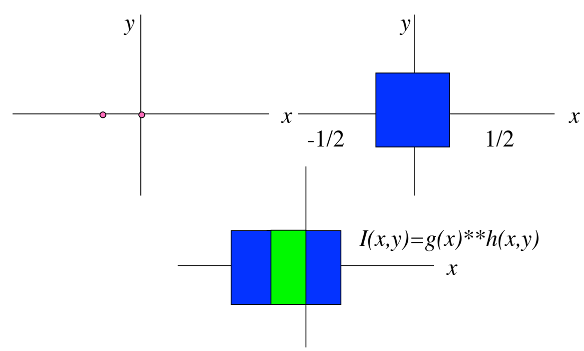
For a space invariant linear system, the superposition integral becomes a convolution integral.

$$
\begin{aligned}
I(x_2,y_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi,\eta)h(x_2,y_2;\xi,\eta)d\xi d\eta \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi,\eta)h(x_2-\xi,y_2-\eta)d\xi d\eta \\
&= g(x_2,y_2)**h(x_2,y_2)
\end{aligned}
$$

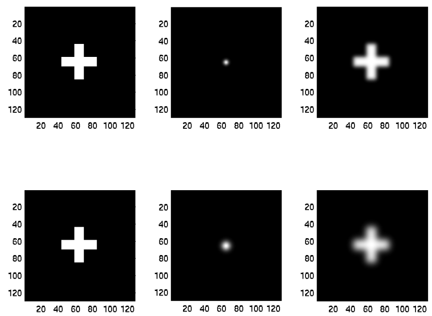
where ** denotes 2D convolution. This will sometimes be abbreviated as *, e.g. $I(x_2, y_2)= g(x_2, y_2)*h(x_2, y_2)$.

2D Convolution Example

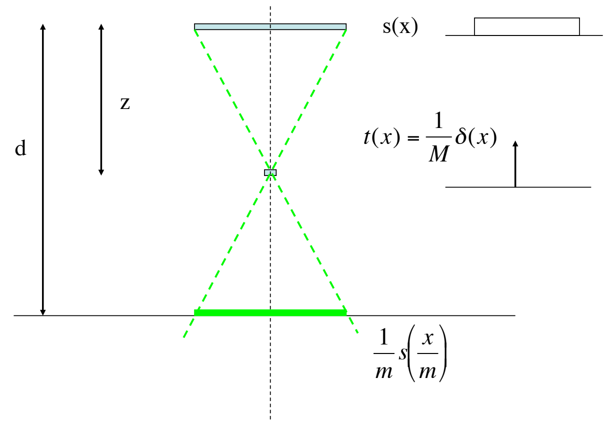
$g(x)= \delta(x+1/2,y) + \delta(x,y)$ $h(x)=rect(x,y)$



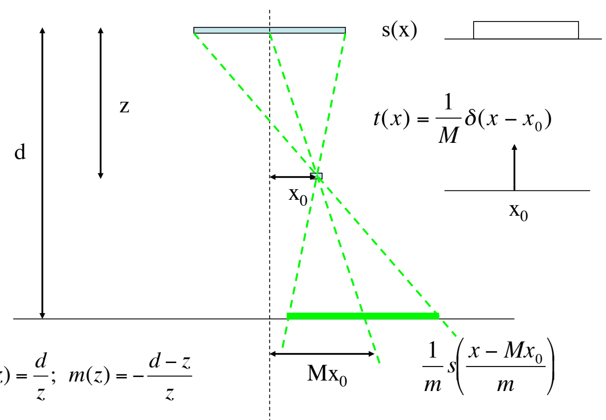
2D Convolution Example



X-Ray Imaging



X-Ray Imaging



$$M(z) = \frac{d}{z};\ \ m(z) = -\frac{d-z}{z}$$

X-Ray Imaging

For off-center pinhole object, the shifted source image can be written as

$$s\left(\frac{x - Mx_0}{m}\right) = s\left(\frac{x}{m}\right) * \frac{1}{M}\delta\left(\frac{x - Mx_0}{M}\right)$$

$$= s(x/m) * t\left(\frac{x}{M}\right)$$

For the general 2D case, we convolve the magnified object with the impulse response

$$I(x,y) = t\left(\frac{x}{M},\frac{y}{M}\right) ** \frac{1}{m^2} s\left(\frac{x}{m},\frac{y}{m}\right)$$

Note: we have ignored obliquity factors etc.

X-Ray Imaging

$m = 1; M = 2$

$$\frac{1}{m} s\left(\frac{x}{m}\right) * t\left(\frac{x}{M}\right) = rect(x/10) * rect(x/20)$$

$$= ???$$

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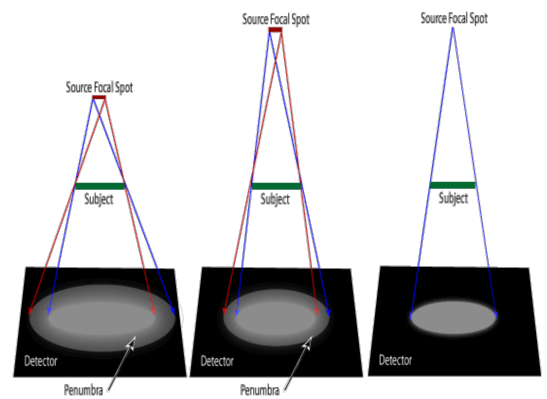

https://www.nde-ed.org/EducationResources/CommunityCollege/Radiography/Graphics/Penumbra1.gif

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Summary

1. The response to a linear system can be characterized by a spatially varying impulse response and the application of the superposition integral.
2. A shift invariant linear system can be characterized by its impulse response and the application of a convolution integral.

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